

Lecture Notes, Unit 1

General caveat about online notes: These notes are provided in case you miss anything. Reading them on their own is poor substitute for attending class, for a number of reasons:

- (i) What is taught in this course are *processes* for solving certain problems. These notes do not show the processes, but only the end results, and cannot substitute for seeing the processes in action, and may be difficult to understand on their own.
- (ii) These notes constitute only what I'm likely to write or project on the screen; they do not contain nearly everything I say in lecture that it might be helpful for you to know.
- (iii) These notes are from last time I taught the course, and I reserve the right to change things this semester.

A. BASIC DEFINITIONS

Logic is the science of the correctness or incorrectness of reasoning,
or, more to the point,

Logic is the study of the evaluation of arguments.

A **statement** is a declarative sentence, or part of a sentence, that can be either true or false.

How many statements are there in the example below?

Today is the third day of classes this semester, and today is the second day of Logic class.

(The answer is 3.)

A **proposition** is what it is meant by a statement, the *idea* or *notion* it expresses, which might be the same for different sentences (e.g. translations from one language into another.)

An **argument** is a collection of statements or propositions, some of which are *intended* to provide support or evidence in favor of one of the others.

The **premises** of an argument are those statements or propositions in it that are intended to provide the support or evidence.

The **conclusion** of an argument is that statement or proposition for which the premises are intended to provide support. (In short, it is the point the argument is trying to make.)

(*Important note:* premises are always *intended* to provide support or evidence for the conclusion, but they don't always succeed! It's still an argument, and there are still premises and a conclusion, even if the premises don't really provide any support at all.)

B. EXAMPLE ARGUMENTS

Identify the conclusions (underlined)

1. God is defined as the most perfect being. A perfect being must have every trait or property that it's better to have than not to have. It is better to exist than not to exist.
Therefore, God exists.

2. Hillary Clinton must be a communist spy. She supports socialized health care, and everyone who supports socialized health care is a communist spy.

3. It has rained more than 15 inches per year in Amherst every year for the past 30 years.
So you can safely bet it will rain more than 15 inches in Amherst this year.

4. Professor Chappell said that the ratio of female to male students in the class was exactly 3:1. This means that there are 112 female students in the class, because there are 148 students in the class total.

5. The Encyclopaedia Britannica has an article on symbiosis. It stands to reason that the Encyclopedia Americana has an article on symbiosis as well, since the two reference works tend to cover the same topics.

6. 1 is prime. 3 is prime. 5 is prime. 7 is prime. Therefore, all odd integers between 0 and 8 are prime.

7. Jason isn't an NRA member. Almost 90% of NRA members are republicans, and Jason isn't a republican.

C. DEDUCTION AND INDUCTION; VALIDITY AND SOUNDNESS

- Distinction often taught wrongly in high school.
- NOT the distinction of reasoning from general to specific/specific to general.
- Has to do with the strength of intended evidence.

An argument is **deductive** if the author intends it to be so strong that it is *impossible* for the premises to be true and the conclusion to be false, or in other words, that the conclusion follows *necessarily* from the premises.

Another Important Note: Just because the author intends the argument to be this strong, doesn't mean it actually is!

- However the intention itself is enough to make it deductive.
- The argument might be invalid: more on this below.

An argument is **inductive** if the author intends it only to be so strong that it is *improbable* that the premises could be true and the conclusion false, or in other words, that the conclusion is *likely* if the premises are true.

(The author only has to *intend* that the premises make the conclusion likely. It doesn't actually have to be the case.)

An inductive argument for which it *actually* is true that the conclusion is probably true if the premises are true is **strong**.

An inductive argument for which this isn't the case is called **weak**.

This course deals almost entirely with deductive logic. To learn inductive logic, take a course on statistics or scientific method.

Examples 1, 2 and 6 are deductive.
Examples 3, 4, 5 and 7 are inductive. (4 is

tricky because it relies on not just mathematics but testimonial evidence.)

A deductive argument is **valid** if it actually is impossible for the premises to be true and the conclusion false, or if the conclusion follows necessarily from the premises.

Otherwise, a deductive argument is **invalid**.

A rough test for argument validity:

- First *imagine* that the premises were true—whether or not they actually are.
- Ask yourself, without appealing to any other knowledge you have, could you still *imagine* the conclusion being false?
- If you can, the argument is invalid.
- If you can't, then the argument is valid.

Validity does not have to do with the *actual* truth or falsity of the premises.

It only has to do with what would follow from them if they *were* true.

A valid argument can have false premises.

Example 1:

*All toasters are items made of gold.
All items made of gold are time-travel devices.
Therefore, all toasters are time-travel devices.*

- May be hard to imagine the premises as true.
- Not hard to see that if they were true, the conclusion would have to be as well.

The validity of an argument has to do with its *process of reasoning*.

Example 1 isn't a good argument. So there's more to an argument's being a good one than validity.

- A good argument must also have true premises.

An argument with (all) true premises is called **factually correct**.

A **sound** argument is an argument that is *both* valid and factually correct.

An invalid argument may have true or false premises, and a true or false conclusion.

A valid argument may have false premises with either a true or a false conclusion.

The only combination that is ruled out is a valid argument with true premises and a false conclusion. *Sound arguments always have true conclusions.*

- *A good argument is a sound argument.*

D. ARGUMENT FORM

The validity of a deductive argument is determined entirely by its *form*. Consider:

Example 2:

*All tigers are mammals.
No mammals are creatures with scales.
Therefore, no tigers are creatures with scales.*

Example 3:

*All spider monkeys are elephants.
No elephants are animals.
Therefore, no spider monkeys are animals.*

These arguments have the same **form**:

All A are B.
No B are C.
Therefore, No A are C.

