

Lecture Notes, Unit 2

General caveat about online notes: These notes are provided in case you miss anything. Reading them on their own is poor substitute for attending class, for a number of reasons:

- (i) What is taught in this course are *processes* for solving certain problems. These notes do not show the processes, but only the end results, and cannot substitute for seeing the processes in action, and may be difficult to understand on their own.
- (ii) These notes constitute only what I'm likely to write or project on the screen; they do not contain nearly everything I say in lecture that it might be helpful for you to know.
- (iii) These notes are from last time I taught the course, and I reserve the right to change things this semester.

A. INTRODUCTION TO DERIVATIONS

We've learned one way of determining the validity of an argument: truth tables.

Disadvantages to truth tables:

- (1) The size of the tables grows exponentially as the argument gets more complicated.
 - An argument with 6 simple statements would require 64 lines!
- (2) It's alien to our ordinary ways of thinking. When reasoning in our daily lives, we scarcely ever do anything like it.

We'll be learning a new method for showing that an argument is valid in this unit that does not have the above faults.
(It cannot be used to show that an argument is invalid, unfortunately)

The focus is on the notion of *step by step* reasoning.

Consider the following argument form (often called ***modus ponens***):

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

This is obviously valid, regardless of what p and q are, and whether they are simple statements or more complex ones.

Now consider this slightly more complicated argument:

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow D \\ \hline A \\ \hline D \end{array}$$

This argument is not, strictly speaking, of the same form as the one above, and it is not quite as obviously valid.

It is valid, nevertheless, and we can prove that by showing how the conclusion follows from a chain of smaller arguments similar to our first example.

To put it roughly: Since $A \rightarrow B$ and A , we can conclude B .

Now that we know B and already knew $B \rightarrow C$, we can conclude C .

Once we know C , since we know $C \rightarrow D$, we can finally arrive at our conclusion, D .

What we have done is broken up this argument into a number of smaller arguments. We have shown that, by means of a number of small, obviously valid, steps in reasoning, we know that the conclusion must be true if the premises are true. That is, we have proven that the argument is valid.

This method of proving an argument to be valid is just that: a ***PROOF***.

- Very similar to formal proofs you may have had to do in your high school geometry classes, where you start with a "given" and need to reach a certain result.

In this class, we set up these proofs in a very specific format, and we call them ***derivations***, because, through a chain of reasoning, we *derive* the conclusion from the premises.

B. DIRECT DERIVATIONS

Suppose we wanted to construct a derivation for the validity of the following argument:

$$\begin{array}{l} A \rightarrow (A \rightarrow B) \\ \underline{A} \\ B \end{array}$$

To construct a formal proof, begin by numbering the premises and writing them out.

We also write the abbreviation 'Pr' on the right to remind us that these are the *premises* with which we began.

1. $A \rightarrow (A \rightarrow B)$	Pr
2. A	Pr

We then introduce a new line for the conclusion. However, we write the word "SHOW" before it, to indicate that this is what we are trying to derive, and not something we are already taking as true.

1. $A \rightarrow (A \rightarrow B)$	Pr
2. A	Pr
3. SHOW: B	

Underneath this we will construct our step-by-step derivation.

The derivation should always be indented slightly, as we will draw a box around the derivation when we finish.

The first step of this derivation looks like this:

1. $A \rightarrow (A \rightarrow B)$	Pr
2. A	Pr
3. SHOW: B	
4. $A \rightarrow B$	1,2 \rightarrow O

What we have written at line 4 follows from the first two premises by *modus ponens*.

We had an if-then statement at line 1, and the if-part of that statement at line 2, so we concluded the then-part at line 4.

On the right we wrote "1,2" to indicate that the justification of this new line comes from lines 1 and 2.

The " \rightarrow O" means that we arrived at this line by means of *modus ponens*.

We learn abbreviations for a number of different derivation steps as we go along.

We still have not gotten to our conclusion.

Now that we have line 4, and keeping in mind what we had in line 2, we can perform another *modus ponens*.

1. $A \rightarrow (A \rightarrow B)$	Pr
2. A	Pr
3. SHOW: B	
4. $A \rightarrow B$	1,2 \rightarrow O
5. B	2,4 \rightarrow O

Again we write " \rightarrow O" on the right because this was another *modus ponens*, but notice now we made use of lines 2 and 4 in our reasoning instead of 1 and 2.

In doing a derivation, every new line we introduce should be "justified" by some of the earlier lines. However, we can never make use of SHOW: lines in justifying a new line.

In this case, it was OK for us to appeal to 1 and 2, and to appeal to 4 once we had shown it, but we couldn't have appealed to line 3, since that is what we were trying to prove!

What we have at 5, B, is what we were trying to show at line 3. (QED, or "Quod erat demonstrandum" as they say in Latin.)

Therefore, we have done what we were trying to do. In finishing, we draw a box around our derivation (lines 4 and 5), cross off "SHOW" since we have *shown* it, and write "DD" for "Direct Derivation" on the right to show that we have successfully derived our conclusion.

1. $A \rightarrow (A \rightarrow B)$ Pr
2. A Pr
3. SHOW: B DD
4. $A \rightarrow B$ 1,2 \rightarrow O
5. B 2,4 \rightarrow O

You do not need to draw the entire box, but can draw any portion of it that includes the left side. *(In the online notes, only the left side is ever drawn.)*

Our first example would look like this:

1. $A \rightarrow B$ Pr
2. $B \rightarrow C$ Pr
3. $C \rightarrow D$ Pr
4. A Pr
5. SHOW: D DD
6. B 1,4 \rightarrow O
7. C 2,6 \rightarrow O
8. D 3,7 \rightarrow O

The rule of *modus ponens* is a very natural and powerful rule. You will be using it more than any other inference rule.

