On the Desired Rate of Capacity Utilization

Michalis Nikiforos*

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Abstract

This paper examines the endogeneity (or lack thereof) of the rate of capacity utilization in the long run within the context of the controversy surrounding the Kaleckian model of growth and distribution. We argue that the proposed long-run dynamic adjustment, proposed by Kaleckian scholars, lacks a coherent economic rationale. We provide economic justification for the adjustment of the desired rate of utilization towards the actual rate on behalf of a cost-minimizing firm, after examining the factors that determine the utilization of resources. The cost minimizing firm has an incentive to increase the utilization of its capital if the rate of the returns to scale decreases as its production increases. We show that there are evidence in the theory and the empirical research that justify this behavior of returns to scale. In that way the desired rate of utilization becomes endogenous.

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*Department of Economics, New School for Social Research, 6 East 16th Street, New York, NY, 10003. I would like to thank Duncan Foley, Peter Skott, Lance Taylor, Luca Zamparelli, Laura Barbosa de Carvalho, Christian Schoder and Jonathan Cogliano for useful comments and suggestions. The usual disclaimer applies. Financial support from the Greek State Scholarships Foundation is gratefully acknowledged.
1 Introduction

The Kaleckian model of growth and distribution is a standard analytical tool of non-mainstream macroeconomics. The attractiveness of its theoretical framework lies in the combination of the distribution of income and the existence of classes with the principle of effective demand. In that sense, it is able to combine the Keynesian emphasis on demand with classical ideas of political economy. Finally, it has the flexibility to accommodate different views and approaches, and, as a result, it is no accident that it has been the field for many recent debates among different economic traditions.

At the same time, the analytical framework of the model has been subjected to severe critique. The most fundamental argument of this critique is that in the long run the rate of capacity utilization has to return to its normal, desired or target rate. Since firms determine their desired rate of utilization under the cost minimizing principle, there is no reason to change this desired rate, unless the underlying reasons associated with the cost minimization problem change. The deviation of the actual rate from the desired rate is not one of these. Therefore, the results of the Kaleckian model apply only in the short run. In the long run we either have to find a way for the actual level of utilization to adjust to the exogenous desired level (within the model’s framework), or abandon the model in favor of other formulations where—in the long run—the actual level of utilization is equal to its exogenous desired level. Consequently in the long run the Keynesian characteristics of the model cease to exist; there is not space for the paradox of cost or the paradox of thrift. This critique is reinforced by the Federal Reserve data, where the utilization of capacity gravitates around a desired rate of around 80 percent.

The “Kaleckian side” has conceded that in the long run the two rates must equalize. However they argue that it is the desired rate of utilization which adjusts to the actual rate through the so-called hysteresis effect. Although this argument is formally correct it lacks a coherent economic rationale.

The main contribution of the present paper is that it explains why a firm that optimizes its behavior will tend to change its utilization in the face of a change in the demand for its product. As we explained above, in the literature until now the optimal desired rate of utilization has been thought to be exogenous.

Moreover, we are the first to argue that the Federal Reserve data on capacity utilization—which has been used by both sides in this debate—are not appropriate for answering whether or not the desired of utilization is endogenous in
the long run.

The remainder of the paper is organized as follows. After a brief presentation of a stripped down version of the Kaleckian model, we examine the critique against it and how the scholars supportive of the model have responded. Next, we show that it is wrong to use the data of the Federal Reserve on capacity utilization to examine long run trends of the desired rate of utilization because this series has no trend by construction. Therefore, the reported actual utilization rate represents how much capacity is utilized compared to the desired rate of utilization, but we cannot make any conclusion about the desired rate itself.

The only way to avoid this convention and see the long run trend of capacity utilization is to examine the behavior of the average workweek of capital. In that sense the full capacity is defined as $24 \times 7 = 168$ hours per week. In section 6 we present several efforts to estimate the average workweek of capital, which show that there has been an increase of the workweek of capital, and therefore of utilization. If we look at the utilization of capital from this point of view, it seems that it is far from stationary.

In section 7 we explore the factors that determine the optimal level of capacity utilization by a firm. We show that the firm will tend to utilize its capital more—adopt a double shift system—as the output grows, if there are increasing returns to scale and the rate of the returns to scale decreases. In the following section we examine how the theory of economies of scale can provide justification for this kind of behavior of economies of scale.

Finally, in section 9 we put the pieces of the puzzle together and we show how the desired rate of utilization becomes endogenous at the macro level.

\section{The Basic Setup of the Model}

The fundamental ideas of the Kaleckian model of growth and distribution go back to the writings of the classical political economists, John Maynard Keynes and—of course—Michal Kalecki (e.g.1971). In its contemporary form it has been developed by Joseph Steindl (e.g. 1952), Rowthorn (1981), Taylor (1983, 1990, 2004), Dutt (1984, 1990), Amadeo (1986) and Marglin and Bhaduri (1990), Bhaduri and Marglin (1990). It is worth noting that the model under examination and its broader analytical framework has received different names. Two of the most common names within the literature are “Structuralist” and “Post-Keynesian”.
Its basic setup is built around the concepts of demand and distribution. The demand schedule is determined by the saving behavior of its members—workers and capitalists—and the investment behavior of the firms. The total income of the economy is distributed between wages and profits.

Investment (normalized for capital stock) can be defined as \( g^i = I(\pi, u) \), where \( \pi \) is the profit share, \( Y \) and \( \bar{Y} \) is output and potential output respectively and finally \( u = Y/\bar{Y} \) is capacity utilization with \( I_\pi > 0 \) and \( I_u > 0 \). The first partial derivative explains the effect of a higher profit share on investment. For Kalecki, higher realized profitability means higher profit expectations, which have a positive effect on investment. Moreover higher profitability allows the firm to finance a bigger part of its investment through internal funds and eases the access to the capital markets. The effect of higher utilization on investment is positive because firms want to hold excess capacity to face an unexpected rise in demand, thus a higher degree of utilization will induce accumulation (Steindl, 1952). We can also think of this positive effect in terms of the acceleration principle.

On the other hand, total saving (normalized for the capital stock) is \( g^s = S(\pi, u) \). \( S_u \) is positive. The saving propensity of the capitalists is assumed to be higher than the saving propensity of the workers and therefore \( S_\pi \) is also positive.

For the purposes of the present paper we will assume the following functional form for the investment function

\[
g^i = \gamma + \alpha(u - u_d) + \beta\pi
\]

where \( u_d \) is the desired rate of utilization and \( \alpha, \beta > 0 \). The only difference of this formulation with the generic one above is that investment does not react to the level of utilization per se but to the deviation of the level of utilization from its desired rate for the same reasons outlined above. This kind of investment function has been proposed by Steindl (1952) and Amadeo (1986) and more recently has been used by Lavoie (1995, 1996) and Dutt (1997).

Moreover, for reasons of convenience, we will assume that the workers do not save, and the saving behavior of the economy boils down to the familiar Cambridge equation

\[
g^s = sv = s\pi pu
\]

\(^1\)The subscript stands for the partial derivative for this variable.
where $s$ is the saving rate of the capitalists, $r$ is the profit rate and $\rho = \bar{Y}/K^{ip}$ is the ratio of the potential output to the capital stock in place.

The equilibrium level of utilization ($u^*$) will be such as to equate the total saving and total investment $g^t = g^*$ for the exogenously given distribution. From equation (1) and (2) it is easy to see that

$$u^* = \frac{\gamma - \alpha u_d + \beta \pi}{s \pi \rho - \alpha}$$  \hspace{1cm} (3)

and therefore the equilibrium level of the growth rate is

$$g^* = s \pi \rho u^* = s \pi \rho \frac{\gamma - \alpha u_d + \beta \pi}{s \pi \rho - \alpha}$$  \hspace{1cm} (4)

The equilibrium is stable if $s \pi \rho > \alpha$, that is if savings react more than investment to changes of utilization; what is usually called Keynesian stability condition.

From equations (3) and (4) we can see that the paradox of thrift holds, since $\partial u^*/\partial s$ and $\partial g^*/\partial s$ are both negative. On the other hand the effect of a change in distribution to the level of utilization and growth depends on the relative magnitude of the reaction of saving ($S^* = spu^*$) and investment ($I^* = \beta$) to changes in distribution. In our demand driven economy a decrease of saving stimulates demand and thus output, while a decrease of investment decreases demand and output. If saving reacts more than investment to a change in the wage share ($spu^* > \beta$) a redistribution of income against capitalists will tend to increase utilization and the growth rate. In this case $\partial u^*/\partial \pi$ and $\partial g^*/\partial \pi$ are both negative. This is what is called a stagnationist, wage-led, or under-consumptionist economy. If $spu^* < \beta$ we are under an exhilarationist, profit-led regime where the redistribution in favor of the capitalists leads to higher output—$\partial u^*/\partial \pi$ and $\partial g^*/\partial \pi$ are positive.