All arguments with this form are valid, including both examples.

Example 3 is not factually correct, and hence not sound, but it is valid. Contrast:

Consider instead:

Example 4:

*All Jedis are one with the Force.
Yoda is one with the Force.
Therefore, Yoda is a Jedi*

Example 5:

*All basketballs are round.
The Earth is round.
Therefore, the Earth is a basketball.*

These both arguments have form:

*All A are F.
X is F.
Therefore, X is an A.*

All arguments with this form are *invalid*.

#4 may seem like a good argument because all the premises and the conclusion are true (at least in fiction),
- but note that the conclusion isn't *made true* by the premises;
- it *could be* possible for the premises to be true and the conclusion false.
- #4 is invalid, and all invalid arguments are unsound.

Studying different argument forms will be one of the primary focuses of this course.

The Counterexample Method

A recap of two points:

- *Whether a deductive argument is valid or invalid is determined by its form.*
- *Valid arguments with true premises always have true conclusions.*

This means:

- *If an argument has true premises and a false conclusion, it must be invalid.*

Together, these principles lead to this interesting result:

- *For every argument, if you can find another argument that (1) has the same form, and (2) has true premises and a false conclusion, then both arguments are invalid.*

The basketball/Earth example can be used as a **counterexample** to show the invalidity of the less obviously invalid Jedi/Yoda example.

Introduction to Logic

Unsure whether an argument is valid? Try to construct a simple counterexample:

Example 6:

*All bandersnatches are toves.
Some borogoves are toves.
Therefore, some bandersnatches are borogoves.*

Counterexample:

Example 7:

*All fish are aquatic animals.
Some mammals are aquatic animals.
Therefore, some fish are mammals.*

If an argument is valid, it will be impossible to find a counterexample.

There are no counterexamples to this argument:

Example 8:

*All bandersnatches are toves.
Some borogoves are bandersnatches.
Therefore, some toves are borogoves.*

E. SYMBOLIC LOGIC; SENTENTIAL LOGIC

Symbolic logic is the study of the evaluation of arguments through the use of mathematically-inspired logical notation.

Mathematical symbols such as + and = stand for the mathematical concepts of *addition* and *equality*.

In symbolic logic, we use signs such as ~ and v to stand for such logical concepts as *negation* and *disjunction*.

(I will be explaining these shortly.)

Sentential logic, also called **propositional logic**, is the simplest species of symbolic logic.

Sentential logic has signs standing for the concepts which we would express in English with words or phrases such as:

and, or, but, if, only if, unless, not, yet (etc.)

These words are understood as **statement connectives**:

They are words that can be used to make more complex (or "molecular") statements out of simple ("atomic") statements.

I live in Amherst and I hate living there.
If Gigli was a good movie, then I'm crazy.
You shouldn't take this class unless you are prepared to work hard.

In sentential logic...

- we use uppercase letters to abbreviate simple statements, and
- we represent complex statements with simple statements joined together by symbols.

So the first example becomes: L & H

L abbreviates the whole statement *I live in Amherst*, and

H abbreviates the whole statement *I hate living here*.

The '&' is used in place of 'and'.

Truth Functions

Sentential logic borrows the notion of a *function* from math.

Crudely put, a function in math is something that takes one or more numbers as "input", and yields another number as "output".

- When the function *square root of*, takes the number 4 as "input", the "output" is the number 2.

- When the function *addition* takes the numbers 5 and 7 as "input", the "output" is the number 12. ($5 + 7 = 12$)

Sentential logic deals with functions that operate on **truth** and **falsity** rather than on numbers.

- The truth-value of a complex statement is a "function of" the truth-values of its parts.

Amherst is in Massachusetts and Boston is in Massachusetts.

Amherst is in Massachusetts and Chicago is in Massachusetts.

Introduction to Logic

A complex statement combined with *and* is true if both sides are true, and false if either side is false.

A = Amherst is in Massachusetts (TRUE)
B = Boston is in Massachusetts (TRUE)
C = Chicago is in Massachusetts (FALSE)

So A & B is TRUE.
And A & C is FALSE.

It's similar to a form of mathematics with only two numbers ("Boolean algebra").

Negation

The simplest truth function is negation. It is written ' \sim '. (This sign is called a *tilde*.)

The truth-value of $\sim A$ is always the opposite of the truth-value of A. If A is true, then $\sim A$ is false, and if A is false, then $\sim A$ is true.

p	$\sim p$
T	F
F	T

The sign " \sim " is used to translate"
not
it is not true that
it is false that
it is not the case that, etc.

Other logic books might use the signs ' \neg ' or ' $\bar{}$ ' instead of ' \sim '.

Note, however, that the logical concept of *negation* is not the same as the mathematical concept of *negative*.

To say that I am NOT 8 feet tall is not the same as to say that I am -8 feet tall!

Conjunction

Conjunction is another simple truth function, except that it has two inputs, unlike negation, which only has one.

Conjunction is written '&'. This sign is called an *ampersand*.

A & B is true if both A and B are true and it is false, otherwise.

Strictly speaking, with 2 inputs there are four possibilities altogether. Here is a chart:

p	q	$p \& q$
T	T	T
T	F	F
F	T	F
F	F	F

The two sides of a conjunction are called **conjuncts**.

Conjunction is used to translate :

and
but
moreover
however
(a)though
yet
and similar words.

Some other logic books sometimes use the signs ' \bullet ' or ' \wedge ' instead of "&".