It is worth noting that it applies not only when the if-parts and then-parts (“antecedents” and “consequents” in logical terminology) of if-then statements are simple, but also when they are complex. Examples:

1. $\sim(A \vee B) \rightarrow (C \ \& \ \sim D)$ Pr
 2. $(E \vee F) \rightarrow \sim(A \vee B)$ Pr
 3. $E \vee F$ Pr
 4. SHOW: $C \ \& \ \sim D$ DD
 5. $\sim(A \vee B)$ 2,3 \rightarrow O
 6. $C \ \& \ \sim D$ 1,5 \rightarrow O
-
1. $(P \vee Q) \rightarrow (R \leftrightarrow S)$ Pr
 2. $T \rightarrow [(P \vee Q) \rightarrow \sim M]$ Pr
 3. $\sim M \rightarrow [(R \leftrightarrow S) \rightarrow N]$ Pr
 4. T Pr
 5. $P \vee Q$ Pr
 6. SHOW: N DD
 7. $R \leftrightarrow S$ 1,5 \rightarrow O
 8. $(P \vee Q) \rightarrow \sim M$ 2,4 \rightarrow O
 9. $\sim M$ 5,8 \rightarrow O
 10. $(R \leftrightarrow S) \rightarrow N$ 3,9 \rightarrow O
 11. N 7,10 \rightarrow O

[Yes, it’s OK to use the same premise twice.]

This eleven-step derivation may seem very long and complicated. However, compare it to having to do a truth table for the same argument.

For these five premises and one conclusion, the table would have 128 rows and 28 columns, for a total of 3,584 T’s and F’s!!

C. THE SIMPLEST INFERENCE RULES

We will begin by learning the four simplest valid inference or derivation rules.

The first two involve two ways of using or “breaking up” an if-then statement, and they are both abbreviated as “ \rightarrow O” for “ARROW OUT”.

$p \rightarrow q$	$p \rightarrow q$	\rightarrow O
\underline{p}	$\underline{\sim q}$	
q	$\sim p$	

The first one is the famous *modus ponens*, which we’ve already been using.

The other is called *modus tollens* in Latin.

An example of an English argument that uses this mode of reasoning is:

If Macavity stole the diamonds, then Macavity’s hair was left at the scene of the crime. Macavity’s hair was not left at the scene of the crime. Therefore, Macavity did not steal the diamonds.

Simply put, if an if-then statement is true, but the then-part is false, the if-part must be false as well.

Modus ponens is Latin for “affirming mode”. *Modus tollens* is Latin for “denying mode”.

Here are some derivations that make use of this form of “ \rightarrow O”.

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1. $X \rightarrow Y$ Pr
2. $Y \rightarrow Z$ Pr
3. $\sim Z$ Pr
4. SHOW: $\sim X$ DD
5. $\sim Y$ 2,3 \rightarrow O
6. $\sim X$ 1,5 \rightarrow O

1. $(M \vee N) \rightarrow Q$ Pr
2. $\sim Q$ Pr
3. $(R \& S) \rightarrow (M \vee N)$ Pr
4. SHOW: $\sim(R \& S)$ DD
5. $\sim(M \vee N)$ 1,2 \rightarrow O
6. $\sim(R \& S)$ 3,5 \rightarrow O

Here are some that use both forms of " \rightarrow O".

1. $A \rightarrow B$ Pr
2. $\sim A \rightarrow C$ Pr
3. $\sim B$ Pr
4. SHOW: C DD
5. $\sim A$ 1,3 \rightarrow O (MT)
6. C 2,5 \rightarrow O (MP)

1. $\sim(P \vee Q) \rightarrow \sim S$ Pr
2. $S \rightarrow \sim\sim S$ Pr
3. S Pr
4. SHOW: $\sim\sim(P \vee Q)$ DD
5. $\sim\sim S$ 2,3 \rightarrow O (MP)
6. $\sim\sim(P \vee Q)$ 1,5 \rightarrow O (MT)

This last example shows that our rules are very strict. *Modus tollens* takes the form:

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q} \\ \sim p \end{array}$$

We have to plug things in uniformly for " p " and " q " here.

If " q " already begins with a negation, then we need a double negation to apply the rule.

Similarly, if " p " already begins with a negation, the thing we get in the end will be a double negation.

We can't simply drop these off. We need a separate rule for that, which we will learn later.

The next two rules are rules for using or breaking up an OR-statement, and they are abbreviated as "vO" for "WEDGE OUT".

$p \vee q$	$p \vee q$	vO
$\underline{\sim p}$	$\underline{\sim q}$	
q	p	

Simply put, if we know an OR-statement is true, and we know that one of the two sides ("disjuncts") is false, we know the other side (the other disjunct) must be true.

Here's an example in English.

Either the US team will win the gold medal or the Canadian team will win the gold medal. The US team will not win the gold medal. Therefore, the Canadian team will win the gold medal.

Here are some derivations that make use of these rules.

- | | |
|---------------|--------|
| 1. $U \vee C$ | Pr |
| 2. $\sim U$ | Pr |
| 3. SHOW: C | DD |
| 4. C | 1,2 vO |

- | | |
|--|--------|
| 1. $\sim(L \& H) \vee S$ | Pr |
| 2. $(L \& H) \vee (P \leftrightarrow Q)$ | Pr |
| 3. $\sim S$ | Pr |
| 4. SHOW: $P \leftrightarrow Q$ | DD |
| 5. $\sim(L \& H)$ | 1,3 vO |
| 6. $P \leftrightarrow Q$ | 2,5 vO |

Here are some derivations that make use of all the rules covered so far.

- | | |
|--------------------------------------|---------------------|
| 1. $A \rightarrow (B \rightarrow C)$ | Pr |
| 2. $B \vee D$ | Pr |
| 3. A | Pr |
| 4. $\sim C$ | Pr |
| 5. SHOW: D | DD |
| 6. $B \rightarrow C$ | 1,3 \rightarrow O |
| 7. $\sim B$ | 4,6 \rightarrow O |
| 8. D | 2,7 vO |

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1.	$P \rightarrow [(Q \rightarrow R) \vee S]$	Pr
2.	$\sim S$	Pr
3.	$\sim S \rightarrow \sim R$	Pr
4.	$\sim R \rightarrow P$	Pr
5.	SHOW: $\sim Q$	DD
6.	$\sim R$	2,3 $\rightarrow O$
7.	P	4,6 $\rightarrow O$
8.	$(Q \rightarrow R) \vee S$	1,7 $\rightarrow O$
9.	$Q \rightarrow R$	2,8 $\vee O$
10.	$\sim Q$	6,9 $\rightarrow O$

Pitfalls to Avoid

The following argument forms are invalid and must never be used!

$p \rightarrow q$	$p \rightarrow q$	$p \vee q$	$p \vee q$
q	$\sim p$	p	q
p	$\sim q$	$\sim q$	$\sim p$

These may appear superficially similar to the real rules, but a little reflection shows that the slight differences matter.

The first two are *not* cases of " $\rightarrow O$ " and the latter two are *not* cases of " $\vee O$ ".

