Introduction to Logic

Lecture Notes, Unit 3

General caveat about online notes: These notes are provided in case you miss anything. Reading them on their own is poor substitute for attending class, for a number of reasons:
(i) What is taught in this course are *processes* for solving certain problems. These notes do not show the processes, but only the end results, and cannot substitute for seeing the processes in action, and may be difficult to understand on their own.
(ii) These notes constitute only what I'm likely to write or project on the screen; they do not contain nearly everything I say in lecture that it might be helpful for you to know.
(iii) These notes are from last time I taught the course, and I reserve the right to change things this semester.

A. LIMITATIONS OF SENTENTIAL LOGIC

In sentential logic, a letter (A, B, C, etc.) is used to represent an *entire* simple statement.

Hence, it cannot represent common features between different simple statements.

Example 1:
George W. Bush has driven drunk.

This statement shares common elements with other statements:

Examples 2-5:
George W. Bush is a politician.
(Same subject, different predicate.)
Lindsay Lohan has driven drunk.
(Same predicate, different subject.)
Everyone has driven drunk.
(Same predicate, general subject.)
Someone has driven drunk.
(Same predicate, indefinite subject.)

Sentential logic treats these all as simple statements (e.g., G, P, T, E, S, etc.), and so loses any connection between them.

However, this argument is valid intuitively:

George W. Bush has driven drunk.
Therefore, someone has driven drunk.

However the argument:

$$\frac{G}{S}$$

is obviously invalid in sentential logic.

We now introduce a new style of symbolic logic, called **predicate logic**.

B. SUBJECTS AND PREDICATES

In predicate logic, we use *lowercase* letters (except x, y and z, which we'll explain later) to represent *specific individual* persons, places or things.

These are **subjects** of statements.

English	Predicate logic
Amsterdam	a
Kevin	k
Britney Spears	b
the Pope	p
the number five (or just 5)	f

Warning: plural nouns are *not* translated with lowercase letters.

We use *uppercase* letters to represent things *that can be said about* the things above.

(We no longer use them for whole statements.)

These are called ***predicates***.

English	Predicate logic
_____ is hungry	H
_____ plays tennis	T
_____ has driven drunk	D
_____ is prime	P
_____ loves …	L

We put these together, capital letters first, to form statements.

Statement	English meaning
Hk	Kevin is hungry.
Hb	Britney Spears is hungry.
Tb	Britney Spears plays tennis.
Pf	5 is prime.
Lpb	The pope loves Britney Spears.

Monadic and Polyadic Predicates

Some predicates apply to more than one thing. With these, the order usually matters.

(Such predicates are called **polyadic** or relational.)

‘Lpb’ means:
The pope loves Britney Spears.
‘Lbp’ means:
Britney Spears loves the pope.

In principle, predicates can apply to any number of things:

Use ‘B; to mean
_____ *is between ……… and _ _ _ _ _.*

Then ‘Bsfn’ might mean
Seven is between five and nine.

‘and’ with polyadic predicates

In unit 1, we saw that sometimes when “and” occurs in a sentence between two names, it is possible to break it up.

Example 1:
Chandler and Monica are hungry.

This really means:
Chandler is hungry and Monica is hungry.
And becomes: Hc & Hm

However,
Example 2:
Chandler and Monica are engaged.

This does not mean the same as:
Chandler is engaged and Monica is engaged.

Instead, it means:
Chandler is engaged to Monica.

In predicate logic, we can write this as:
Ecm

In general, when statements with composite subjects can’t be broken up into two statements, a polyadic predicate is involved.

C. COMPOUND STATEMENTS

The logical connectives v, →, ↔, & and ~ are also used in predicate logic, as before.

Example 1:
Peter is a Republican only if he is ignorant.
Becomes: Rp → Ip

Example 2:
Unless Peter plays tennis, then Alison loves him only if he does not drive drunk.
Becomes: ~Tp → (Lap → ~Dp)

Example 3:
If neither Ross nor Phoebe are Hungry, then Monica will not cook.
Becomes: (~Hr & ~Hp) → ~Cm OR
~(Hr v Hp) → ~Cm

Sometimes “~” can be used to translate prefixes like “non”, “un”, “in” and “im”.

Example 4:
Alison is unkind if and only if Peter is impolite.
Becomes: ~Ka ↔ ~Pp

D. VARIABLES AND QUANTIFIERS

Not all statements are about specific individuals, but about all or some unspecified members of a group:

All dentists floss.
Some politicians are alcoholics.
Everyone loves Björk.

To capture these statements, we borrow the idea of **variables** from algebra.

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Mathematicians use variables to make statements about all numbers or some unknown number.

We do something similar, except our variables don't just stand for numbers, they can stand for *anything at all*.