Disjunction

Disjunction is yet another simple truth function. It is written ' \vee '. This sign is usually called a *wedge*.

$A \vee B$ is true if either A or B is true, or if both are true, and it is false if both A and B are false.

Chart:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The two sides of a disjunction are called **disjuncts**.

Disjunction is often used to translate

or
either... or...
unless

Introduction to Logic

However, we use the word 'or' in different ways.

The sign 'v' always leaves open the possibility that both sides are true, and this is not always true of our word 'or'.

Contrast:

Either the Yankees will be AL champs or the Mets will be NL champs.

(Still true if both are respective champs.)

Either the Yankees will be AL champs or the Red Sox will be AL champs.

(Here there's no possibility of both being true.)

It's the first form of 'or' that we write as 'v': also called the **inclusive** meaning of 'or'.

Implication

Implication, or more specifically, **material implication**, is another simple truth-function.

It is written ' \rightarrow '

This sign is called an arrow.

(This is not implication in the sense of "hints".)

$A \rightarrow B$ is false if A is true and B is false, and is true in all other cases.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implications are also called **conditionals**.

The first part of a conditional is called the **antecedent**, and the second is called the **consequent**.

Implication is most often used to translate:
if... then...

*only if
implies that*

However, it works differently than the English "if... then...".

The truth-value of " $A \rightarrow B$ " is always only determined by the truth-values of A and B. There doesn't need to be any causal or conceptual connection between A and B.

If Al Gore is the president, then a Republican is running the country.

This seems false, but it is true if written as:
 $A \rightarrow R$

If Kevin grew up in Milwaukee, then he used to live in Minnesota.

This seems false, but since both parts are true, we regard it as true if written as:
 $G \rightarrow L$

Some other logic books use the signs ' \supset ' or ' \Rightarrow ' instead of ' \rightarrow '.

Equivalence

Our last simple truth-function is equivalence, or sometimes, **material equivalence**.

It is written ' \leftrightarrow '.

This sign is called a *double arrow*.

" $A \leftrightarrow B$ " is true if A and B have the same truth-value (both true or both false), and false if A and B have different truth-values.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Equivalences are also called **biconditionals** or **material biconditionals**.

Equivalence is usually used to translate:

*... if and only if ...
... just in case ...
... iff ...*

Introduction to Logic

Again, not a perfect translation:

Al Gore is president if and only if a Green Party member runs the country.

Seems false, but is true if taken as :

$$A \leftrightarrow G$$

Some other logic books use the sign '≡' instead of '↔'.

Complex Statements

Sometimes a complex statement is made up of more than two simple or atomic statements. Consider:

The election was held on November 7th 2000, and either Bush won the election or Gore won the election.

If you think 'N Sync was good or you think the Backstreet Boys were talented, then you're crazy.

I hate Justin Timberlake, but if you like Fergie, then if you don't like Britney Spears, then we can still be friends.

How to we translate these sentences? We do so using multiple connectives. These become:

$$\begin{aligned} E \ \& \ (B \vee G) \\ (N \vee B) \ \rightarrow \ C \\ H \ \& \ [L \ \rightarrow \ (\sim B \ \rightarrow \ F)] \end{aligned}$$

Why do we need parentheses?
For the same reason we need them in math.

There's a difference between $(12 \div 3) \div 4$ and $12 \div (3 \div 4)$. The first is 1, the second is 16!

Let us imagine A, B and C are true and X, Y and Z are false.

Then $(A \vee B) \ \& \ Y$ is FALSE.
But $A \vee (B \ \& \ Y)$ is TRUE.

And $\sim Y \vee C$ is TRUE.
But $\sim(Y \vee C)$ is FALSE.

Determining the Truth Value of a Complex Statement

Again, assume A, B and C are true and X, Y and Z are false.

Work from the inside brackets outwards (just like you would if you were doing an arithmetical problem.)

Examples:

$$\sim A \vee (B \ \& \ C)$$

TRUE

$$\sim(Y \vee Z) \ \& \ (A \leftrightarrow Y)$$

FALSE

$$\sim[C \ \rightarrow \ (A \vee Y)] \ \rightarrow \ X$$

TRUE

$$\sim[\sim(C \vee \sim \sim A) \ \& \ B]$$

TRUE

$$A \vee \sim A$$

TRUE

$$Y \vee \sim Y$$

TRUE

Negations apply to just what immediately follows them, unless they are followed by a parenthesis, and then they apply the entire part of the statement inside the scope of that parenthesis.

Calculate the result of a negation '∼' prior to any other truth function inside the same number of parentheses.

The connective whose truth-function you calculate last (the one in the least parentheses) is called the statement's **main connective** or **main operator**.

F. TRUTH TABLES

Suppose you don't know the truth-values of the simple constituents of a complex statement.

Then, you can draw a chart of all the

Introduction to Logic

possibilities. We call these charts **truth tables**.

Let us consider the statement: $P \rightarrow (Q \rightarrow P)$

There are four possibilities:

- (1) P and Q are both true,
- (2) P is true, Q is false,
- (3) Q is true but P is false,
- (4) P and Q are both false.

Let us make a chart in which we put these truth-values under P and Q in a little table. So it looks like this:

$P \rightarrow (Q \rightarrow P)$		
T	T	T
T	F	T
F	T	F
F	F	F

Notice the second 'P' column the same as the first. These 'P's abbreviate the same statement, so they have the truth-value.)

Now we can start determining the truth-value of the whole statement based on all the possible truth-values of the parts.

We start by looking at the arrow inside the parentheses, since that's where you would start calculating if you knew the truth-values.