Aside from the demand schedule, where the distribution of income is the exogenous variable, we can define the distributive schedule where the causality runs in the other direction. The distributive schedule expresses how output is distributed among wage and profit earners, and how distribution reacts to changes in utilization. By definition the profit share share is equal to $\pi = 1 - \psi = 1 - \bar{\omega}$, where $\psi$ is the wage share $\omega$ is the real wage and $x$ is labor productivity. The question then is how these different components behave for different levels of capacity utilization and how they interact and form the nominal wages, prices,

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For the equilibrium to make sense it must also be true that $\gamma - \alpha u_d + \beta \pi > 0$. 

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productivity and distribution. The answer to this question involves different approaches to the macroeconomic debate.

A growing recent literature (e.g. Tavani et al., 2011, Assous and Dutt, 2010, Nikiforos and Foley, 2011) examine the implications of a non-linear behavior of distribution for different levels of utilization. However, in this paper we will assume that the distribution of income is exogenously determined, hence

$$\pi = \bar{\pi} = 1 - \bar{\psi}$$  \hspace{1cm} (5)

Foley and Michl (1999) refer to this assumption as the *Classical conventional wage share*, due to the view held by the classical economists that the distribution of income—at least in the long run—is exogenously determined at the subsistence level of the workers (which in turn is conventional).

The equilibrium level of distribution and capacity utilization is the outcome of the interaction of the demand and the distributive schedules, “the functional distribution of income and effective demand jointly determine economic activity” (Foley and Taylor, 2006, p.75). As a result of the exogenous distribution assumption, the impact of a change in distribution on utilization and growth can be inferred by equations (3) and (4).

3 The controversy around the Kaleckian model

3.1 The critique

The Kaleckian model has been criticized on many levels. For example Skott (2008a,b) questions the assumption that saving reacts more than investment to changes of income, what we called *Keynesian stability condition*, and Steedman (1992, p.125) poses some “questions concerning the Kaleckian theory of pricing and the closely related theory of distribution”.

However, the most persistent critique is related to the inability of the Kaleckian model to equate the *actual* rate of capacity utilization ($u^*$) with the exogenously given desired—or normal or planned—rate ($u^d$). Committeri (1986, p.170), referring to the contributions of Rowthorn (1981) and Amadeo (1986) writes that “there is the possibility of utilization being different from its normal degree, even in states of equilibrium (and indeed, actual and normal utilization would coincide only by a mere fluke).” Auerbach and Skott (1988, p. 52) claim
that a steady growth path with $u^* \neq u^d$ is ruled out, i.e. $u^* = u^d$ along the steady growth path\textsuperscript{3} and they add in the next page that “it is inconceivable that utilization rates should remain significantly below the desired level for any long period”.

The failure of the Kaleckian model to equate the actual to the desired rate of capacity utilization has led many to consider it as relevant only in the short run. For example Dumenil and Levy (1993) provide a mechanism for the convergence of the economy from the short term Keynesian/Kaleckian equilibrium “with any capacity utilization rate” to the long run classical equilibrium “with a normal capacity utilization rate”.

Heinz Kurz (1986) explores the concept of the normal rate of utilization. Quoting Sraffa (1960) he argues that the normal rate of utilization “will be exclusively grounded on cheapness”. In other words, each firm will chose how much to utilize its capital based on the principle of cost minimization. Kurz uses the example of a firm that can produce a certain amount of output either by employing a certain amount of capital and labor in a single-shift mode of operation, or by employing a second shift and using only half the capital. The wage of labor in the second shift is higher than in the first because of social norms and conventions (e.g. the wage premium earned by workers who work during the night). The firm will choose the system of operation—and thus the level of utilization of capital—which is more profitable (or which is less costly). Under Kurz’s setup the level of the normal rate qualifies as exogenous and structurally given. Utilization will only change in response to technological changes or changes in the norms that determine the relative cost of labor between the two shifts.

In conclusion, the critique can be summarized in the following arguments: i) in the long run utilization cannot be different from its desired rate, ii) the desired rate is determined on the basis of the cost-minimizing principle and iii) the desired rate for a firm that minimizes its cost is exogenously given. Thus—in the long run—the rate of utilization gravitates around a structurally given and exogenous desired rate of utilization. The critique is serious. If it is correct, the conclusions of the model are only short-run. In the long term, we either have to turn back to the classical results, where there is no room for the paradox of thrift and the paradox of cost—a higher saving rate leads to higher growth and a lower real wage and wage share is always related to lower profit rate and

\textsuperscript{3}Auerbach and Skott (1988) use $u$ and $u^*$ to symbolize the actual and the desired rate of utilization respectively. The change has been made for reasons of consistency.
growth—or we have to seek other formulations which can potentially establish Keynesian results. Comitteri (1986) and Dumenil and Levy (1993) are in favor of the first approach while Skott (2008a,b) support the latter.

3.2 The Kaleckian response

In response to this critique the proponents of the Kaleckian model have argued that in the long run it is the desired rate of utilization that converges towards the actual rate and not the other way around. Amadeo (1986, p.148) says “we should be prepared to examine the possibility of utilization being an endogenous variable even in the long period”. A few pages later (p. 155) he adds: “Indeed one may argue that if the equilibrium degree is systematically different from the planned degree of utilization, entrepreneurs will eventually revise their plans, thus altering the planned degree. If, for instance, the equilibrium degree of utilization is smaller than the planned degree ($u^* < u_d$), it is possible that entrepreneurs will reduce $u_d$”.4

More formally, what he proposes is an adjustment process described by the following dynamic equation

$$\dot{u}_d = \mu (u^* - u_d)$$

(6)

where $\mu > 0$ and $\dot{u}_d$ is the derivative of $u_d$ with respect to time.

In a series of articles Lavoie (1995, 1996), Lavoie et al. (2004) and more recently Hein et al. (2010) are more explicit. They argue that the firms apart from the capital/capacity they own and they operate according to the principle of cost minimization—as described by Kurz (1986)—have “some plants or segments of plant [that] remain idle in normal positions”(Lavoie et al., 2004, p.133). This kind of complete idleness of a part of capital is maintained by firms in order to face the uncertainty of future demand. They conclude (in the same page) that “the normal rate of capacity utilization, in that context, is thus a convention [emphasis added], which may be influenced by historical experience or strategic considerations related to entry deterrence. Although firms may consider the normal rate of capacity utilization as a target, macroeconomic effective demand effects might hinder firms from achieving this target, unless the normal rate is itself a moving target influenced by its past values”(Lavoie et al., 2004). Similar arguments, with such a distinction of capacity between capacity that is oper-

4 Amadeo (1986) uses $u_n$ for the planned degree of utilization.
ated under the principle of cost minimization and capacity that stays idle as a buffer for a potential future increase in demand can be found in all the papers mentioned before, authored or co-authored by Marc Lavoie.

Finally, Amitava Dutt (1997, p.247) explains the endogeneity of the desired rate of utilization in terms of “strategic considerations of the firms”. The firms will reduce their desired utilization rate if they “expect a higher rate of entry than at present” and they consider the entry threat to be “proportional to the investment rate”, thus \( \dot{u}_d = \mu'(g_0 - g^*) \), which with the help of equations (1), (3) and (4) can be transformed into equation (6)\(^5\).

Based on this interpretation equations (3) and (4) define a short run equilibrium, where the desired rate of utilization is exogenous. In the long run the desired rate becomes endogenous and behaves according to equation (6). To “close” the system Lavoie (1995, 1996) and Dutt (1997) supplement equation (6) with the following dynamic equation

\[
\dot{\gamma} = \theta [g^* - (\gamma + \beta \pi)]
\]

where \( \gamma + \beta \pi \) represents the expected rate of accumulation and \( \theta > 0 \). If the actual rate of accumulation exceeds the expected rate firms revise their expectations about the growth rate upwards; an argument with intense Harrodian flavor\(^6\). This dynamic equation can be rewritten as

\[
\dot{\gamma} = \theta [a(u^* - u_d)]
\]

Equations (6) and (8) define a 2 \( \times \) 2 system of dynamic equations. Substituting the equilibrium values of utilization it becomes

\[
\dot{u}_d = \mu(\frac{2 - \alpha u_d + \beta \pi}{s \pi \rho - \alpha} - u_d)
\]

\[
\dot{\gamma} = \theta(\frac{2 - \alpha u_d + \beta \pi}{s \pi \rho - \alpha} - u_d)
\]

An extensive analysis of this system is beyond the scope of this paper. In brief, the Jacobian of this system is zero. For stability, it is required that the trace of the Jacobian is negative, that is \( \mu s \pi \rho > \alpha \theta \). The sufficient condition for this to hold—because of the Keynesian stability condition—is \( \mu > \theta \), that is the adjustment of the desired utilization rate is faster than the adjustment of the desired growth rate. The system has an infinite number of equilibria.