Another problem that must be avoided is applying the rules to parts of lines.

These rules only apply when the whole statement is of the right form.

" $\rightarrow O$ " cannot be used if the main connective of a statement is not the arrow.

One cannot go from
 $(P \rightarrow Q) \vee R$
 and
 $\sim Q$
 to either $\sim P$ or to R !

Rules must be applied to whole lines.

D. BASIC DIRECT DERIVATION RULES

So far we have learned $\rightarrow O$ and $\vee O$. To review:

$p \rightarrow q$	$p \rightarrow q$	
p	$\sim q$	$\rightarrow O$
q	$\sim p$	

$p \vee q$	$p \vee q$	
$\sim p$	$\sim q$	$\vee O$
q	p	

We now add:

$p \& q$	$p \& q$	$\& O$
p	q	

p		
q		$\& I$
p		

$p \leftrightarrow q$	$p \leftrightarrow q$	$\leftrightarrow O$
$p \rightarrow q$	$q \rightarrow p$	

$p \rightarrow q$		
$q \rightarrow p$		$\leftrightarrow I$
$p \leftrightarrow q$		

p	p	$\vee I$
$p \vee q$	$q \vee p$	

p	$\sim \sim p$	DN
$\sim \sim p$	p	

It is worth noting that $\vee I$, $\& O$, $\leftrightarrow O$ and DN require only one line for justification, whereas $\vee O$, $\rightarrow O$, $\& I$, and $\leftrightarrow I$ require two.

The **OUT** rules are rules for using statements of a certain form (with a certain main operator).

The **IN** rules are rules for deriving or getting statements of a certain form.

With all of these rules in place, we can prove the validity of a wide variety of different arguments.

Here are some sample derivations.

- | | |
|--------------------------------------|---------------------|
| 1. $(F \vee G) \rightarrow (H \& J)$ | Pr |
| 2. F | Pr |
| 3. SHOW: H | DD |
| 4. F \vee G | 2 vI |
| 5. H & J | 1,4 \rightarrow O |
| 6. H | 5 &O |

- | | |
|----------------------------------|---------------------|
| 1. $(A \& B) \rightarrow \sim C$ | Pr |
| 2. A | Pr |
| 3. $D \vee C$ | Pr |
| 4. $A \rightarrow B$ | Pr |
| 5. SHOW: D & B | DD |
| 6. B | 2,4 \rightarrow O |
| 7. A & B | 2,6 &I |
| 8. $\sim C$ | 1,7 \rightarrow O |
| 9. D | 3,8 vO |
| 10. D & B | 6,9 &I |

- | | |
|--------------------------------------|----------------------|
| 1. $(S \rightarrow T) \& (U \vee W)$ | Pr |
| 2. $\sim T$ | Pr |
| 3. $\sim U$ | Pr |
| 4. $(\sim S \& W) \rightarrow K$ | Pr |
| 5. SHOW: K | DD |
| 6. $S \rightarrow T$ | 1 &O |
| 7. $U \vee W$ | 1 &O |
| 8. $\sim S$ | 2,6 \rightarrow O |
| 9. W | 3,7 vO |
| 10. $\sim S \& W$ | 8,9 &I |
| 11. K | 4,10 \rightarrow O |

- | | |
|---|-------------------------|
| 1. $\sim D \rightarrow (F \rightarrow E)$ | Pr |
| 2. $(E \rightarrow F) \vee D$ | Pr |
| 3. $\sim D$ | Pr |
| 4. SHOW: $E \leftrightarrow F$ | DD |
| 5. $F \rightarrow E$ | 1,3 \rightarrow O |
| 6. $E \rightarrow F$ | 2,3 vO |
| 7. $E \leftrightarrow F$ | 5,6 \leftrightarrow I |

These next examples show that DN is often used to set up \rightarrow O or vO:

- | | |
|--|-----------------------|
| 1. $(P \leftrightarrow Q) \vee \sim R$ | Pr |
| 2. R | Pr |
| 3. Q | Pr |
| 4. SHOW: P | DD |
| 5. $\sim \sim R$ | 2 DN |
| 6. $P \leftrightarrow Q$ | 1,5 vO |
| 7. $Q \rightarrow P$ | 6 \leftrightarrow O |
| 8. P | 3,7 \rightarrow O |

Step 6 step cannot be done without DN first!

- | | |
|---|-------------------------|
| 1. $(A \rightarrow B) \rightarrow \sim(C \vee D)$ | Pr |
| 2. $(E \leftrightarrow F) \rightarrow D$ | Pr |
| 3. $E \rightarrow F$ | Pr |
| 4. $F \rightarrow E$ | Pr |
| 5. SHOW: $\sim(A \rightarrow B)$ | DD |
| 6. $E \leftrightarrow F$ | 3,4 \leftrightarrow I |
| 7. D | 2,6 \rightarrow O |
| 8. $C \vee D$ | 7 vI |
| 9. $\sim \sim(C \vee D)$ | 8 DN |
| 10. $\sim(A \rightarrow B)$ | 1,9 \rightarrow O |

E. CONDITIONAL DERIVATION (CD)

We have both IN and OUT rules for all our logical operators, except for the arrow.

There is no \rightarrow I rule!

This makes deriving the conclusion of arguments with SHOW: lines that contain arrows nearly impossible.

We now introduce a new way of proving if-then statements. It is not simply a new derivation rule, but a new style of proof.

The basic idea of **conditional derivation** is that in order to prove that $p \rightarrow q$, i.e., that if p is true, then q is true, we can take the strategy of assuming temporarily that p is true and deriving q from that assumption.

If we can prove that q is true by assuming p , we know that if p were true, q would have to be true as well, and so, that $p \rightarrow q$.

Conditional derivation will work when (and only when) our problem contains an if-then statement in the SHOW line. Consider:

- | | |
|----------------------------|----|
| 1. $P \rightarrow Q$ | Pr |
| 2. $Q \rightarrow R$ | Pr |
| 3. SHOW: $P \rightarrow R$ | |

We can't prove this only using the other rules. Instead, at line 4, we assume that P is true, then we change what we're trying to prove.

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Once we assume P, we introduce a new show line, because *now* we're going to try to show R on the basis of that assumption.

This gets set up like this:

1. $P \rightarrow Q$	Pr
2. $Q \rightarrow R$	Pr
3. SHOW: $P \rightarrow R$	CD
4. P	Ass
5. SHOW: R	

["CD" for "conditional derivation", and "Ass" for "Assumption."]