- This part is false when Q is true and P is false, and true otherwise. Let's fill this in.

$P \rightarrow (Q \rightarrow P)$		
T	T	T
T	F	T
F	T	F
F	F	F

The right half of the statement is false only under possibility (3) above.

The whole statement would only be false when P is true and the right half is false. If we look closely, we notice that this doesn't happen under any of the possibilities.

In other words, no matter what the truth-values of P and Q are, the whole statement is

true.

Let us fill in all T's under the main connective:

P	\rightarrow	(Q	\rightarrow	P)
T	T	T	T	T
T	T	F	T	T
F	T	T	F	F
F	T	F	T	F

Since the column under the main connective tells you the truth-value for the whole statement, it is good to highlight it in some way (in a different color, or put a box around it, etc.)

By doing the "truth table" for this statement, we realized that it is TRUE regardless of the truth-values of P and Q.

Sometimes philosophers put it like this: this statement is true in *all possible worlds*.

Logicians call a statement, like this, that is true, no matter what, a **tautology**.

They are not only true, they are *necessarily* true.

Here's a simple truth table, resulting in a tautology.

P	\vee	$\sim P$
T	T	F
F	T	T

It's pretty easy to recognize that this is true no matter what.

It has two rows, because, since only one letter is involved, there are only two possibilities that matter: P is true, or P is false.

P	$\&$	$\sim P$
T	F	F
F	F	T

This statement is *false* no matter whether P is true or false.

Introduction to Logic

It is *false* in all possible worlds.

In other words, it is *logically impossible* for it to be true.

Logicians call statements like this **self-contradictory**, because they could only be true if something were *both* true and false, which nothing can be!

Besides tautologies and contradictions, some statements are true for some possibilities, and false for other possibilities.

We call these statements **contingent**: they are neither necessarily true nor impossible, logically speaking.

Here is a truth-table for a contingent statement:

P	Q	(P → Q)
T	T	T
T	F	F
F	T	T
F	F	T

This is true if P and Q are both true, and false otherwise.

Constructing Truth Tables

We can construct a truth table for any statement.

It is important to include a row for every possible combination of truth-values for the simple letters.

Mathematically speaking, the size of the table doubles with each new letter we add.

For one simple statement, we need 2 rows; for two simples, we need 4 rows; for three simples, we need 8 rows; for four simples, we'd need 16 rows, and so on.

(Luckily, I won't make you do any tables bigger than that.)

For math geeks, if *n* is the number of simple

statements in our statement, the number of rows needed is 2^n .

Once you've figured out how many rows you need, you do the following:

- You find the first letter to occur: make the first half of the rows Ts and the second half Fs.
- For the next letter, make half of the rows you made Ts for the previous letter into Ts for that column, and the other half Fs; repeat the same number until finishing the table.
- And so on until under your last letter, where you should alternate Ts and Fs.

For example:

P	Q	R	(Q ↔ (¬P & R))
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	F	T
F	T	T	F
F	F	F	T
F	F	T	F

Notice that the second 'P' column needs to match the first, and doesn't count as a fourth letter.

We then fill it in the bits to see that the statement is contingent.

P	Q	R	S	(Q ↔ (¬P & R))
T	T	T	F	T
T	T	F	F	F
T	F	T	F	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F
F	T	F	T	T
F	F	T	F	F

Consider how we set up this table:

$$[(A \rightarrow B) \vee (B \rightarrow A)] \& [(C \leftrightarrow D) \vee (C \leftrightarrow \sim D)]$$

It has four distinct simple letters, so it'll need 16 rows.

Introduction to Logic

[[$(A \rightarrow B) \vee (B \rightarrow A)$] & [[$(C \leftrightarrow D) \vee (C \leftrightarrow \sim D)$]]							
T	T	T	T	T	T	T	T
T	T	T	T	T	F	T	F
T	T	T	T	F	T	F	T
T	T	T	T	F	F	F	F
T	F	F	T	T	T	T	T
T	F	F	T	T	F	T	F
T	F	F	T	F	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	T	T	T	T
F	T	T	F	T	F	T	F
F	T	T	F	F	T	F	T
F	T	T	F	F	F	F	F
F	F	F	F	T	T	T	T
F	F	F	F	T	F	T	F
F	F	F	F	F	T	F	T
F	F	F	F	F	F	F	F

Consider the two statements:

$$\sim(P \& Q)$$

$$\sim P \& \sim Q$$

Do these say the same thing?

Well, let's fill in the truth-table for the two halves and see if they ever differ in truth-value.

$\sim(P \& Q)$	$\sim P \& \sim Q$
F	T
T	T
T	F
T	F
F	T
F	F
F	F
F	F

Here we see that for the second and third rows, the first statement is true, but the second is false.

These two statements are not equivalent.

However, let us consider the following two:

P	\rightarrow	Q	Q	\vee	\sim	P
T	T	T	T	T	F	T
T	F	F	F	F	F	T
F	T	T	T	T	T	F
F	F	F	F	T	T	F

Wow, these are true for all the same rows.

In cases like the above, it's important to set it up like one big truth-table, and not just do two separate tables.

The Q at the end of the first must have the same column as the Q at the beginning of the second.

Also, sometimes the statements to compare have a different number of simple statements. You must count the total number of letters in the two statements put together and make the "combined table" the appropriate size.

Truth Tables for Arguments

Truth-tables can also be used to determine whether an argument is logically valid or invalid.