\(^5\)Implicitly it is assumed that \( g_0 = \gamma + \beta \pi \).

\(^6\)See Harrod (1939).
The steady state depends on the initial equilibrium and the path of economy towards it.

The important feature of this formulation is that the economy remains demand driven in the long run: the paradox of thrift and the paradox of cost continue to hold. A higher saving rate will lead the economy to a steady state with a higher level of utilization and growth rate. Interestingly, in the long run the economy cannot be profit-led. A higher profit share decreases the steady state level of utilization and growth.

3.3 Why the response is unconvincing

From a formal point of view the arguments of the preceding section answer the critique regarding the impossibility of a long run deviation of the actual rate of utilization from the desired one. The actual and the desired rate are equal in the long run. However, they fall short in explaining why the desired rate of utilization behaves in the way it is described in equation (6). Stated differently, why a deviation of the actual utilization from the desired rate would induce the entrepreneurs to revise their desired rate? It is not clear why in the long run a firm will desire a lower level of utilization in response to a recession and vice versa.

The argument of the conventional desired rate of utilization is not convincing for two reasons. First, there is no particular rationale behind distinguishing excess capacity in the form of lower than full speed of operation, or lower than full time of operation (one shift instead of two or three shifts), or in the form of totally idle capacity, of “some plants or segments of plant remaining idle”. If a firm operates at 80% of capacity in order to be able to respond to an unexpected increase in demand or to deter the entry to possible competitors it can do it either by lowering the speed of its operation to 80% of what it would be, or by operating its plants 80% of the time it would otherwise operate them, or by keeping a 20% of its productive capacity idle, or by a combination of all three of them. There is no general a priori reason—theoretical or based on the actual experience—that makes the last method superior to the two first. Under certain certain technologies it would probably be more profitable for the firm to choose the idleness method, but this would depend on certain characteristics of an industry and is not a general rule. On the contrary we could think of reasons

\[7\text{The argument of low utilization as an entry deterrence mechanism is made by Spence (1977).}\]
why the firm would favor the first two methods compared to the third one (e.g. adjustment costs for hiring labor).

Moreover, even if this claim were true, the utilization rate does not become a convention. The need of the firm to face unexpected increases in demand is an *objective* and *non-conventional* reason for keeping a part of its capacity idle. A behavior of the desired utilization rate as described in equation (6) based on the need of the firm to face unexpected demand, would mean that when the actual rate of utilization is lower than the desired rate, the firm expects more volatile demand and thus decreases its desired rate of utilization, but it is hard to see why this would happen.

## 4 Capacity and Capital utilization

Before proceeding to the analysis of the thesis of this paper it would be useful to open a parenthesis and discuss briefly the concept of capacity utilization and then its relation with capital utilization.

The difficulty with the concept of capacity utilization arises because of the ambiguous meaning of *capacity*. How can one define capacity? Different answers have been given to this question. The most straightforward definition of capacity is the *engineer capacity*, the maximum level of output obtained if we use the quasi-fixed factors of production 24 hours per day and 7 days per week at the maximum possible speed of operation. The full capacity of a firm whose only quasi-fixed production factor is a factory is the output that would be produced if this factory was working 168 hours per week at full speed. We can also define a *statistical concept of capacity*. This is usually done by deriving a peak-to-peak trend of output or applying a filter (e.g. Hodrick and Prescott, 1997) to the time series of output.

Besides these two concepts, we can define capacity as an economic (as opposed to engineer or statistical) concept. Two alternative definitions have been employed. As *preferred capacity*, it is usually defined the level of capacity specified with the cost-minimization principle. On the other hand *practical capacity (or full production)* is defined as the level of production where the variable (non-quasi-fixed) inputs are used at their maximum level. These definitions are not identical but are close both from a theoretical and an empirical point of view. In the next section we show that the change in the questionnaires of the Census from the one definition to the other led to a small discrete change in the re-
ported utilization (around four percentage points). These definitions are close to what we have called desired or normal utilization.

Within the context of these definitions the importance of the distinction between capacity and capital utilization becomes more clear. Many times the two terms are used interchangeably with or without always realizing it. However, we should note that there are important differences between them. The utilization of capital expresses how much of the capital stock, as a distinct input of production, is utilized, while capacity utilization measures how much output is produced vis-à-vis how much output could be produced. Capital and capacity utilization would be the same only if capital is the only quasi-fixed factor of production.

In this paper we will not make a distinction between the two concepts. In other words we will assume that capital is the only quasi-fixed input in production. With a Leontief-type production function and with the assumption of elastic supply of labor, an assumption almost universal in the related literature, the ratio of potential output to capital in place is equal to the ratio of capital services (or utilized capital) to actual output. As a result \( \frac{\bar{Y}}{K} = \frac{Y}{K^*} \), so the two definitions of utilization coincide \( (K^* \text{ stands for the capital services}) \). Therefore in the series of articles and books which discuss the issue of the long run rate of actual and desired utilization, explicitly or implicitly, the two definitions are used interchangeably. We will follow the same path but we should keep in mind the difference between the two and the prerequisites for their coincidence.

5 Data on Utilization

5.1 The Federal Reserve Data on Utilization

The Federal Reserve Board (FRB) data on capacity utilization have been used by both sides in the debate. Lavoie et al. (2004) filter the data with the Hodrick-Prescott (HP) filter and they argue that the fluctuations of the HP-filtered series prove that the desired rate adjusts as described in equation (6). They provide an econometric justification for their claim using these data.

\[8A \text{a detailed discussion of the different definitions is provided among others by Klein (1960), Berndt and Morrison (1981), Morrison (1985), Bresnahan and Ramey (1994), Mattey and Strongin (1997) and Corrado and Mattey (1997).} \]

\[9The \text{ data can be found at http://www.federalreserve.gov/releases/g17/caputl.htm.}\]

\[10They \text{ use data on capacity utilization from Statistics Canada, which are practically the same with those of the FRB.}\]
In figure 1 we present the FRB capacity utilization series for the US economy for the period 1948 to 2007. It is hard to see how these data support the claim of an endogenous utilization rate. From the fitted lines it becomes obvious that the rate of capacity utilization tends to gravitate around a constant rate over a prolonged period of time, around 83% in the period 1948 to 1980 to around 79% in the period 1980 to 2007. This change can be attributed to a change of the structural characteristics behind the desired rate of utilization. For example, Spence (1977) would argue that this shift is the result of an increase in the concentration in the market and as a result the firms need to lower their utilization rate to deter the entry of the competitors. However, we do not have to go that far. Morin and Stevens (2004, p.9), in a paper describing the construction of the FRB capacity utilization index\textsuperscript{11}, argue that a big part—if not the whole—of this decrease is due to changes in the definition of capacity in the questionnaires of the surveys which are used to construct the index. These changes led to a “discrete shift” of the index around “4 percentage points”. It is probably no coincidence that this is the difference between the two horizontal fitted lines.

Similarly, in the case of the HP filtered series of utilization, we observe a remarkable stability of the desired rate of utilization—if we interpret the HP-series as representing the desired rate. Over the whole period there are some minor fluctuations, but they are not enough to support the hypothesis of an endogenous rate of utilization. If we HP-filter the series for the period before and after 1980 we will end up with almost horizontal lines, as in the case of the linear fits, which, taken together with the change of the estimated utilization rates because of the questionnaire, leads to the conclusion of a constant desired rate of utilization.

Therefore, if one relies on the capacity utilization index of the Federal Reserve, the argument of an endogenous rate of capacity utilization seems unwarranted. Instead, there seems to exist an exogenous desired rate of about 80%, around which the actual rate gravitates.

However, the FRB data is inappropriate to judge whether the desired rate of utilization is (or is not) endogenous in the long run. This becomes clear if we pay a little closer attention to the way these data are constructed. For that purpose the paper by Morin and Stevens (2004) is very useful. The index is based on the Survey of Plant Capacity (SPC) which is conducted by the

\textsuperscript{11}The online documentation of the Industrial Production and Capacity Utilization in the website of the Federal Reserve (2009) is a short version of this paper.
Figure 1: Capacity Utilization from the FED dataset. Annual data, for the period 1948-2007.