Now, we do a little "proof within a proof". In other words, we try to show R, using line 4 (P) as an assumption. This uses DD rules.

(Note that line 5 is a SHOW line; we cannot use it in our derivation.) We end up drawing a little box within a box. With P as an assumption, the mini-derivation of R is quite easy.

1. $P \rightarrow Q$	Pr
2. $Q \rightarrow R$	Pr
3. SHOW: $P \rightarrow R$	CD
4. P	Ass
5. SHOW: R	DD
6. Q	1,4 \rightarrow O
7. R	2,6 \rightarrow O

We have now shown R in the normal way. We can cross off that "SHOW":

1. $P \rightarrow Q$	Pr
2. $Q \rightarrow R$	Pr
3. SHOW: $P \rightarrow R$	CD
4. P	Ass
5. SHOW: R	DD
6. Q	1,4 \rightarrow O
7. R	2,6 \rightarrow O

Where are we now?

We have assumed P, and have successfully shown R on the basis of that assumption.

We are now entitled to regard ourselves as having shown $P \rightarrow R$ and can cross off the "SHOW" at line 3.

That's the way CD works.

The final problem looks like this:

1. $P \rightarrow Q$	Pr
2. $Q \rightarrow R$	Pr
3. SHOW: $P \rightarrow R$	CD
4. P	Ass
5. SHOW: R	DD
6. Q	1,4 \rightarrow O
7. R	2,6 \rightarrow O

One thing that is important to remember is that, once a new SHOW line is introduced, your task has changed. You should focus on proving what it says to prove at the new SHOW line, and forget for the moment about trying to get the final conclusion.

Here are some additional examples.

1. $P \rightarrow \sim Q$	Pr
2. SHOW: $(Q \& R) \rightarrow (S \vee \sim P)$	CD
3. Q & R	Ass
4. SHOW: $S \vee \sim P$	DD
5. Q	3 &O
6. $\sim \sim Q$	5 DN
7. $\sim P$	1,6 \rightarrow O
8. $S \vee \sim P$	7 vI

1. $E \leftrightarrow G$	Pr
2. $H \vee E$	Pr
3. SHOW: $\sim H \rightarrow (G \& E)$	CD
4. $\sim H$	Ass
5. SHOW: $G \& E$	DD
6. E	2,4 vO
7. $E \rightarrow G$	1 \leftrightarrow O
8. G	6,7 \rightarrow O
9. G & E	6,8 &I

Sometimes when we introduce a new SHOW line as part of a CD, the new SHOW line will itself be an if-then statement. If so, we may do a conditional derivation within a conditional derivation.

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1.	$A \rightarrow D$	Pr
2.	$(D \& \sim B) \rightarrow C$	Pr
3.	SHOW: $A \rightarrow (\sim B \rightarrow C)$	CD
4.	A	Ass
5.	SHOW: $\sim B \rightarrow C$	CD
6.	$\sim B$	Ass
7.	SHOW: C	DD
8.	D	1,4 $\rightarrow O$
9.	D & $\sim B$	6,8 &I
10.	C	2,9 $\rightarrow O$

1.	$B \vee (C \rightarrow E)$	Pr
2.	$\sim D \rightarrow \sim(E \& A)$	Pr
3.	SHOW: $(A \& \sim B) \rightarrow (C \rightarrow D)$	CD
4.	A & $\sim B$	Ass
5.	SHOW: $C \rightarrow D$	CD
6.	C	Ass
7.	SHOW: D	DD
8.	$\sim B$	4 &O
9.	$C \rightarrow E$	1,8 $\vee O$
10.	E	6,10 $\rightarrow O$
11.	A	4 &O
12.	E & A	10,11 &I
13.	$\sim\sim(E \& A)$	12 DN
14.	$\sim\sim D$	2,13 $\rightarrow O$
15.	D	14 DN

We can use CD whenever we wish to prove a conditional (if-then) statement.

It does not need to be the final conclusion of our argument; we may do it as an intermediate step. However, we do need to have the conditional we wish to prove as a SHOW line.

We are allowed to introduce new SHOW lines whenever we wish.

Once we cross-off the "SHOW" of the newly introduced "SHOW" line we can then use it as though it were a normal line.

However, once a box has been drawn around some lines, we cannot use those lines again in our derivation. Examples:

1.	$[P \rightarrow (P \vee Q)] \rightarrow R$	Pr
2.	SHOW: R	DD
3.	SHOW: $P \rightarrow (P \vee Q)$	CD

4.	P	Ass
5.	SHOW: $P \vee Q$	DD
6.	$P \vee Q$	4 $\vee I$
7.	R	1,3 $\rightarrow O$

Here, we used CD in proving the SHOW line at line 3, but the ultimate conclusion, the SHOW line at line 2, was not proven by CD, but by $\rightarrow O$, which is DD rule, so we write "DD" at 2, and "CD" at line 3.

This basic procedure allows us to use multiple CDs, combined with $\leftrightarrow I$, to prove biconditional (iff) statements.

We first prove the conditional one way, and then the other. Then we put them together.

1.	$\sim B \& C$	Pr
2.	SHOW: $(A \vee B) \leftrightarrow A$	DD
3.	SHOW: $(A \vee B) \rightarrow A$	CD
4.	A $\vee B$	Ass
5.	SHOW: A	DD
6.	$\sim B$	1 &O
7.	A	4,6 $\vee O$
8.	SHOW: $A \rightarrow (A \vee B)$	CD
9.	A	Ass
10.	SHOW: $A \vee B$	DD
11.	A $\vee B$	9 $\vee I$
12.	$(A \vee B) \leftrightarrow A$	3,8 $\leftrightarrow I$

1.	$R \leftrightarrow Q$	Pr
2.	$Q \leftrightarrow P$	Pr
3.	SHOW: $P \leftrightarrow R$	DD
4.	SHOW: $P \rightarrow R$	CD
5.	P	Ass
6.	SHOW: R	DD
7.	$P \rightarrow Q$	2 $\leftrightarrow O$
8.	$Q \rightarrow R$	1 $\leftrightarrow O$
9.	Q	5,7 $\rightarrow O$
10.	R	8,9 $\rightarrow O$
11.	SHOW: $R \rightarrow P$	CD
12.	R	Ass
13.	SHOW: P	DD
14.	$Q \rightarrow P$	2 $\leftrightarrow O$
15.	$R \rightarrow Q$	1 $\leftrightarrow O$
16.	Q	12,14 $\rightarrow O$
17.	P	14,16 $\rightarrow O$
18.	$P \leftrightarrow R$	4,11 $\leftrightarrow I$

Lastly, there is a special rule, called “R” for “Repetition”, that we can use if, for some reason, we need to repeat something we already know on a later line—e.g., if needed on a line within a sub-proof. Example:

1. A		Pr
2. SHOW: B → A		CD
3. B		Ass
4. SHOW: A		DD
5. A		1 R

This is rule is not needed very often.