Whew! Now go home and fill it in and you get:

[[$(A \rightarrow B) \vee (B \rightarrow A)$] & [[$(C \leftrightarrow D) \vee (C \leftrightarrow \sim D)$]]															
T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	T	T	T	T	T	T	T	F	T	T	T	T	T	F
T	T	T	T	T	T	T	T	F	F	T	T	F	T	F	T
T	T	T	T	T	T	T	T	F	T	F	T	F	F	T	F
T	F	F	T	F	T	T	T	T	T	T	T	F	F	T	F
T	F	F	T	F	T	T	T	F	F	T	T	F	T	F	T
T	F	F	T	F	T	T	T	F	T	F	T	F	F	T	F
F	T	T	T	F	F	T	T	T	T	T	T	F	F	T	F
F	T	T	T	F	F	T	T	F	F	T	T	F	T	T	F
F	T	T	T	F	F	T	T	F	T	F	T	F	F	T	F
F	T	F	T	F	T	T	T	T	T	T	T	F	F	T	F
F	T	F	T	F	T	T	F	F	T	T	T	T	T	F	F
F	T	F	T	F	T	T	F	F	T	T	F	T	F	T	F
F	T	F	T	F	T	T	F	T	F	T	F	F	T	F	F

Lo and behold, a tautology.

Logical Equivalence

We can also use truth tables them to compare two statements.

Two statements are **logically equivalent** if they necessarily have the same truth-value, or have the same truth-value for every row of a combined truth table.

Introduction to Logic

Consider this argument:

$P \rightarrow Q$
 $R \rightarrow Q$
 $R \leftrightarrow P$

Is it valid? Again, we do one big truth table, as if we comparing for logical equivalence.

(We must use 8 rows, While each statement only contains 2, there are 3 overall.)

P	→	Q	;	R	→	Q	/	R	↔	P
T	T	T		T	T	T		T	T	T
T	T	T		F	T	T		F	F	T*
T	F	F		T	F	F		T	T	T
T	F	F		F	T	F		F	F	T
F	T	T		T	T	T		T	F	F*
F	T	T		F	T	T		F	T	F
F	T	F		T	F	F		T	F	F
F	T	F		F	T	F		F	T	F

(Here I have used a '/' to separate the premises from the conclusion, and ';' to separate the premises from one another.)

Recall the definition of validity:

- an argument is valid if it is **impossible** for the premises to be true and the conclusion still be false;
- our chart shows all the possibilities.

Can the premises of this argument be true while the conclusion is false?

Yes. See the rows marked by '*'.

Hence, this argument is invalid.

To use truth tables to determine the status of an argument, after you fill it in, look for a row in which *all the premises are true*, but *the conclusion is false*:

- if there is such a row, then the argument is invalid.
- if there is no such row, the argument is valid.

Note: a valid argument can have rows in which some, but not all, of the premises are true and the conclusion is false, or in which the conclusion is true while the premises are false.

-Only *all true* premises and a *false* conclusion pose a problem for an argument's validity.

Let us do another argument:

P	∨	Q	;	P	→	R	;	~R	→	~Q	/	R
T	T	T		T	T	T		F	T	F		T
T	T	T		T	F	F		T	F	F		F
T	T	F		T	T	T		F	T	T		T
T	T	F		T	F	F		T	F	T		F
F	T	T		F	T	T		F	T	F		T
F	T	T		F	T	F		T	F	F		F
F	F	F		F	T	T		F	T	T		F
F	F	F		F	T	F		T	F	T		F

Here, there are no rows in which all premises are true and the conclusion false.

So the argument is valid.

This procedure only tells you whether an argument is valid, not whether it is sound. To know that, you'd need to know which row of the table was "actuality".

Let's apply this argument to a real argument, written in English:

If there is a God (G), then God created everything in the universe (C).

If God created everything in the Universe (C), then everything in the universe is good (E).

It's not the case that everything in the universe is good (E).

Therefore, there is not a God (G).

G	→	C	;	C	→	E	;	~E	/	~G
T	T	T		T	T	T		F	T	F
T	T	T		T	F	F		T	F	F
T	F	F		F	T	T		F	T	F
T	F	F		F	T	F		T	F	F
F	T	T		T	T	T		F	T	T
F	T	T		T	F	F		T	F	T
F	T	F		F	T	T		F	T	T
F	T	F		F	T	F		T	F	T

This argument is valid, but is it sound? We could be here all day debating that.

G. SENTENTIAL LOGIC TRANSLATIONS

The first step is to assign a letter to each simple statement:

Example 1:

If we are going dancing, then Jessica should change her shoes and Mark should take a shower.

Use: 'W' for "we are going dancing"
'J' for "Jessica" should change her shoes" and
'M' for "Mark should take a shower"

The final translation is: $W \rightarrow (J \ \& \ M)$

Usually we'll use the first letter from a given simple statement to represent that statement.

However, there are exceptions.

- You must use different letters for different statements.
(Pick something that will help you remember the difference.)

Example 2:

Either I'll buy a Chevy, or I'll buy a Ford.

We might use C to abbreviate "I'll buy a Chevy", and F to abbreviate "I'll buy a Ford."

So we get: $C \vee F$

- Use the same letter twice only when used for the same simple statement.

Example 3:

Either the Orchard Hill bus is late, or the Orchard Hill bus is not late and the schedule is outdated.

This might become: $O \vee (\sim O \ \& \ S)$

Translating Conjunctions

Logical symbols translate those words that are not parts of simple statements.

The following words are all typically translated with the conjunction sign '&':

and, but, yet, although, however, moreover, (even) though, furthermore, whereas

These words differ in connotation, but from the standpoint of logic, they are equivalent.