The letters *x*, *y* and *z* are variables.

They are used in place of a lowercase letter like 'a' or 'b' for some specific thing.

If "Hk" means "Kevin is hungry", then "Hx" means something like "it is hungry", where we don't know who or what that 'it' is.

Such statements containing variables are ambiguous as is.

To make them unambiguous, we use **quantifiers**. These are new logical symbols:

(1) **The Universal Quantifier**
∀x means *for all things x, it is true that …*

(2) **The Existential Quantifier**
∃x means *there exists at least one thing x such that…*

So ∃xHx
means
There exists at least one thing x such that x is hungry.
or, more loosely
Something is hungry.

And ∀xBx
means
for all things x it is true that x is beautiful.
or, more loosely
Everything is beautiful.

• ∀xBx means that *all* of Ba, Bb, Bc, Bd, Be, etc., are true. [Al is beautiful, Britney is beautiful, Chicago is beautiful, etc.]
• ∃xBx means that at least one of Ba, Bb, Bc, Bd, Be., etc., are true.

<u>Using Quantifiers with Other Logical Signs</u>

Quantifiers are most useful when combined together with other logical signs.

<u>Universal Affirmative Statements</u>
A statement making an assertion about <u>every</u> member of some group is translated with a universal quantifier and a conditional:

All dentists floss.
Every dentist flosses.
Any dentist flosses.
Each dentist flosses.

These all become: ∀x(Dx → Fx)
I.e., *for all things x, if x is a dentist, then x flosses.*
This requires the truth of Da → Fa, Db → Fb: note these don't require the if-parts to be true.

Statements merely about "everything" (or "everyone") can be translated more simply.

Everything flosses. ∀xFx

<u>Particular Affirmative Statements</u>
Any statement asserting that some (unspecified) member(s) of a group is translated with a universal quantifier and conjunction sign:

Some politicians play tennis.
A few politicians play tennis.
At least one politician plays tennis.

These all become ∃x(Px & Tx)
I.e., *there is at least one x such that x is politician and x plays tennis.*

Other statements can be more simply translated.

Something is a tiger.
Tigers exist.

These become: ∃xTx

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Universal Negative Statements
A statement that something is true of *no* things in a certain group can be seen in one of two ways.

No politicians are ethical.

This can either be paraphrased as

for all x, if x is a politician, then x is NOT ethical.
i.e., ∀x(Px → ~Ex)

Or as:
It is NOT the case that there exists at least one x such that x. is a politician and x is ethical.
i.e., ~∃x(Px & Ex)

These two formulations are logically equivalent.

Particular Negative Statements
Here, we deny a certain predicate of some members of a group.

Some politicians are unethical.

This means roughly the same as:
Not all politicians are ethical.

And can be paraphrased as:
There exists at least one x such that x is politician and x is not ethical.
i.e., ∃x(Px & ~Ex)
Or as:
It is NOT the case that for all x, if x is politician, then x is ethical.
i.e., ~∀x(Px → Ex)

These formulations are also logically equivalent.

Watch parentheses and order carefully!
~∀x(…) and ∀x~(…) are not the same.
Nor are ∃x~(…) and ~∃x(…).
Nor are ∀x(… → ____) and (∀x … → ___), etc.

Some Example Translations

All tigers are mammals.
∀x(Tx → Mx)
No tigers play golf.
∀x(Tx → ~Gx)
Some politicians have not driven drunk.
∃x(Px & ~Dx)
Some golf players love Björk.
∃x(Gx & Lxb)
Nothing is moving.
∀x~Mx
Something is not moving.
∃x~Mx
If the *Mona Lisa* is not beautiful, then nothing is beautiful.
~Bm → ∀x~Bx
George W. Bush is happy only if no politicians are ethical.
Hg → ∀x(Px → ~Ex)

Combined Predicates

More than two predicates can be involved in a quantified statement.

Often they should be joined with the conjunction sign '&'. Consider:

Example 1:
Every rockstar is popular and sexy.
Becomes: ∀x[Rx → (Px & Sx)]
I.e. for all x, if x is a rockstar, then x is popular and x is sexy.

Sometimes such complex predicates are shortened into adjective phrases like "popular rockstar" or "orange vegetables", as in:

Example 2:
All popular rockstars are sexy.
Becomes: ∀x[(Px & Rx) → Sx]

Example 3:
Some carrots are orange vegetables.
Becomes: ∃x[Cx & (Ox & Vx)]

Complex predicates like these are put together in parentheses, and are not broken up, even when negations are involved.