F. INDIRECT DERIVATION (ID)

Indirect derivation, like conditional derivation, is a wholly new way of setting up a proof.

Indirect derivation is used to show that something couldn’t possibly be true, because if it were true, it would lead to something absurd or impossible.

To do an ID, we first assume that something is true. We then show that this assumption leads to a *contradiction*, i.e., it leads to something being both true and false.

We then conclude that what we assumed to be true must not be true.

We now introduce a special symbol, ‘✖’, which we use to signify “the absurd” or the “impossible”.

We then allow ourselves to introduce this in a proof, whenever we get a contradiction, by means of the following rule (✖-IN):

p	
$\sim p$	✖I
✖	

The basic structure of an indirect proof is to assume the opposite of what we’re trying to prove, and derive a contradiction. I.e., to show $\sim p$, we assume p , and show ✖.

Let us take as our example:

1. A → B	Pr
2. A → ~B	Pr
3. SHOW: ~A	

We set this up by assuming P and adding a new SHOW: line.

1. A → B	Pr
2. A → ~B	Pr
3. SHOW: ~A	ID
4. A	Ass
5. SHOW: ✖	DD

We can show ✖ by showing any contradiction. (Any contradiction will do. It need not be the opposite of what we assumed.)

1. A → B	Pr
2. A → ~B	Pr
3. SHOW: ~A	ID
4. A	Ass
5. SHOW: ✖	DD
6. B	1,4 →O
7. ~B	2,4 →O
8. ✖	6,7 ✖I

Having successfully shown that a contradiction results when we assume A is true, we can safely conclude that A is false, by ID.

1. A → B	Pr
2. A → ~B	Pr
3. SHOW: ~A	ID
4. A	Ass
5. SHOW: ✖	DD
6. B	1,4 →O
7. ~B	2,4 →O
8. ✖	6,7 ✖I

Indirect Derivation is very powerful. It can be used for almost any type of problem, and even when what we’re trying to prove is complex.

Here are some additional examples.

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1.	$(P \leftrightarrow Q) \vee \sim A$	Pr
2.	$A \rightarrow P$	Pr
3.	$\sim Q$	Pr
4.	SHOW: $\sim A$	ID
5.	A	Ass
6.	SHOW: \times	DD
7.	P	2,5 \rightarrow O
8.	$\sim \sim A$	5 DN
9.	$P \leftrightarrow Q$	1,8 vO
10.	$P \rightarrow Q$	9 \leftrightarrow O
11.	$\sim P$	3,10 \rightarrow O
12.	\times	7,11 \times I

(There are quite often multiple ways of doing these problems. E.g., on this last one, at line 11, we could have used 7 and 10 to get Q instead of 3 and 10 to get $\sim P$. This would also have given us a different contradiction (lines 3 and 11). Either way you do it is fine.)

1.	$P \rightarrow (N \ \& \ M)$	Pr
2.	$Q \vee \sim N$	Pr
3.	SHOW: $\sim(P \ \& \ \sim Q)$	ID
4.	$P \ \& \ \sim Q$	Ass
5.	SHOW: \times	DD
6.	P	4 &O
7.	$\sim Q$	4 &O
8.	$N \ \& \ M$	1,6 \rightarrow O
9.	$\sim N$	2,7 vO
10.	N	8 &O
11.	\times	9,10 \times I

There are, strictly speaking, two forms of indirect proof. The above are all examples of *assuming* that something is *true* in order to prove that it is *false*.

We can also *assume* that something is *false* in order to *prove* that it is *true*.

This form works almost exactly the same.

Here are two examples.

1.	$P \vee Q$	Pr
2.	$Q \rightarrow P$	Pr
3.	SHOW: P	ID
4.	$\sim P$	Ass
5.	SHOW: \times	DD

6.	Q	1,4 vO
7.	P	2,6 \rightarrow O
8.	\times	4,7 \times I
1.	$S \rightarrow (M \vee N)$	Pr
2.	$\sim S \rightarrow (S \ \& \ P)$	Pr
3.	SHOW: $M \vee N$	ID
4.	$\sim(M \vee N)$	Ass
5.	SHOW: \times	DD
6.	$\sim S$	1,4 \rightarrow O
7.	$S \ \& \ P$	2,6 \rightarrow O
8.	S	8 &O
9.	\times	6,8 \times I

We can do indirect derivations within conditional derivations (or, more rarely, vice-versa).

1.	$P \rightarrow [Q \vee (R \ \& \ \sim P)]$	Pr
2.	SHOW: $P \rightarrow Q$	CD
3.	P	Ass
4.	SHOW: Q	ID
5.	$\sim Q$	Ass
6.	SHOW: \times	DD
7.	$Q \vee (R \ \& \ \sim P)$	1,3 \rightarrow O
8.	$R \ \& \ \sim P$	5,7 vO
9.	$\sim P$	8 &O
10.	\times	3,9 \times I

1.	$R \rightarrow \sim[P \rightarrow (P \ \& \ Q)]$	Pr
2.	Q	Pr
3.	SHOW: $\sim R$	ID
4.	R	Ass
5.	SHOW: \times	DD
6.	$\sim[P \rightarrow (P \ \& \ Q)]$	1,4 \rightarrow O
7.	SHOW: $P \rightarrow (P \ \& \ Q)$	CD
8.	P	Ass
9.	SHOW: $P \ \& \ Q$	DD
10.	P & Q	2,7 &I
11.	\times	6,7 \times I

(Remember we can introduce SHOW lines whenever we wish.)

Often, it is helpful to do two indirect derivations in order to prove that a conjunction (AND-statement) is true.