Example 1:

Peter is intelligent but he voted for Bush.
Becomes: $I \ \& \ V$

Example 2:

Philosophers loves truth whereas rhetoricians love eloquence.

Becomes: $P \ \& \ R$

Some of these words can occur at the beginning of an English sentence.

We still put the sign '&' between the two conjuncts.

Example 3:

Even though Peter is intelligent, he voted for Bush.

Also Becomes: $I \ \& \ V$
Not: $\& \ I \ V$

Example 4:

Although Quebec is in Canada, the people in Quebec speak French.

Becomes: $C \ \& \ F$

Translating Disjunctions

The sign ' \vee ' is usually used to translate 'or' or 'either... or...'.

Example 1:

Jenna bought a copy of the book from the bookstore, or she downloaded it off the internet.

Becomes: $B \vee D$

Example 2:

Either Peter overslept or he forgot about the meeting.

Becomes: $O \vee F$

Recall that 'v' is used for the inclusive sense of 'or'.

- If context makes it clear that the case where both disjuncts are true is excluded, the translation needs to be more complex.

Example 3:

Either you can keep dating Sanjukta, or you can keep dating Kalinda (but not both).

Becomes: $(S \vee K) \& \sim(S \& K)$

Or alternatively: $S \leftrightarrow \sim K$

Translating Negations

'~' translates any phrase used to negate a sentence, such as:

not, it is not the case that..., it is false that..., it is not true that...

Example 1:

It is not true that Boston is boring.

Becomes: $\sim B$

Because 'not' can occur in the middle of a sentence, and can even be hidden in a ___n't word, it can be difficult to spot.

Example 2:

It isn't raining.

Becomes: $\sim R$

(with 'R' standing for "It is raining")

Other words like 'no' are sometimes used to make negations:

Example 3:

George W. Bush is no friend to the environment.

Becomes: $\sim G$

(with G standing for "George W. Bush is a friend to the environment".)

Translating Conditionals

The sign ' \rightarrow ' is used to translate all of the following phrases:

if, if... then..., only if, provided that, on the condition that..., ...in case..., ...implies that...

Conditionals are tricky since the order of the sides matters (unlike the others connectives).

All of these get translated as $A \rightarrow K$:

- *If Anna will come out tonight, then Ken will watch the baby.*

- *Anna will come out tonight only if Ken will watch the baby.*

- *Anna's coming out tonight implies that Ken will watch the baby.*

- *Provided that Anna will come out tonight, Ken will watch the baby.*

- *On the condition that Anna will come out tonight, Ken will watch the baby.*

But these all get translated as $K \rightarrow A$:

- *Anna will come out tonight if Ken will watch the baby.*

- *Anna will come out tonight provided that Ken will watch the baby.*

- *Anna will come out tonight on the condition that Ken will watch the baby.*

- *Anna will come out tonight in case Ken will watch the baby.*

A conditional rules out the possibility of one half being true while the other half is false.

If it rules out the possibility ($p \& \sim q$) then write $p \rightarrow q$, but if it rules out the possibility ($\sim p \& q$), then write $q \rightarrow p$.

Translating Biconditionals

The sign ' \leftrightarrow ' can be used to translate the phrases:

... if and only if... and ... just in case ...

Example 1:

Annemarie will make it to the conference if and only if her car is running well.

Becomes: $A \leftrightarrow C$

Example 2:

Kevin will have a date this weekend just in case hell freezes over.

Becomes: $K \leftrightarrow H$

Hidden Conjunctions, Hidden Disjunctions

Sometimes what appears, grammatically, to be a simple statement can actually be used to assert multiple statements.

Consider:

Massachusetts and Connecticut are in New England.

The 'and' here does not connect whole statements, but simply two names.

However, this is really just a short form of:
Massachusetts is in New England, and Connecticut is in New England.

Therefore, translate this as $M \ \& \ C$.

Similarly:

She got a ride from either Fred or Eileen.

Can be seen as shorthand for:

Either she got a ride from Fred or she got a ride from Eileen.

So it can be translated: $F \vee E$

This can't always be done. Consider:

Naomi and Kathy are roommates.

Can this be rewritten this way?

Naomi is a roommate and Kathy is a roommate.

This may also be true, but it does not say exactly the same thing.

So it is inappropriate to write "Naomi and Kathy are roommates" as $N \ \& \ K$.

In each case, ask yourself whether the change is possible or not. If it is, then use the "&" or " \vee " in your translation.

Some Other Statement Connectives

Neither... nor...

A statement of the form: *neither p nor q* is translated as:

$$\sim(p \vee q)$$

$$\sim p \ \& \ \sim q$$

(either translation is fine)

Example 1:

Neither the sun shone nor the stars twinkled.

Becomes: $\sim(S \vee T)$

Like 'not', sometimes these words occur in the middle of what they are used to negate.

Example 2:

Minnesota is neither an Eastern nor a Western state.

Becomes: $\sim(E \vee W)$

(With E abbreviating "Minnesota is an Eastern state" and W abbreviating "Minnesota is a Western State".)

Unless

A statement of the form *p unless q* is translated as:

$$p \vee q$$

$$\sim q \rightarrow p$$

(either translation is fine)

A statement of the form *unless p, q* is translated as:

$$p \vee q$$

$$\sim p \rightarrow q$$

(either translation is fine)

Hence, "unless" means roughly the same as "if not" and "or".

Example 1:

Unless you stop starting at me, I'll throw a taco at you.

Becomes: $\sim S \rightarrow T$

Example 2:

Sarah still works at the library unless she's been fired.