Example 4:
No hamsters are endangered animals.
Becomes: $\sim\exists x[Hx \& (Ex \& Ax)]$
Or: $\forall x[Hx \rightarrow \sim(Ex \& Ax)]$

Example 5:
Some intelligent students are not pathetic geeks.
Becomes: $\exists x[(Ix \& Sx) \& \sim(Px \& Gx)]$
Or: $\sim\forall x[(Ix \& Sx) \rightarrow (Px \& Gx)]$

Things can't always be done this way.
- A "computer programmer" isn't someone who is a computer and is a programmer, but someone who programs computers.
- A "basketball player" isn't someone who is a basketball and a player, etc.

Disjunctive Combined Predicates

Sometimes disjunctions (or-clauses) are used to translate certain combined predicates.

Usually this will be obvious:

Example 1:
Anything that's plaid or striped is ugly.
Becomes: $\forall x[(Px \vee Sx) \rightarrow Ux]$

Example 2:
All athletes are either quick or strong.
Becomes: $\forall x[Ax \rightarrow (Qx \vee Sx)]$

With negations…

Example 3:
Some professors are not either quick or strong.
Becomes: $\exists x[Px \& \sim(Qx \vee Sx)]$
Or: $\sim\forall x[Px \rightarrow (Qx \vee Sx)]$

Interestingly, some statements that use the word 'and' in English can actually be translated with 'v'. Consider:

Example 4:
All doctors and lawyers are rich.

This is NOT: $\forall x[(Dx \& Lx) \rightarrow Rx]$

This says that *for all x, if x is BOTH a doctor AND a lawyer then x is rich*.

This is not what we want.

We want to say that *for all x, if x is a doctor OR a lawyer, then x is rich*.

Correct translation: $\forall x[(Dx \vee Lx) \rightarrow Rx]$

So where did the 'and' come from in the English version?
The above is logically equivalent to:
$\forall x(Dx \rightarrow Rx) \& \forall x(Lx \rightarrow Rx)$

'Only'

Only-statements, like *every*- and *some*-statements, are translated with quantifiers.

What does it mean to say that:
Only As are Bs ?

There are different ways to think about it.

On one way of thinking of it, it means that:
No non-As are Bs.

Example 1:
Only cats purr.
Becomes: $\forall x(\sim Cx \rightarrow \sim Px)$
I.e., *for all x, if x is not a cat, then x does not purr*.

This is logically equivalent to: $\forall x(Px \rightarrow Cx)$
Or: $\sim\exists x(Px \& \sim Cx)$

But NOT: $\forall x(Cx \rightarrow Px)$
(Every cat purrs.)

Therefore, 'only' is not the same as 'every' or 'all'. However, it can be thought of as the converse of 'every' and 'all'.

All As are Bs. = Only Bs are As.
$\forall x(Ax \rightarrow Bx)$
or $\forall x(\sim Bx \rightarrow \sim Ax)$
or $\sim\exists x(Ax \& \sim Bx)$

Ambiguities Involving 'Only'

'Only' creates ambiguities when complex predicates are involved.

Example 2:
Only happy cats purr.

This might be read as just about cats:
among cats, only the happy ones purr,
OR it might be about everything:
the only things that purr at all are happy cats.

The first reading becomes:
$\forall x[(Cx \& Px) \rightarrow Hx]$
or $\forall x[\sim Hx \rightarrow \sim(Cx \& Px)]$
or $\sim\exists x[(Cx \& Px) \& \sim Hx]$

The second reading becomes:
$\forall x[Px \rightarrow (Hx \& Cx)]$
or $\forall x[\sim(Hx \& Cx) \rightarrow \sim Px]$
or $\sim\exists x[Px \& \sim(Hx \& Cx)]$

'The Only'

These can also be tricky since *'the only' does not mean the same as 'only'.*

Usually, *only A are B* means the same as *the only B are A*.

Example 1:
The only likable teachers are philosophers.
Becomes: $\forall x[(Lx \& Tx) \rightarrow Px]$
Or: $\forall x[\sim Px \rightarrow \sim(Lx \& Tx)]$
Or: $\sim\exists x[\sim Px \& (Lx \& Tx)]$

Only is the converse of *all*. *The only* is the converse of *only*.
Therefore, *all* and *the only* mean the same thing!

'The'

Normally, statements using 'the' are about some specific thing, and are translated with constant letters rather than variables.

Example 1:
The dog ran away.
Becomes: Rd

However, sometimes statements with 'the' are really about every member of a group.

Example 2:
The dolphin is a mammal.
Becomes: $\forall x(Dx \rightarrow Mx)$

Missing Quantifiers

Sometimes an explicit quantifier word is simply left off a statement in English.
- These can mean different things, in different contexts.

Often, they mean the same as 'every' or 'all' statements; sometimes they mean the same as 'some' statements.