1.	$A \rightarrow (B \ \& \ \sim B)$	Pr
2.	$C \vee [(D \vee \sim C) \rightarrow C]$	Pr

3.	SHOW: C & ~A	DD
4.	SHOW: C	ID
5.	~C	Ass
6.	SHOW: ✖	DD
7.	(D v ~C) → C	2,5 vO
8.	D v ~C	5 vI
9.	C	7,8 →O
10.	✖	5,8 ✖I
11.	SHOW: ~A	ID
12.	A	Ass
13.	SHOW: ✖	DD
14.	B & ~B	1,12 →O
15.	B	14 &O
16.	~B	14 &O
17.	✖	15,16 ✖I
18.	C & ~A	4,11 &I

1.	Q → P	Pr
2.	~Q → R	Pr
3.	SHOW: P v R	ID
4.	~(P v R)	Ass
5.	SHOW: ✖	DD
6.	SHOW: ~Q	ID
7.	Q	Ass
8.	SHOW: ✖	DD
9.	P	1,7 →O
10.	P v R	9 vI
11.	✖	4,10 ✖I
12.	R	2,6 →O
13.	P v R	12 vI
14.	✖	4,13 ✖I

G. NEW DIRECT DERIVATION RULES (THE NEGATION RULES)

Just using the rules we have already learned, we can demonstrate the validity of any valid argument in sentential logic.

However, if we stick only to what we have so far, some problems require making moves that are rather strange and awkward.

Consider the following problem:

1.	Q → P	Pr
2.	~Q → R	Pr
3.	SHOW: P v R	

There is no obvious way to do this problem.

The conclusion is not a conditional statement, so it seems to make little sense to do CD.

We could try an ID.

If we assume ~(P v R) and try to get a contradiction, however, we get stuck immediately.

It is possible to do this problem with our rules so far, but it requires some very unusual and creative moves:

To make our lives easier, we introduce some special rules dealing with negations of complex statements.

$\frac{\sim(p \vee q)}{\sim p}$	$\frac{\sim(p \vee q)}{\sim q}$	$\sim vO$
$\frac{\sim(p \rightarrow q)}{p \& \sim q}$	$\sim \rightarrow O$	
$\frac{\sim(p \& q)}{p \rightarrow \sim q}$	$\sim \& O$	
$\frac{\sim(p \leftrightarrow q)}{\sim p \leftrightarrow q}$	$\sim \leftrightarrow O$	

With these rules, the above proof simplifies to:

1.	Q → P	Pr
2.	~Q → R	Pr
3.	SHOW: P v R	ID
4.	~(P v R)	Ass
5.	SHOW: ✖	DD
6.	~P	4 ~vO
7.	~R	4 ~vO
8.	~Q	1,6 →O
9.	R	2,8 →O
10.	✖	7,9 ✖I

Some other derivations that use these rules.

1.	$(\sim A \rightarrow B) \rightarrow C$	Pr
2.	SHOW: $B \rightarrow C$	CD
3.	B	Ass
4.	SHOW: C	ID
5.	$\sim C$	Ass
6.	SHOW: \times	DD
7.	$\sim(\sim A \rightarrow B)$	1,5 $\rightarrow O$
8.	$\sim A \ \& \ \sim B$	7 $\sim \rightarrow O$
9.	$\sim B$	8 $\& O$
10.	\times	3,9 $\times I$

1.	$(P \leftrightarrow Q) \rightarrow M$	Pr
2.	$M \vee P$	Pr
3.	$\sim(\sim M \ \& \ \sim Q)$	Pr
4.	SHOW: M	ID
5.	$\sim M$	Ass
6.	SHOW: \times	DD
7.	$\sim(P \leftrightarrow Q)$	1,5 $\rightarrow O$
8.	$\sim P \leftrightarrow Q$	7 $\sim \leftrightarrow O$
9.	P	2,5 $\vee O$
10.	$Q \rightarrow \sim P$	8 $\leftrightarrow O$
11.	$\sim \sim P$	9 DN
12.	$\sim Q$	10,11 $\rightarrow O$
13.	$\sim M \rightarrow \sim \sim Q$	3 $\sim \& O$
14.	$\sim \sim Q$	5,13 $\rightarrow O$
15.	\times	12,14 $\times I$

SHOW line	Form	Try	Assume	Show
simple statement	A	ID	$\sim A$	\times
negation	$\sim p$	ID	p	\times
disjunction	$p \vee q$	ID	$\sim(p \vee q)$	\times
conditional	$p \rightarrow q$	CD	p	q

For 'and' and 'iff' statements, break the problem into two parts, and put them together at the end with $\&I$ or $\leftrightarrow I$.

SHOW line	Form	Try	Assume	Show
conjunction	$p \ \& \ q$	DD		
	SHOW: p	ID	$\sim p$	\times
	SHOW: q	ID	$\sim q$	\times
biconditional	$p \leftrightarrow q$	DD		
	SHOW: $p \rightarrow q$	CD	p	q
	SHOW: $q \rightarrow p$	CD	q	p

This should tell you how to set up any problem:

- Look at your SHOW line, figure out which kind of derivation works best and then make your assumption and write your new SHOW line.
- Look at your new SHOW line and repeat the procedure until you get SHOW \times .
- Now, use the DD rules to attempt to get a contradiction.

This will work for almost all problems.

Examples:

1.	$A \rightarrow B$	Pr
2.	$\sim C \rightarrow D$	Pr
3.	$\sim B \vee \sim D$	Pr
4.	SHOW: $C \vee \sim A$	ID
5.	$\sim(C \vee \sim A)$	Ass
6.	SHOW: \times	DD
7.	$\sim C$	5 $\sim \vee O$
8.	$\sim \sim A$	5 $\sim \vee O$
9.	A	8 DN
10.	B	1,9 $\rightarrow O$
11.	D	2,7 $\rightarrow O$
12.	$\sim \sim B$	10 DN
13.	$\sim D$	3,12 $\vee O$
14.	\times	11,13 $\times I$

H. STRATEGIES AND RULES OF THUMB

When you begin a derivation, you might be unsure of the best way to go about it. Should you do a CD, ID or just try it by DD? Would you need to do some combination of these? When should you do one technique within another?

The simplest rule is: *when in doubt, try an indirect derivation.*

This is because all problems *can* be done with ID. Some problems are shorter if you don't use ID, but if you can't see that far ahead, it almost never hurts to try.