United States Census Bureau\textsuperscript{12}. In the questionnaires of the SPC the plant managers are asked to specify the “full production capability of their plant—the maximum level of production that this establishment could reasonably expect to attain under normal and realistic operating conditions fully utilizing the machinery and equipment in place”. Among the instructions they are given is to “assume number of shifts, hours of plant operations, and overtime pay that can be sustained under normal conditions and a realistic work schedule\textsuperscript{13}”. The results of this questionnaire are then processed and aggregated in order to produce the series we present in figure 1.

Let us put ourselves in the shoes of a plant manager who answers the questionnaire. Assume that over a period of years our plant works 5 days per week, 8 hours per day. Under these normal and realistic conditions the number of shifts is one and the hours of plant operations per week is forty. We can also assume that the full production capacity of the plant under these normal conditions is 100 units. The plant manager wants to be able to face unexpected

\textsuperscript{12}A copy of the questionnaire can be found online at http://bhs.econ.census.gov/bhs/pca/pdf/10_mqc2.pdf
\textsuperscript{13}Emphasis in the original.
demand increases (or to deter the entry of the competitors in the market), so the plant is working on average at the 80% of its full capacity. Note that there is no general reason why the manager either does not “run” the plant at its full speed, or lets the workers go home earlier, or keeps a part of the plant idle. Therefore, on average the production of the plant will be 80 units vis-à-vis a full production capability of 100 units, that is utilization of capacity around 80%. Of course when the economy is doing well and the demand is high the plant manager will increase the speed of production, or will not let the workers leave earlier—they might even work overtime—or he will utilize the idle part of the plant. In these “fat cow” years, when he fills the SPC questionnaire the production will be higher than 80 units and therefore the utilization will be higher than 80%. In years of economic downturn for the same reasons the utilization will be lower than 80%.

Imagine that our firm is doing well and after a period of high demand the plant manager decides to add a second shift. Over a period of years the plant works 5 days per week, 16 hours per day. Under the new normal and realistic conditions the number of shifts is two and the hours of plant operations per week is eighty. The full production capability of this plant under the new normal conditions is 200 units (we abstract from any kind of economies of scale or other reasons that would make this product to be different than 200). The plant manager still worries about competition and being able to face unexpected demand. That is why he “runs” his plant on average at the 80% of its full capability. Therefore, on average the production of the plant will be 160 units vis-à-vis a full production capability of 200 units, that is utilization of capacity around 80%. The cyclical fluctuations effects still apply and have the same effects.

If after a few years the demand for the products of the firm permanently decreases and the manager decides to drop the second shift, we will be back to the original situation.

The conclusion of this simple example is that the FRB index of capacity utilization by construction gravitates around a structural exogenous level of utilization and by construction is stationary. In the online documentation of the Federal Reserve (2009) it is made explicit that “a major aim is that the Federal Reserve utilization rates be consistent over time so that, for example, a rate of 85 percent means about the same degree of tightness that it meant in the past”. In that sense the FRB utilization index is a proxy for the deviation of $u^*$ from $u_d$ and gives us no information about the $u_d$ itself.
In order to examine the behavior of the utilization over the long run we have to rely on other measures, which can capture if the plant “runs” for one or two (or three) shifts, if it runs during the weekends etc. The obvious method to do that is to compare the number of hours the plant works with the maximum hours it can work. The maximum hours the plant can work within one week period is $24 \times 7 = 168$ hours. Therefore, a more appropriate measure of utilization for our purpose is the ratio of hours worked by the plant per week over 168. In that case the utilization of our fictional plant of the previous paragraphs would have increased from an average of $80\% \times \frac{40}{168} \approx 20\%$ to $80\% \times \frac{80}{168} \approx 40\%$. As we will see in the next section this measure is not without problems but it is more appropriate for our discussion. In the next section we present various attempts to measure utilization, which capture the amount of time the capacity is utilized.

6 Other Measures of Utilization

A first attempt of measuring capital utilization as the ratio of the time the plant worked over an absolute amount of time (number of hours per week or year) was made by Foss (1963). He used data on power equipment from the Census of Manufactures and estimated the theoretical maximum of electrical consumption of the machinery-in-place for a one-year period. He then estimated the utilization rate of the machinery by comparing this theoretical maximum to the actual consumption of electricity (available from the Census of Mineral Industries). Foss finds that the “equipment utilization ratio from 1929 to an approximately comparable high employment year in the 1950’s [1955] shows an increase of almost 45%”. Foss’ methodology has several drawbacks related with the assumptions he made to estimate his theoretical maximum. However two main conclusions, which would be confirmed by later studies, came out: i) the capital equipment lies idle most of the time and ii) there is a clear upwards trend in the utilization of capital equipment.

Foss (1981a,b) uses a more direct method. He utilizes data from the 1929 Census of Manufactures and the 1976 Survey of Plant Capacity undertaken by the Census Bureau on weekly plant hours worked by manufacturing plants, which he then aggregates. The results are summarized in table 1. We can see that over the period 1929 to 1976 the average workweek of capital increased around 25 percent. If we take into account that 1929 was a year at the peak of
<table>
<thead>
<tr>
<th>Variant</th>
<th>1929</th>
<th>1976</th>
<th>Change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^a$</td>
<td>66.5</td>
<td>81.8</td>
<td>23.0</td>
</tr>
<tr>
<td>$B^b$</td>
<td>91.9</td>
<td>110.3</td>
<td>20.0</td>
</tr>
<tr>
<td>$C^c$</td>
<td>—</td>
<td>—</td>
<td>24.7</td>
</tr>
</tbody>
</table>

Source: Foss (1981a,b)

$^a$Hours for each industry (at a four and two digit level) weighted by employment in each year.
$^b$Hours for each industry (at a four and two digit level) weighted by capital in each year.
$^c$Hours for each industry at the four digit level constructed as in $B$ and aggregated using as weights the gross fixed capital stock for 1954 in 1972 prices.

Table 1: Alternative measures of average weekly plant hours in manufacturing and their change, 1929 to 1976

the economic cycle while 1976 was close to the bottom the increase is higher.
Foss stresses that this increase occurred in the face of a decline in the average workweek of a labor “from a customary 50 hours per week in 1929 to a 40-hour standard in 1976”.

In a later study Foss (1984) makes use of the Area Wage Surveys of BLS, covering metropolitan areas for high employment periods, 1959-60, 1969 and 1978-79. The percentage of workers employed on second and third shifts for all manufacturing industries combined is used to interpolate the average weekly plant hours (of table 1). The estimates between “pairs of endpoints” is derived by straight line interpolation. Thus the annual estimates between 1929 and 1976 constitute a “high employment trend-line”. The results are presented on the left hand side of the dashed line in figure 2.

Certain industries, because of some particular characteristics of theirs, usually either work only one shift (like apparel), or 3-shift (like Petroleum). The latter are usually called continuous industries. The continuous (24 hours a day) utilization of the capital in these industries is usually related to very high costs of stopping and restarting production. Foss finds that if we excluded these two types of industries the increase in average weekly plant hours reaches 32.4 percent between 1929 and 1976.

Finally Foss (1995) is able to construct annual indices based on actual annual observations of utilization for the period 1976 to 1988. The results are depicted on the right hand side of the dashed line of figure 2. The series presents the expected cyclical fluctuations.

The Average Workweek of Capital (AWW) has been also estimated by other
researchers. Taubman and Gottschalk estimate AWW for the period 1952 to 1968. The logic behind their effort is that the amount of the services of the capital stock can change either by varying the speed of operation or by varying the time the capital stock operates. Assuming that the speed of operation remains constant, the most common method of altering the time of operation of the capital stock is by changing the number of shifts it operates. Therefore, the amount of capital services can be estimated using the information of how many workers are employed in each of the three shifts\(^{14}\). Orr (1989), using the same methodology, extends the estimation for the period until 1984.

Their estimates are presented in figure 3a. It is clear that the utilization of capital presents pro-cyclical fluctuations and that there is a definite upward trend of the utilization of capital. Both papers run simple linear regressions and find statistically significant positive trends.

The same methodology is also followed by Shapiro (1986) for estimating the average workweek of capital for the period 1952 to 1982. He follows Taubman and Gottschalk until 1968. For the period 1969 to 1982 he is using his own estimates of national level data on shift-work and the workweek of labor. As a

\(^{14}\)The necessary data come from the Area Wage Surveys of the Bureau of Labor Statistics.
(a) Index for the average workweek of capital in manufacturing (1966=100) based on Taubman and Gottschalk (1971) and Orr (1989).