Becomes: $\sim F \rightarrow W$

Necessary and Sufficient Conditions

To say that *p is a sufficient condition for q* is to say that p guarantees the truth of q, or that if p, then q.

So *p is sufficient for q* or
p is a sufficient condition for q
becomes: $p \rightarrow q$

Example 1:

Your passing every exam is a sufficient condition for your passing the course.

Becomes: $E \rightarrow C$

(With 'E' abbreviating "you pass every exam" and 'C' abbreviating "you pass the course".)

That *p is a necessary condition for q* means that p is required to be true if q is, or that if p is not true, then q cannot be either.

So we translate

p is a necessary condition for q, or

p is necessary for q, as

$\sim p \rightarrow \sim q$

This is the same as: $q \rightarrow p$

Example 2:

George's buying a lottery ticket is a necessary condition for George's winning the lottery.

Becomes: $\sim B \rightarrow \sim W$

or $W \rightarrow B$

To say that *p is both a necessary and sufficient condition for q* is to combine both things; this can be translated with ' \leftrightarrow '.

Example 3:

Your averaging above 50% on the exams is a necessary and sufficient condition for your passing the course.

Becomes: $A \leftrightarrow C$

Translating More Complex Statements

For these, we need to combine approaches:

Example 1:

You'll ENJOY Scary Movie, provided you have a SENSE of humor and you LIKE horror movies.

Becomes: $(S \ \& \ L) \rightarrow E$

Example 2:

Unless ALISON and ERIN go the party, I don't WANT to go.

Becomes: $\sim(A \ \& \ E) \rightarrow \sim W$

Parentheses

Knowing where to put parenthesis can be very difficult.

Try to make use of all of the following:

- context
- common sense
- hints from punctuation

Example 3:

It's not true that I am BALD, and I RESENT the insult.

Becomes: $\sim B \ \& \ R$

Example 4:

It's not true that I'm BALD and I'm LAZY.

Becomes: $\sim(B \ \& \ L)$

Example 5:

If they go to the STORE at 7pm then they'll arrive HOME at 9pm, but I'll be GROWING hungry by 8pm.

Becomes: $(S \rightarrow H) \ \& \ G$

Example 6:

If they go to the STORE at 7pm, they'll arrive HOME at 9pm and they will MISS the start of the movie.

Becomes: $S \rightarrow (H \ \& \ M)$

Translating Arguments

Often you'll need to translate an entire argument into logical notation, not just an isolated statement.

Two tips for translating arguments:

- Identify the conclusion first. Remember it isn't always the last sentence.
- Use the same letter for the same simple statement, and different letters for different simple statements whether or not they occur in the same premise/conclusion.

Example 1.

If the man in black murdered Inigo Montoya's father, then the man in black has six fingers on his right hand. The man in black does not have six fingers on his right hand, so the man in black did not murder Inigo Montoya's father.

What's the conclusion?

It's the underlined sentence. Let's translate that first.

Introduction to Logic

Then we do the premises.

$M \rightarrow S$; $\sim S$ / $\sim M$

Where 'M' stands for "the man in black murdered Inigo Montoya's father."

And 'S' stands for "the man in black has six fingers on his right hand".

Example 2.

Marijuana will not be legalized in the next decade unless the medical community will support its legalization. Thus a medical use for marijuana will be found soon if marijuana will be legalized in the next decade, because the medical community will support its legalization only if a medical use is found soon.

First, we identify the conclusion. The premises come next.

$\sim S \rightarrow \sim L$; $S \rightarrow U$ / $L \rightarrow U$

We could then do a truth-table to determine its validity.

$\sim S$	\rightarrow	$\sim L$;	S	\rightarrow	U	/	L	\rightarrow	U
F	T	F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	F	F	F
F	T	T	F	T	T	F	T	T	F	T
F	T	T	F	T	F	F	F	F	F	F
T	F	F	T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T	F	F	F
T	F	T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	F	F	T	F	F

The argument is valid.

Some More Examples

The OZONE layer becoming depleted is a sufficient condition for SKIN cancer rates increasing to epidemic levels, although our POLITICIANS do not seem concerned with the problem.

$(O \rightarrow S) \& \sim P$

HUMAN life on Earth will not perish unless either we POISON ourselves with pollution or a large ASTEROID collides with Earth.

$\sim(P \vee A) \rightarrow \sim H$

CHINA and RUSSIA will reduce their nuclear arsenals only if neither the UNITED States nor BRITAIN increase their tariffs.

$(C \& R) \rightarrow (\sim U \& \sim B)$

The EBOLA virus is deadly, but it will become a MAJOR threat to humanity if and only if becomes AIRBORNE and a VACCINE is not developed.

$E \& [M \leftrightarrow (A \& \sim V)]$

If EVOLUTIONARY biology is correct, then HIGHER life forms arose by chance, but if higher life forms arose by chance, then it is not the case that there is any DESIGN in nature and divine PROVIDENCE is a myth.

$(E \rightarrow H) \& [H \rightarrow (\sim D \& P)]$

SHEILA is the best candidate, but she won't WIN the election unless a MIRACLE occurs.

$S \& (\sim M \rightarrow \sim W)$

A UNITED Nations-lead military being created is a necessary condition for there existing PEACE in the 21st century.

$\sim U \rightarrow \sim P$

Neither RAIN nor SNOW will prevent your postal carrier from delivering the mail.

$\sim R \& \sim S$

IKE and TINA Turner won't both sing at the concert.

$\sim(I \ \& \ T)$

I'll get TENURE only if I PUBLISH and get good teaching EVALUATIONS, but I'll neither publish nor get good teaching evaluations unless I DELETE all the games on my computer.