To make things more specific, when it doubt, I recommend constructing a derivation using the following strategies, depending on what kind of statement in your SHOW line:

1.	$(\sim P \vee Q) \rightarrow P$	Pr
2.	$(R \& P) \rightarrow \sim R$	Pr
3.	SHOW: $P \& \sim R$	DD
4.	SHOW: P	ID
5.	$\sim P$	Ass
6.	SHOW: ✖	DD
7.	$\sim P \vee Q$	5 vI
8.	P	1,7 \rightarrow O
9.	✖	5,8 ✖I
10.	SHOW: $\sim R$	ID
11.	R	Ass
12.	SHOW: ✖	DD
13.	$R \& P$	4,10 &I
14.	$\sim R$	2,13 \rightarrow O
15.	✖	11,14 ✖I
16.	$P \& \sim R$	4,10 &I

[Remember you can use a SHOW: line after SHOW: has been crossed out.]

1.	$P \leftrightarrow (\sim R \rightarrow R)$	Pr
2.	SHOW: $(P \vee \sim Q) \leftrightarrow (Q \rightarrow R)$	DD
3.	SHOW: $(P \vee \sim Q) \rightarrow (Q \rightarrow R)$	CD
4.	$P \vee \sim Q$	Ass
5.	SHOW: $Q \rightarrow R$	CD
6.	Q	Ass
7.	SHOW: R	ID
8.	$\sim R$	Ass
9.	SHOW: ✖	DD
10.	$\sim \sim Q$	6 DN
11.	P	4,10 vO
12.	$P \rightarrow (\sim R \rightarrow R)$	1 \leftrightarrow O
13.	$\sim R \rightarrow R$	11,12 \rightarrow O
14.	R	8,13 \rightarrow O
15.	✖	8,14 ✖I
16.	SHOW: $(Q \rightarrow R) \rightarrow (P \vee \sim Q)$	CD
17.	$Q \rightarrow R$	Ass
18.	SHOW: $P \vee \sim Q$	ID
19.	$\sim(P \vee \sim Q)$	Ass
20.	SHOW: ✖	DD
21.	$\sim P$	19 \sim vO
22.	$\sim \sim Q$	19 \sim vO
23.	$(\sim R \rightarrow R) \rightarrow P$	1 \leftrightarrow O
24.	$\sim(\sim R \rightarrow R)$	21,23 \rightarrow O
25.	$\sim R \& \sim R$	24 $\sim \rightarrow$ O
26.	$\sim R$ 25	&O
27.	$\sim Q$	17,26 \rightarrow O
28.	✖	22,27 ✖I
29.	$(P \vee \sim Q) \leftrightarrow (Q \rightarrow R)$	3,16 \leftrightarrow I

I. WHAT TO DO WHEN YOU GET STUCK: "DESPERATE MEASURES" STRATEGIES

Following the guidelines suggested above will get you through just about any problem. But there is a certain kind of problem where you still might get stuck in the middle, and you'll have to try something desperate to get out of being stuck.

Here is an example of that kind of problem.

1.	$(P \rightarrow R) \vee (Q \rightarrow S)$	Pr
2.	$P \& Q$	Pr
3.	SHOW: $R \vee S$	

If you follow the guidelines, you'll begin by starting an ID:

1.	$(P \rightarrow R) \vee (Q \rightarrow S)$	Pr
2.	$P \& Q$	Pr
3.	SHOW: $R \vee S$	ID
4.	$\sim(R \vee S)$	Ass
5.	SHOW: ✖	DD

At this point, there are *some* things we can do...

1.	$(P \rightarrow R) \vee (Q \rightarrow S)$	Pr
2.	$P \& Q$	Pr
3.	SHOW: $R \vee S$	ID
4.	$\sim(R \vee S)$	Ass
5.	SHOW: ✖	DD
6.	P	2 &O
7.	Q	2 &O
8.	$\sim R$	4 \sim vO
9.	$\sim S$	4 \sim vO

But, uh oh, now what? There doesn't seem to be anywhere to go from here.

When you get stuck like this, you need to ask yourself: *what would be helpful? what resources do you have that you would like to make use of, but can't?*

Usually this will lead you to one or more lines that are just begging to be used. In this case, it's line 1, which hasn't been used so far.

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Line 1 is an OR statement, which means, if we were going to make use of it, we'd have to make use of the vO rule.

To make use of the vO rule, we'd have to have that one of the two sides of the OR-statement is false. Let's try to prove just that!

That is, let's try to prove $\sim(P \rightarrow R)$. Let's go ahead and write that in as a new SHOW line. Then we can do it by ID.

Lo and behold, it works! Once we have it, we can finish the rest of the derivation.

1.	$(P \rightarrow R) \vee (Q \rightarrow S)$	Pr
2.	P & Q	Pr
3.	SHOW: $R \vee S$	ID
4.	$\sim(R \vee S)$	Ass
5.	SHOW: \times	DD
6.	P	2 & O
7.	Q	2 & O
8.	$\sim R$	4 \sim VO
9.	$\sim S$	4 \sim VO
10.	SHOW: $\sim(P \rightarrow R)$	ID
11.	$P \rightarrow R$	Ass
12.	SHOW: \times	DD
13.	R	6,11 \rightarrow O
14.	\times	8,13 \times I
15.	$Q \rightarrow S$	1,10 vO
16.	S	7,15 \rightarrow O
17.	\times	9,16 \times I

Basically, when you get stuck, look for a way you can set up either doing a vO step (like we did here at line 15), or a \rightarrow O, by proving either the negation of half of an OR-statement, or by proving the if-part of an if-then statement, or the negation of a then-part of an if-then statement.

Introduce a new SHOW line if necessary, so you can do an ID or CD for the thing that you need to set up the desired move.

Here are some more examples of such "desperate measures" strategies.

1.	$(D \vee \sim D) \rightarrow (A \rightarrow B)$	Pr
2.	SHOW: $\sim B \rightarrow \sim A$	CD
3.	$\sim B$	Ass
4.	SHOW: $\sim A$	ID
5.	A	Ass
6.	SHOW: \times	DD
7.	SHOW: $D \vee \sim D$	ID
8.	$\sim(D \vee \sim D)$	Ass
9.	SHOW: \times	DD
10.	$\sim D$	8 \sim VO
11.	$\sim\sim D$	8 \sim VO
12.	\times	10,11 \times I
13.	$A \rightarrow B$	1,7 \rightarrow O
14.	B	5,13 \rightarrow O
15.	\times	3,14 \times I

1.	$(P \rightarrow Q) \vee (R \leftrightarrow \sim R)$	Pr
2.	SHOW: $\sim P \vee Q$	ID
3.	$\sim(\sim P \vee Q)$	Ass
4.	SHOW: \times	DD
5.	$\sim\sim P$	3 \sim VO
6.	$\sim Q$	3 \sim VO
7.	SHOW: $\sim(P \rightarrow Q)$	ID
8.	$P \rightarrow Q$	Ass
9.	SHOW: \times	DD
10.	$\sim P$	6,8 \rightarrow O
11.	\times	5,10 \times I
12.	$R \leftrightarrow \sim R$	1,7 vO
13.	SHOW: R	ID
14.	$\sim R$	Ass
15.	SHOW: \times	DD
16.	$\sim R \rightarrow R$	12 \leftrightarrow O
17.	R	14,16 \rightarrow O
18.	\times	14,17 \times I
19.	$R \rightarrow \sim R$	12 \leftrightarrow O
20.	$\sim R$	13,19 \rightarrow O
21.	\times	13,20 \times I

Put these strategies together with the general rules of thumb considered earlier, and you should be able to do any problem you come across!