(b) The average workweek of capital in manufacturing (1952=100) from Shapiro (1986).

(c) The average workweek of capital in manufacturing from Beaulieu and Mattey (1998).

Figure 3: Measurements of the Average Workweek of Capital
result there is a serious downward break in the series in 1968 which interrupts the positive trend. Therefore, the overall series estimated by Shapiro present a weaker trend. The fact that the series begin from a year at the peak of the cycle and end at a year at the bottom of the cycle also contribute to that. Shapiro's estimates are presented in figure 3b below.

Finally, (Beaulieu and Mattey, 1998, p. 210) use the Survey of Plant Capacity (SPC), which contains series on the number of days per week and the number of hours per day the plant was in operation. The basic unit of observation is the product of these two series. They calculate the workweek of capital with different weighting techniques (employment per shift, employment, book value of capital, SPC sampling weights) for the period 1974-1992\(^{15}\). The weighting scheme is important regarding the level and the trend of the workweek of capital. However, their conclusions are similar to the previous estimates. The trend of the utilization is positive\(^{16}\) and it presents pro-cyclical fluctuations. We present their estimates with fixed employment per shift weights in figure 3c.

From all the above efforts for estimating the workweek/utilization of capital we conclude that utilization aside from the expected pro-cyclical fluctuations, has a clear upward trend over time. This clearly comes in stark contrast with the horizontal trend of the FRB data on utilization.

The remaining question to be answered is if there is a way to link the cyclical fluctuations of utilization with its trend. Until now in the literature the two phenomena are treated separately. In the next section we provide a brief review of the theory of capacity utilization and then we show how a firm which minimizes its cost will tend to increase its long run rate of utilization as a reaction to short-run fluctuations of demand.

7 The Theory of Utilization

7.1 Early writings on the utilization of capital

Karl Marx was among the first to deal with the issue of the utilization of capital. In *Capital* he provides the main insights that still form the backbone of the theory of capital utilization. He analyzes the working day of capital within the framework of the labor theory of value and his theory of exploitation. Marx ar-

\(^{15}\)Due to lack of data when they use the book value of capital they do it only for the period 1974-1985.

\(^{16}\)When they use the book value of capital the positive trend is statistically insignificant. This is most probably related to the particular period of estimation (1974-1985).
gues that since the (fixed) capital exists in order to “absorb labor and, with every drop of labor, a proportional quantity of surplus labor...Capitalist production drives, by its inherent nature, towards the appropriation of labor throughout the whole of the 24 hours in the day. But since it is physically impossible to exploit the same individual labor-power constantly during the night as well as the day, capital has to overcome this obstacle. An alteration becomes necessary, between the labor-powers used up by day and those used up by night” (Marx, 1976, ch. 10, p.367).

Thus the shift system is the means which allows the capitalists to appropriate surplus value throughout the 24 hours. A careful interpretation of this would lead to the conclusion that the working day of capital is determined by the capitalists—given the established norms of the working day and the technology—in their effort to maximize the surplus value or the rate of the surplus value.

The working day is a recurrent theme in Marx’s work, always in relation to the his theory of production of surplus value. However, the distinction between the working day of capital and labor is not always explicit.

John Stuart Mill (1864, ch. IX, p.176) in his *Principles of Political Economy* argues that “the only economical mode of employing” the machines is to keep them “working through the twenty-four hours”. Kurz (1986, p.42) interprets this statement as Mill arguing that the firm under the profit maximizing principle will run its machines through the twenty-four hours and therefore “Mill’s opinion cannot be sustained”. It is more probable that the meaning of this short comment by Mill is that the machinery is wasted if it is not utilized and this waste is non-“economical”. A century later Georgescu-Roegen (1972, p. 284) stresses—along the same lines—that capital “idleness is the worst form of economic waste and a great hamper to economic progress”.

Finally, the young Alfred Marshall in the essay *The Future of the Working Classes* Marshall (1925) argues that society would benefit from the “diminution of hours of manual labor”. He recognizes that “for every hour, during which

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17For example in chapter 15 of Volume 1 of *Capital* he examines the working day in relation to the introduction of the machinery in production. In the same chapter (p. 533-544) Marx deals with the speed of production, which as we will see below, is another constituent of the modern theory of utilization of capital.

18In this chapter Mill discusses the “Production on large and production on a small scale”. He comments a passage of Charles Babbage’s *On the economy of machinery and manufactures* (1832, p.174) and explains that the larger the scale production “the further the division of labor may be carried”.

19The essay was written and originally published in 1873.
his untiring machinery is lying idle, the capitalist suffers loss” (p. 113). The solution he proposes is the adoption of the multiple shift system, so as both the working day of labor is short and the capital does not sit idle. Note that the idea of idle capital as a form of economic waste is also present in this argument of Marshall. In his last book, *Industry and Trade* (1920) he reflects on the British society and the “place of Britain in the world” after World War I. He argues in favor of extracting “as much work as possible out of his plant”. Again his proposal is “to work in shifts, so as to keep the plant at work for twice as long as the normal working day, then wages will be raised automatically far above their present level”.

### 7.2 Recent contributions to the theory of capital utilization

The insights provided in these early writing have become more explicit in more recent contributions, which examine the determination of capital utilization.

The problem of the choice of the optimal utilization on behalf of a firm is similar in nature with the more familiar choice of the capital labor ratio. For example, the choice of the optimal capital-labor ratio is related (among others) to the trade-off between increasing labor productivity and decreasing capital productivity as we move from one capital labor ratio to the other. Similarly, the choice of the level of capital utilization is related to the trade-off between the lower total capital cost of a higher level of capital utilization and the higher wage which is required as a compensation for working non-normal hours.

The similarity of the two problems was recognized by Marris (1964). He says (p.5) “We may therefore restate the production function by saying that annual output now depends on three variables in place of two: instantaneous employment, quantity of capital and the rate of utilization”. In the traditional production theory the firm has to choose only the optimal capital-labor ratio. Implicitly this is tantamount to assuming that the utilization of capital is always maintained at a constant level. Therefore capital utilization adds “another dimension” to the choice problem of the firm. This second dimension can be thought of either in continuous terms, e.g. how much time during a day capital is utilized, where time is a continuous variable (this is the approach used by

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*20* A necessary condition for that is the set of techniques of production and the production function to be “well-behaved”, in the sense that higher capital productivity is accompanied with lower labor productivity. The production function as the description of the existence of infinite well behaved techniques is explained in Foley and Michl (1999).
Therefore, the question is what are the factors that determine the level of utilization. A standard approach within the literature is to distinguish the factors which are related to cyclical fluctuation and others which are not. Winston (1974b) says “idle capital is explained as a consequence of unwanted accidents and adversities that occur after [ex post] a plant is built, or as a result of rational ex ante investment planning”. In Marris’ words it is “methodologically convenient to distinguish” between them. This approach, at least at a first sight, is in accord with the critics of the Kaleckian model that there is a set of factors that determine the long run, ex ante, desired level of utilization and another set of factors that explain the short run, ex post, actual level.

The ex post unwanted utilization is usually explained in terms of cyclical fluctuations in demand (Winston (1974b) is actually using the word “Keynesian”), unexpected shocks or simply to mistakes of judgement on behalf of the entrepreneurs. Therefore, the literature is mostly concerned with the factors that determine the ex ante level of utilization.

However, as we will see later, no matter how convenient it is to distinguish the two sets of factors, there is not always a clear cut way to do that and, as a result, the ex post level of utilization is also related to the actual level.

7.3 The Ex Ante Decision on Capital Utilization

The literature on capacity utilization is well summarized by Winston (1974b). In his article he mentions a series of determinants of the optimal capital utilization, ex ante. A first reason for desired excess capacity is related to the market structure. The excess capacity is used as an entry deterrent on behalf of the monopolist or oligopolist. A low level of utilization of capacity would make unprofitable the entry into the market. Different dimensions of this argument are provided by Kaldor (1935), Chamberlin (1962), Spence (1977) and Cowling (1981).

Another reason for intended excess capacity is “rhythmic” variations of demand. Winston and Betancourt and Clague (1981) give the example of a pizza restaurant. Since the demand for pizza is usually limited around lunch and dinner time it is profitable for the owner of a pizzeria to leave its “plant” idle when there is no demand (after midnight till late in the morning).
Except for “rhythmic” variations in demand, intended unutilized capital can be explained with rhythmic variations in the prices of inputs. If the price of an input is predictably higher during a certain period it might probably be profitable for a firm to let its capital idle during this period and produce when the price of this input is low. The first of these inputs that comes to someone’s mind is labor. For reasons, related to several social norms people prefer to work during the normal working hours and the wage for working outside of these normal hours is higher. Firms have to pay a utilization differential, a premium over the regular wage to workers who work outside of these normal hours, during an afternoon or a night shift or during the weekend. In most countries this differential is granted by the legislation. Marx was the first to realize the importance of these social norms for the working day of capital. The higher labor cost during “abnormal” hours may induce the firms to leave their capital idle during these hours and utilize it only when labor is relatively cheap.