$[T \rightarrow (P \ \& \ E)] \ \& \ [\sim D \rightarrow (\sim P \ \& \ \sim E)]$

Although narcotic drug use is DETRIMENTAL to the welfare of many, if we want to KEEP our streets safe and REDUCE our police expenditures, we must either LEGALIZE drug use or not ENFORCE the existing laws.

$D \ \& \ [(K \ \& \ R) \rightarrow (L \vee \sim E)]$

Ross and Rachel will get back together on the condition that Ross takes full responsibility for the break-up and does not sleep with the girl from the copyshop again.

$(R \ \& \ \sim S) \rightarrow G$

If death is like a deep SLEEP or there is an AFTERLIFE, then there is not any reason to FEAR death unless GOD is cruel.

$(S \vee A) \rightarrow (\sim C \rightarrow \sim F)$

If it is not true that KEN will watch the baby if ANNA goes out tonight, then we will need to find a BABYSITTER.

$\sim(A \rightarrow K) \rightarrow B$

Peter is a GENTLEMAN and a SCHOLAR, but he will get a JOB if and only if his BRIBING a university official is a necessary and sufficient condition for his getting a job.

$(G \ \& \ S) \ \& \ [J \leftrightarrow (B \leftrightarrow J)]$

I have FREEWILL to choose my actions if and only if I am not COERCED, provided that

it is false that all human action is DETERMINED by brain chemistry.

$\sim D \rightarrow (F \leftrightarrow \sim C)$

I will be in DEBT next month if my TAX refund doesn't arrive before the 28th, but if my tax refund arrives before the 28th, then I'll go to CANCUN with you over Spring Break if you ASK.

$(\sim T \rightarrow D) \ \& \ [T \rightarrow (A \rightarrow C)]$

H. REVIEW PROBLEMS

The first exam will consist of the following:

- 2-4 true/false questions about validity and soundness (like 1A)
- 2-4 syllogisms to identify as valid or invalid, factually correct or not factually correct, and sound or sound (like 1B)
- 2-4 truth-tables to determine whether a statement is a tautology, contradiction or contingent statement (like 2B/3A)
- 2-4 truth-tables to determine whether an argument is valid or invalid (like 3C)
- 6-10 translations of English statements into the notation of sentential logic (like 4A-4D)
- 1-3 "combined problems" involving translating an entire argument and using a truth table to check its validity or invalidity.

The online practice exam provides a perfect example of the format of the exam.

REVIEW: True/False

1. All valid arguments are sound.
2. All sound arguments are valid.
3. All arguments with all true premises and true conclusions are valid.
4. All valid arguments with true conclusions are sound.
5. All invalid arguments with false premises have true conclusions.

(Answers: 1. False, 2. True, 3. False, 4. False, 5. False.)

Introduction to Logic

REVIEW: Syllogisms

valid or invalid?
factually correct or not?
sound or unsound?

No novels are books.
Some books are refrigerators.
Therefore, all novels are refrigerators.
(Invalid, not fact. corr., unsound.)

All poets are authors.
All novelists are authors.
Therefore, some poets are novelists.
(Invalid, fact. corr., unsound.)

All diamonds are gems.
Some gifts are not gems.
Therefore, some gifts are not diamonds.
(Valid, fact. corr., sound.)

All camels are snowmobiles.
Some staplers are camels.
Therefore, some staplers are snowmobiles.
(Valid, not fact. corr., unsound.)

REVIEW: Truth Tables for Statements

1. $\sim[(P \rightarrow \sim P) \vee (Q \rightarrow P)]$

$\sim[(P \rightarrow \sim P) \vee (Q \rightarrow P)]$	P	$\sim P$	Q	$\sim Q$
F	T	F	T	T
F	T	F	T	F
F	T	F	F	T
F	T	F	F	F
F	F	T	T	T
F	F	T	T	F
F	F	T	F	T
F	F	T	F	F

(SELF-CONTRADICTION)

2. $\sim(P \& Q) \leftrightarrow (\sim P \leftrightarrow \sim Q)$

$\sim(P \& Q)$	\leftrightarrow	$(\sim P \leftrightarrow \sim Q)$
F	T	F
F	T	T
T	F	F
T	F	T
F	T	F
F	T	T
T	F	F
T	F	T

(CONTINGENT)

REVIEW: Truth Tables for Arguments

3. $\sim(P \& \sim Q) ; Q \rightarrow P / P \leftrightarrow Q$

$\sim(P \& \sim Q)$	$Q \rightarrow P$	$P \leftrightarrow Q$
T	T	T
F	T	F
T	F	F
T	F	F

(VALID)

4. $P \rightarrow (Q \vee \sim R) ; P \rightarrow Q / \sim(P \rightarrow R)$

$P \rightarrow (Q \vee \sim R)$	$P \rightarrow Q$	$\sim(P \rightarrow R)$
T	T	F
T	T	T
T	F	F
T	F	F
F	T	F
F	T	F
F	F	F
F	F	F

(INVALID)

REVIEW: Combined Problems

The kangaroo will be saved from extinction only if sports shoe manufacturers decline to use kangaroo hides in their products. After all, if sports shoe manufacturers decline to use kangaroo hides in their products, then Australian hunters will cease killing millions of kangaroos yearly. Moreover, it is not the case that both Australian hunters will cease killing millions of kangaroos yearly and the kangaroo not be saved from extinction.

$S \rightarrow A$	$\sim(A \& \sim K)$	$K \rightarrow S$
T	T	T
T	F	T
T	F	F
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(INVALID)