J. SOME REVIEW PROBLEMS

Here are some additional derivations for review purposes.

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1.	$M \leftrightarrow N$	Pr	1.	$K \rightarrow [(M \vee N) \rightarrow (P \& Q)]$	Pr
2.	SHOW: $\sim N \leftrightarrow \sim M$	DD	2.	$L \rightarrow [(Q \& K) \rightarrow (S \& \sim N)]$	Pr
3.	SHOW: $\sim N \rightarrow \sim M$	CD	3.	SHOW: $(K \& L) \rightarrow \sim N$	CD
4.	$\sim N$	Ass	4.	$K \& L$	Ass
5.	SHOW: $\sim M$	DD	5.	SHOW: $\sim N$	ID
6.	$M \rightarrow N$	$1 \leftrightarrow O$	6.	N	Ass
7.	$\sim M$	$4,6 \rightarrow O$	7.	SHOW: \times	DD
8.	SHOW: $\sim M \rightarrow \sim N$	CD	8.	K	$4 \& O$
9.	$\sim M$	Ass	9.	L	$4 \& O$
10.	SHOW: $\sim N$	DD	10.	$(M \vee N) \rightarrow (P \& Q)$	$1,8 \rightarrow O$
11.	$N \rightarrow M$	$1 \leftrightarrow O$	11.	$(Q \& K) \rightarrow (S \& \sim N)$	$2,9 \rightarrow O$
12.	$\sim N$	$9,11 \rightarrow O$	12.	$M \vee N$	$6 \vee I$
13.	$\sim N \leftrightarrow \sim M$	$3,8 \leftrightarrow I$	13.	$P \& Q$	$10,12 \rightarrow O$
			14.	Q	$13 \& O$
1.	$\sim(\sim B \vee \sim C) \rightarrow \sim A$	Pr	15.	$Q \& K$	$8,14 \& I$
2.	SHOW: $A \rightarrow (B \rightarrow \sim C)$	CD	16.	$S \& \sim N$	$11,15 \rightarrow O$
3.	A	Ass	17.	$\sim N$	$16 \& O$
4.	SHOW: $B \rightarrow \sim C$	CD	18.	\times	$6,17 \times I$
5.	B	Ass			
6.	SHOW: $\sim C$	DD	1.	$(M \rightarrow N) \rightarrow (K \vee \sim J)$	Pr
7.	$\sim \sim A$	3 DN	2.	N	Pr
8.	$\sim \sim(\sim B \vee \sim C)$	$1,7 \rightarrow O$	3.	SHOW: $J \rightarrow K$	CD
9.	$\sim B \vee \sim C$	8 DN	4.	J	Ass
10.	$\sim \sim B$	5 DN	5.	SHOW: K	DD
11.	$\sim C$	$9,10 \vee O$	6.	SHOW: $M \rightarrow N$	CD
			7.	M	Ass
1.	$P \vee (Q \& R)$	Pr	8.	SHOW N	DD
2.	SHOW: $(P \vee Q) \& (P \vee R)$	DD	9.	N	2 R
3.	SHOW: $P \vee Q$	ID	10.	$K \vee \sim J$	$1,6 \rightarrow O$
4.	$\sim(P \vee Q)$	Ass	11.	$\sim \sim J$	4 DN
5.	SHOW: \times	DD	12.	K	$10,11 \vee O$
6.	$\sim P$	$4 \sim \vee O$			
7.	$\sim Q$	$4 \sim \vee O$	1.	$F \rightarrow [(C \rightarrow C) \rightarrow G]$	Pr
8.	$Q \& R$	$1,6 \vee O$	2.	$G \rightarrow \{[H \rightarrow (E \rightarrow H)] \rightarrow (K \& \sim K)\}$	Pr
9.	Q	$8 \& O$	3.	SHOW: $\sim F$	ID
10.	\times	$7,9 \times I$	4.	F	Ass
11.	SHOW: $P \vee R$	ID	5.	SHOW: \times	DD
12.	$\sim(P \vee R)$	Ass	6.	$(C \rightarrow C) \rightarrow G$	$1,4 \rightarrow O$
13.	SHOW: \times	DD	7.	SHOW: $C \rightarrow C$	CD
14.	$\sim P$	$12 \sim \vee O$	8.	C	Ass
15.	$\sim R$	$12 \sim \vee O$	9.	SHOW: C	DD
16.	$Q \& R$	$1,14 \vee O$	10.	C	8 R
17.	R	$16 \& O$	11.	G	$6,7 \rightarrow O$
18.	\times	$15,17 \times I$	12.	$[H \rightarrow (E \rightarrow H)] \rightarrow (K \& \sim K)$	$2,11 \rightarrow O$
19.	$(P \vee Q) \& (P \vee R)$	$3,11 \& I$	13.	SHOW: $H \rightarrow (E \rightarrow H)$	CD
			14.	H	Ass
			15.	SHOW: $E \rightarrow H$	CD
			16.	E	Ass
			17.	SHOW: H	DD

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18.					H	14 R
19.					K & ~K	12,13 →O
20.					K	19 &O
21.					~K	19 &O
22.					⊗	20,21 ⊗I

Here's a short, interesting problem that involves proving a conclusion without using *any* premises!

1.					SHOW: $P \rightarrow [(P \rightarrow Q) \rightarrow Q]$	CD
2.					P	Ass
3.					SHOW: $(P \rightarrow Q) \rightarrow Q$	CD
4.					$P \rightarrow Q$	Ass
5.					SHOW: Q	DD
6.					Q	2,4 →O

That's all there is to Unit 2. Time to *dance*.