The rhythmic variation of the input price is not limited to the price of labor. For example, in agriculture one of the most important inputs is heat. Heat’s price is zero during the spring and summer months. During the winter producing an analogous amount of heat—with the construction for example of a greenhouse—is costly. Therefore, for most farmers it is more profitable to have their machinery stay idle during the winter.

We could think of many more similar examples of input price variations. However the essence of the problem can be understood as follows: the firm owns the capital whether it uses it or not. As a result it has to choose between a lower average cost of capital and higher cost of the rhythmic input(s) when capital is utilized a lot and a higher average cost of capital but higher cost of the rhythmic input(s) when the utilization of capital is low on the other. A cost-minimizing firm will determine its optimum level of capital utilization having this trade-off in mind.

Intuitively then it is not hard to list some of the determinants of the utilization of capital. The higher the relative price of capital to the rhythmic price inputs and the capital intensity of the production process the higher the incentive of the firm to utilize its capital. Similarly the bigger is the “amplitude” of the rhythm of the input price (the higher the wage differential for the second shift, or the more expensive to build a green-house) the lower is the incentive.

\[21\] Of course sometimes it is impossible to reproduce the climate conditions of the summer. In this case the idleness of the agricultural machinery is inevitable. This is stressed in Georgescu-Roegen (1969, 1970, 1972).
for utilization.

These conclusions can be made explicit with the help of the simple model in the next section.

7.4 The choice of the number of shifts

The simple model we provide in this section follows the analysis of Betancourt and Clague (1981, especially ch.1) and Marris (1964). Both contributions examine how a firm that minimizes its cost will choose the number of shifts of production. The difference between the two is that while Marris also uses discrete techniques of production Betancourt and Clague use a production function. For our purpose the difference is not very important because Marris deals only with well behaved techniques of production, that is techniques where increased mechanization is accompanied by higher labour productivity, so there exists the trade-off we described above. Apart from continuity, this resembles a well behaved production function. Kurz (1986) adapts a similar underlying framework however he also stresses the implication of techniques of production which are not ‘well-behaved’, the so-called reswitching (p. 49-51). Without any intention to question the importance of reswitching and its implications we will conveniently assume the existence of a well behaved production function.

The firm of our model wants to produce a certain amount of the commodity, $\bar{Q}$, and it has to choose i) the optimal capital labor ratio for its production and ii) the system of production. By system of production we define the number of shifts its capital will be utilized: it chooses if it will produce its output under a single shift system or a double shift system. We can think of a single shift system as a 5-day morning 8-hour shift and the double shift system as a 5-day morning and evening 8-hour shift, so in total under the single shift system the working week is 40 hours, while under the double shift is 80 hours. Obviously there are other systems of operation too. We can have a third system of operation with an additional night shift, so the firm would have a 5-day 24-hour per day working week (in total 120 hours working week), or even a firm with a continuous operation of 168 hours per week. The choice between two systems only is made for reasons of simplicity and clarity of exposition and it does not limit our theoretical predictions, since our results can be easily extended to the cases of more than two systems of operation\textsuperscript{22}. Obviously, the double shift

\textsuperscript{22}A discussion of a firm with more than two systems of operation can be found in Marris (1964) and Betancourt and Clague (1981, ch. 2).
system means higher desired utilization compared to the single shift system.

The firm uses only labor and capital as inputs, which can be combined within a set of techniques, that is defined with the production function \( Q = F(S, L) \), where \( Q \) is the instantaneous flow of output and \( S \) and \( L \) are the instantaneous rates of services of capital and labor respectively.

The utilization of capital can vary only through the time the capital is used, that is, by adopting a workweek of 40 or 80 hours. The speed of operation is constant. That means that the instantaneous rate of services of capital varies proportionally to the capital stock in place (that is \( S = vK \))\(^{23}\).

As we stressed before the main characteristic of increased utilization is the higher cost of labor for the second shift. Firms have to pay a utilization differential which we can define as the ratio of the wage for working in the morning shift, \( w_1 \), and the wage for working in the evening shift, \( w_2 \). The utilization differential is equal to \( \frac{w_2}{w_1} = 1 + \alpha \), where \( \alpha > 0 \).

For simplicity, as Betancourt and Clague (1981), we shall assume that labor can be hired within eight hours shifts and we “shall define our instant to be the eight hour shift, that is, the eight hour shift is the unit of time”.

The firm has to choose between the following two system of production:

\[
\begin{align*}
\bar{Q} &= Q^1 = G(S^1, L^1) \\
\bar{Q} &= Q^2 = G(S^2_1, L^2_1) + G(S^2_2, L^2_2)
\end{align*}
\] (10)

The superscript refers to the system of operation and the subscript to the particular shift within each system. The constant speed of operation of the capital means that \( S^1 = vK^1 \) and \( S^2 = S^2_2 = vK^2 \).

We will also assume that the ratio of capital to labor services in both shifts is constant after the plant is built\(^{24}\), so \( L^1_2 = L^2_2 \). Therefore under the double shift system half of the output will be produced during the first shift and half of it during the second shift. Equation (10) can thus be rewritten as

\[
\begin{align*}
\bar{Q} &= Q^1 = G(vK^1, L^1) \\
\bar{Q} &= Q^2 = G(vK^2, L^2_1) + G(vK^2, L^2_2) = 2G(vK^2, L^2_1)
\end{align*}
\] (11)

The choice of the system of production will be based on which one of them...
is more profitable, or under what system the production of \( \bar{Q} \) costs less. The total cost of production under the first system will be \( C^1 = rK^1 + w_1L^1 \), while under the second \( C^2 = rK^2 + w_1L^2_1 + w_2L^2_2 = rK^2 + (2 + \alpha)w_1L^2_1 \), where \( r \) is the unit cost of capital. We can define the cost ratio of the double shift system over the single shift system as \( \Lambda = \frac{C^2}{C^1} \). It is easy to see that

\[
\Lambda = \left[ \theta \frac{k^2}{k^1} + (2 + \alpha)(1 - \theta) \right] \frac{L^2_1}{L^1} \tag{12}
\]

where \( k^2 = \frac{K^2}{L^2_1} \) and \( k^1 = \frac{K^1}{L^1} \) are the ratio of capital to labor of the first shift under the single and the double shift system respectively and \( \theta \) is the share of capital cost to the total cost of production under the single shift system. The firm will choose the “cheapest” system of production. The double shift system will be chosen as long as \( \Lambda < 1 \).

For reasons of simplicity of exposition and because later we want to focus on the role of the economies of scale on the choice of the system of production we will also assume that our production function is homothetic, therefore \( Q = G[F(S, L)] \), where \( F \) is a homogeneous function of first degree and \( G \) is a positive transformation of \( F \). Based on what we have said so far under the first system we have

\[
\bar{Q} = G[F(S^1, L^1)] = G(F^1) = G[L^1 f(k^1)] \tag{13}
\]

while under the second system

\[
\bar{Q} = 2G[F(S^2_1, L^2_1)] = 2G(F^2) = 2G[L^2_1 f(k^2_1)] \tag{14}
\]

From (13) and (14) we derive

\[
L^1 = \frac{G^{-1}(\bar{Q})}{f(k^1)} \\
L^2_1 = \frac{G^{-1}(1/2\bar{Q})}{f(k^2_1)} \tag{15}
\]

Therefore

\[
\frac{L^2_1}{L^1} = \left( \frac{G^{-1}(1/2\bar{Q})}{G^{-1}(\bar{Q})} \right) \frac{f(k^1)}{f(k^2_1)} \tag{16}
\]

Based on (16) equation (12) is transformed into

\[
\Lambda = \left[ \pi \frac{k^2}{k^1} + (2 + \alpha)(1 - \pi) \right] \frac{f(k^1)}{f(k^2_1)} \varphi(\bar{Q}) \tag{17}
\]
where $\varphi(\bar{Q}) = \frac{G^{-1}(1/2\bar{Q})}{G^{-1}(\bar{Q})}$. This function is related to the economies of scale. $G^{-1}(1/2\bar{Q})$ is the amount of inputs needed to produce $1/2\bar{Q}$ and $G^{-1}(\bar{Q})$ is the amount of inputs needed to produce the whole $\bar{Q}$. If the production process exhibits constant returns to scale the amount of inputs needed to produce half of a certain amount of the output are half of the inputs needed to produce the whole amount, therefore $G^{-1}(1/2\bar{Q}) = 1/2G^{-1}(\bar{Q})$, so $\varphi(\bar{Q}) = 1/2$. On the other hand if we have increasing returns to scale the amount of inputs needed to produce half of a certain amount of output are more than half of the inputs needed to produce the whole amount, therefore $G^{-1}(1/2\bar{Q}) > 1/2G^{-1}(\bar{Q})$, so $\varphi(\bar{Q}) > 1/2$. Mutatis mutandis for decreasing returns.

Based on equation (17) we can now formally derive the intuitive conclusions of the previous section: i) the capital to labor ratio in each shift under the double shift system ($k_2^j$) will be greater than the capital to labor ratio under the single shift system ($k_1^j$), ii) the higher the utilization differential, the higher the cost of the double shift system relative to the single shift system and iii) the higher the share of the wage cost in the total cost under the single shift system ($1-\theta$) the lower will be the cost of the double shift system relative to the single shift system.

Finally, the larger the returns to scale ($\varphi(\bar{Q})$) the higher the ratio $\Lambda$, and, therefore, the higher the cost of the double shift system relative to the single shift system.

### 7.5 Scale of production and utilization

A question which is of interest for our discussion is what is the relation between the choice of the firm between the two systems and the scale of production, in other words what happens to the ratio $\Lambda$ as the output of the firm ($\bar{Q}$) increases. From equation (17) we can see that $\Lambda = \left[\theta \frac{k_2}{k_1^j} + (2 + \alpha)(1 - \theta)\right] \frac{f(k_1^j)}{f(k_1)} \varphi(\bar{Q}) = c\varphi(\bar{Q})$ where $c = \left[\theta \frac{k_2}{k_1^j} + (2 + \alpha)(1 - \theta)\right] \frac{f(k_1^j)}{f(k_1)}$. Because of the homotheticity of the production function $c$ is invariant to changes of $\bar{Q}$, so $\frac{\partial \Lambda}{\partial \bar{Q}} = c\frac{\partial \varphi}{\partial \bar{Q}}$. The sufficient condition for the double shift system to become more attractive as the production increases ($\frac{\partial \Lambda}{\partial \bar{Q}} < 0$) is $\frac{\partial \varphi}{\partial \bar{Q}} < 0$. By the definition of $\varphi(\bar{Q})$ we can see
that
\[ \frac{\partial \varphi}{\partial \bar{Q}} < 0 \iff \frac{G^{-1}(\bar{Q})}{G^{-1}(1/2\bar{Q})} \frac{G'[G^{-1}(\bar{Q})]}{G'[G^{-1}(1/2\bar{Q})]} < 2 \]  \tag{18}

The first fraction on the left hand side of the inequality is the inverse of \( \varphi(\bar{Q}) \), while the second one is equal to the marginal product at the amount of inputs producing \( \bar{Q} \) over the marginal product at the amount of inputs producing \( 1/2\bar{Q} \).

The sufficient condition for the satisfaction of this inequality, is that the rate of the degrees of scale of the production function decreases. Our production function can be characterized for its returns to scale based on the level of \( j \) in the equation \( t^j \bar{Q} = G(tF) \). For \( j = 1 \) we have constant returns to scale, for \( j > 1 \) we have increasing returns to scale and vice versa. If \( F \) is the amount of inputs necessary for the production of \( \bar{Q} \) and \( F' \) for the production of \( 1/2\bar{Q} \), then the LHS of inequality 18 can be written as \( \frac{F}{F'} \frac{G'(F)}{G'(F')} \). Assuming that at the level of input \( F' \) scales of production \( j' \) prevail, while at \( F \) scales of production \( j \), we can rewrite this fraction using the Euler theorem as \( \frac{G(F')}{jG(F')} = \frac{j\bar{Q}}{j'1/2\bar{Q}} = 2\frac{j}{j'} \). Obviously if we have a constant degree of returns to scale (either if they are constant or increasing or decreasing) \( j = j' \) so \( \frac{\partial \varphi}{\partial \bar{Q}} = 2 \), therefore the scale of the output does not affect \( \Lambda \) and the choice of the system of production. On the other hand if \( j < j' \), that is if the rate of the degrees of scale decreases, \( \frac{\partial \varphi}{\partial \bar{Q}} < 2 \).

Therefore, the entrepreneur will tend to choose a double shift system of operation over a single shift system of operation as the scale of production of her firm increases if the degree of the returns of scale decreases as the scale of production increases. This result is important because it shows that the level of utilization for a cost-minimizing firm depends on the demand for the product of the firm, and, as a result, on its production. Under these aforementioned assumptions about the returns to scale, an increase in the demand for the product of the firm will tend, ceteris paribus, to increase the utilization of its capital.

The next question is if this decrease of the returns to scale is something more than a theoretical sophistry, and if there are reasons to believe that the returns to scale evolve in such a way. We discuss this in the following section.
8 Economies of Scale

In order to examine the behavior of the returns to scale as production increases we have to understand the causes of increasing returns. Within the literature there is a consensus that returns to scale are—at least to a certain degree—due to indivisibilities. Kaldor (1934) in a much cited passage mentions: “It appears methodologically convenient to treat all cases of large-scale economies under the heading “indivisibility.” This introduces a certain unity into analysis and makes possible at the same time a clarification of the relationship between the different kinds of economies. Even those cases of increasing returns where a more-than-proportionate increase in output occurs merely on account of an increase in the amounts of the factors used, without any change in the proportions of the factors, are due to indivisibilities; only in this case it is not so much the “original factors,” but the specialized functions of those factors which are indivisible”. Koopmans (1957) and Eatwell (2008) also explain why indivisibilities are the sole reason behind increasing returns to scale.

The returns to scale are created because in the presence of indivisibilities production can be increased by \( t \) times without increasing the inputs analogously. A simple example given by Georgescu-Roegen (1972, p.285) is the production of bread; compared to the baking of one bread loaf “the baking of a batch of loaves does not require the amplification of all coordinates. The same mixing machine, oven, and building can take care of the multiple task”. In our terminology the “mixing machine, oven, and building” are indivisible inputs to production. Either the baker produces one or a “a batch of loaves”. Obviously, the baker will use a smaller mixing machine, oven and building if the demand for his production is small, compared to a baker that faces very high demand. However, usually they will not be divisible enough to justify the neglect of the benefits provided by a larger scale of production.

These benefits will be exhausted as the scale of production increases. More formally, it can be shown that the rate of returns to scale is equal to the ratio of the average cost to the marginal cost; \( j(Q) = \frac{AC(Q)}{MC(Q)} \), where \( j \) as before is the rate of the returns to scale. The existence of indivisibilities means that \( AC > MC \) and (assuming constant prices of inputs and no substitution) that the average cost is decreasing while the marginal cost stays constant as the production increases, thus the rate of the returns to scale decreases as production increases: \( j'(Q) < 0 \). As we showed in the previous section if the returns to scale behave this way the firm will tend to increase the utilization of its capital.
as the demand for its product increases.

Within the literature on capital utilization it has been Georgescu-Roegen (1969, 1970, 1972) who has paid much attention to the role of indivisibilities in production, and as a cause of increasing utilization when demand increases. To understand his argument we need some background on his production theory. He argues that during the production of any good there are inevitably some idle resources. The degree of this idleness can only be reduced if the demand for the output of the firm, and, as a result, the scale of production increase. From a historical point of view Georgescu Roegen goes as far as to argue that the Industrial Revolution was a result of increase in demand, which allowed the artisan shops of the time to start working with the “factory system” which in turn allowed the further division of labor with the results described by Adam Smith in the first chapters of *The Wealth of Nations*. This is his interpretation of the Smithian proposal that “the division of labor is limited by the size of the market”.

The role of increasing demand as a cause of increasing utilization is also highlighted by Robin Marris in his *Economics of Capital Utilization* (1964). He argues that because of indivisibilities the entrepreneur either has to choose a different technique of production compared to the one that would be optimum if there were no indivisibilities, or underutilize his equipment (or both). He concludes that “the general tendency of effect of restraints on output must inevitably be in the direction of reducing optimum practicable rates of utilization”. As the demand for the product of a firm increases the “optimum practicable rates of utilization” will also increase. In other words the desired rate of utilization will increase.

The causes of the returns to scale are not limited to indivisibilities. Kaldor himself (1966, 1972), three decades after his 1934 article, questions his thesis that all cases of large-scale economies can be treadted under the heading indivisibility. Among others he refers to the returns to scale caused by the “three-dimensional nature of space”. The usual example for this kind of economies of scale—it is given by Kaldor (1972) himself, Koopmans (1957) and Eatwell (2008)—is a cylinder, whose construction cost varies with its diameter ($2\pi r$) while its capacity varies with the area of its top ($\pi r^2$). In this case doubling the inputs (the radius) quadruples the output/capacity and we have a constant rate of degrees of scale equal to 2. However, also in this case, we could have a decreasing rate of returns, if for example a larger diameter would require a thicker plate.
Young (1928), following Adam Smith, emphasized another cause of returns to scale: “with the division of labour a group of complex processes is transformed into a succession of simpler processes, some of which, at least, lend themselves to the use of machinery”. The returns to scale are a result of increased differentiation and the invention of new processes. This phenomenon can take place either at a micro level (division of the product of the same firm into simpler processes) or a macro level (division a certain product among different firms or sectors). Young and Kaldor (1966) seemed to favor the latter. The argue that the benefits of this kind of differentiation will be the “emergence of new subsidiary industries”. Their argument is verified by economic history. At the macro level it is not easy to conclude if the resulting returns to scale are exhausted or not, but we can make some safer conclusions for the micro level. It is clear that the benefits of the division of labor within a firm are exhausted because the number of the simpler production process is finite. In the famous Smithian pin factory it is very clear how the division of the tasks in the production of pins generates significant benefits, but it is also very clear that the number of the different tasks (drawing out the wire, straightening it, cutting it, pointing it, grinding it at the top for receiving the head etc—Smith (1999, p.110) distinguishes eighteen different tasks) is finite and gets exhausted as the extent of the market allows the firm to perform this division of tasks.

Finally, returns to scale can also be the outcome—as Adam Smith (p.112) says—of the “increased dexterity in every particular workman”. More recently, it was Arrow (1962) who explained the benefits of learning by doing. Kaldor (1966, 1972) also includes it in his taxonomy of the causes of the returns to scale. It is hard to make a definite statement about the behavior of the rate of returns to scale due to learning by doing. However it appears plausible that at the micro level, within an individual firm, the rate of the returns if it is not decreasing, at least it is not increasing.

In total, if we take into account all the above causes of returns to scale, together with reasons that would burden the firm with additional cost as the production increases (e.g. increases bureaucracy, more difficult control of production, etc) we can convincingly argue in favor of a rate of returns to scale which at the firm level is decreasing as the demand for the output of the firm and the scale of production is increasing.

The econometric results on this issue do not provide a definite answer. Betancourt and Clague (1981) use data from the United Nations Industrial Development Organization (UNIDO) for France, Japan, India and Israel. They
conclude that for the first two countries the rate of returns to scale is constant, while for India and Israel is decreasing. Other empirical efforts are also ambiguous. Haldi and Whitcomb (1967) and Moore (1959) examine engineering data and find a constant rate of returns to scale. On the other hand Bain (1956) and Pratten (1971) provide interview estimates which show that the benefits due to scale tend to get exhausted as the scale of production increases. Basu (2008) shows that more recent estimations also do not provide a clear answer.

Marris (1964) is more straightforward and clear. He examines his theory of utilization for the UK firms. He confirms from his empirical research that scale restrains in the production of the firm lead—“through indivisibilities”—to underutilization of capital. In the context of our discussion this supports a decreasing rate of returns to scale.

Yet, the most interesting results are provided by the answers of the entrepreneurs themselves when they are asked why they do not utilize enough their capital. In the second page of his book Marris (1964) writes

> “in business inquiries, one of the commonest reasons given for working shifts or not (as the case may be) relates to demand.”

Betancourt and Clague (1981) (also in page 2) mention

> “interviews have shown that that when factory managers have been asked why they are operating only one shift one of the most frequent answer given is that the firm would not be able to sell its product.”

These answers imply that the behavior of the actual firms is similar to the fictional firm, whose rate of returns to scale is decreasing as the demand for their product is increasing.

In conclusion, the discussion of the last two sections shows that the rate of utilization for the individual firm is not determined only by exogenous structural characteristics, like technology, the relative price of capital to the rhythmically price inputs or the degree of monopoly of the market, as it is usually assumed in the literature. On the contrary the cost-minimizing firm adjusts the level of its capacity utilization in the face of changes in the demand for its product. We discuss the implications for the Kaleckian model in the next section.

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26 Emphasis added in both quotes
9 Revisiting the Kaleckian Model

As we mentioned in section 3 the main difficulty with the long run extension of the Kaleckian model is the unconvincing economic narrative behind equation (6). In the two previous sections we showed that as long as the rate of the economies of scale is decreasing the entrepreneurs will have the incentive—under the cost minimizing principle—to meet the increasing demand for the product of their firms not by expanding their plants, but rather by utilizing them more, that is by readjusting their desired rate of utilization upwards. A remaining task then is to relate this conclusion with the macro level.

Looking at the Statistics about Business Size from the U.S. Census Bureau\(^\text{27}\) we can see that the average firm size does not increase in line with the output. Increased aggregate demand at the macro level is accommodated through an increase in the number of the firms. The average firm size seems to increase in periods of high growth rates, periods when the growth rate is higher than the expected growth rate. Formally we could express this micro-macro relation as

\[ \dot{Q} = \zeta (g^* - g_0) \]  

(19)

where \( \zeta \) is a positive parameter, \( Q \) is the demand for the product of the firm and as we mentioned in section 3, \( g_0 \) is the expected rate of accumulation. In a long run equilibrium where \( \dot{Y} = g = g_0 \) the size of the average firm does not change.

In other words, a growth rate higher than the expected rate expands the market for the individual firm and increases the demand for its product. In the face of the increase in demand the firm will increase its product\(^\text{28}\). This increase will tend to materialize through an increase in the (desired) utilization of the capital of the firm.

At the macro level then, the growth of demand and output cause the level of the desired utilization to increase, which confirms that \( \dot{u}_d = \lambda (g^* - g_0) \), where \( \lambda \) is a positive constant. From this equation and by using (3) and (4) it is easy to see that

\(^{27}\)http://www.census.gov/epcd/www/smallbus.html\#Nonemployers

\(^{28}\)(Sraffa, 1926) says in that respect: “Everyday experience shows that a very large number of undertakings and the majority of those which produce manufactured goods work under conditions of individual diminishing costs. Almost any producer of such goods, if he could rely upon the market in which he sells his products being prepared to take any quantity of them from him at the current price, without any trouble on his part except that of producing them, would extend his business enormously”.
\[ \dot{u}_d = \mu(u^* - u_d) \]

where \( \mu = \alpha \lambda \).

Therefore, we end up again with a dynamic system similar to (9). The solution and the stability properties are the same as before.

10 Conclusion

In the preceding sections we argued in favor of an endogenous adjustment of the desired rate of capacity utilization as the economy grows. This adjustment is a possible mechanism for the transfer of the conclusions of the Kaleckian model from the short run to the long run.

Our argument proceeded in a linear fashion. First, we explained why in the long run the actual rate of utilization must be equal to the desired rate. The latter is determined at the firm level and “will be exclusively grounded on cheapness”. We explained why the mechanism that has been proposed in favor of an endogenous desired rate of utilization is not convincing. We also showed why the skepticism against an endogenous long run rate of utilization is reinforced by the FRB data which show a constant long run center of gravitation.

However, as we argued next, these data do not provide a satisfactory answer to the question of the long run trend of the desired rate because by construction they show only the deviations of the actual rate from the desired rate. We propose that one way to face this problem is to examine how much time capacity is utilized \( \text{vis à vis} \) a fixed time interval: a day, a week, a year. If utilization is examined through this prism, the long-run trend of the desired rate of utilization is far from stationary and it presents a significant upward trend.

We then ask ourselves if there are reasons that make the desired rate react positively to changes in demand. We provided a simple model of a firm, which determines the level of utilization of its resources based on the cost minimizing principle. The firm has an incentive to increase the utilization as the production increases if the rate of degrees of scale decreases with the expansion of the scale of production. After examining the theory of the returns to scale we concluded that it is possible the rate of the returns to behave like this, at least at the micro level, since the main source of the economies of scale at this level is related to indivisibilities of different kinds and therefore the benefits of scale are exhausted as the scale of production increases.
We concluded that via this mechanism the firm will tend to increase the utilization of its resources along with its product as the economy grows and it (the firm) faces increasing demand. At a macro level this can be “translated” into an endogenous adjustment of the desired level of utilization towards its actual rate.

References