Example 1:
Frogs are amphibians.
Becomes: $\forall x(Fx \rightarrow Ax)$
(Missing *all.*)

Example 2:
Students work at the library.
Becomes: $\exists x(Sx \& Wxl)$
(Missing *some.*)

It can even mean 'only':
Example 3:
Employees are allowed in.
Becomes: $\forall x(Ax \rightarrow Ex)$

"All and Only"

In those rare cases in which the phrase "all and only" occurs as a single phrase, the statement can be translated with a universal quantifier and a biconditional.

Example 1:
All and only bachelors are allowed in.
Becomes: $\forall x(Bx \leftrightarrow Ax)$

Quantified Statements Joined Together

So far we've mainly been considering single quantified statements, or their negations.

However, entire quantified statements can be joined together with logical connectives like *and*, *or*, *if-then*, and *if and only if*.

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Example 1:
If nothing is permanent, then everything is illusory.
Becomes: ~∃xPx → ∀xIx

Example 2:
No Republicans are environmentalists but some Democrats are environmentalists.
Becomes: ~∃x(Rx & Ex) & ∃x(Dx & Ex)

Example 3:
If some students are confused, then all students are.
Becomes: ∃x(Sx & Cx) → ∀x(Sx → Cx)

Example 4:
If nothing is permanent and nothing is good, then all religions are fictional.
Becomes: (~∃xPx & ~∃xGx) → ∀x(Rx → Fx)

Example 5:
Unless the host cooks or some local restaurant delivers, no partygoer will be happy.
Becomes:
~{Ch v ∃x[(Lx & Rx) & Dx]} → ~∃x(Px & Hx)

Additional Example Translations

No stolen merchandise is returnable.
∀x[(Sx & Mx) → ~Rx]
Not all cheeseheads are drunk idiots.
~∀x[Cx → (Dx & Ix)]
Some cheeseheads are excellent dancers.
∃x(Cx & Ex)
All local researchers are chemists or biologists.
∀x[(Lx & Rx) → (Cx v Bx)]
Some chemists and biologists dance.
∃x(Cx & Dx) & ∃x(Bx & Dx)
Only bats are flying mammals.
∀x[(Fx & Mx) → Bx]
Only rabid dogs bite. (2 readings)
1: ∀x[(Dx & Bx) → Rx]
2: ∀x[Bx → (Rx & Dx)]
The only interesting courses are full.
∀x[(Ix & Cx) → Fx]
Everything is unique if nothing is synthetic.
∀x~Sx → ∀xUx
There are angels only if not everything is physical.
∃xAx → ~∀xPx
All species evolve unless no biologists are competent.
~∀x(Bx → ~Cx) → ∀x(Sx → Ex)
Everything is either mental or physical.
∀x(Mx v Px)
Either everything is mental or everything is physical.
∀xMx v ∀xPx
If all numbers are finite, then every number is either even or odd.
∀x(Nx → Fx) → ∀x[Nx → (Ex v Ox)]

E. TRANSLATIONS WITH POLYADIC PREDICATES

Polyadic predicates are those applied to more than one subject.

These are written with 2 or more names/variables:

Example 1:
Angelina loves Brad.
Becomes: Lab

Recall that for these, order matters.

Normally, we'll make the order match the natural English order:

Example 2:
Brad loves Angelina.
Becomes: Lba

We reverse positions for something written in passive voice (… is ___ed by…):

Example 3:
Brad is loved by Angelina.
Becomes: Lab
(same as "Angelina loves Brad")

Quantifiers with Polyadic Predicates

With polyadic predicates, there are many possible forms of quantification.

English	Translations
Everything loves Fergie.	$\forall xLxf$
Something loves Fergie.	$\exists xLxf$
Fergie loves everything.	$\forall xLfx$
Fergie loves something.	$\exists xLfx$
Nothing loves Fergie.	$\sim\exists xLxf$
	$\forall x\sim Lxf$
Fergie loves nothing.	$\sim\exists xLfx$
	$\forall x\sim Lfx$
Something doesn't love Fergie.	$\exists x\sim Lxf$
	$\sim\forall xLxf$
Fergie does not love something.	$\exists x\sim Lfx$
	$\sim\forall xLfx$
Not everything loves Fergie.	$\sim\forall xLxf$
Everything loves itself.	$\forall xLxx$
etc.	

Reflexive Constructions

When a polyadic predicate will be used along with a reflexive pronoun like 'itself', 'himself' or "herself", we simply repeat the lowercase letter.

Example 1:
Fergie loves herself.
Becomes Lff

Example 2:
Every singer who respects him/herself is happy.
Becomes $\forall x[(Sx \,\&\, Rxx) \rightarrow Hx]$

Constructions with the 'self-' prefix may be treated similarly:

Example 3:
No self-respecting musician admires Sanjaya.
Becomes: $\forall x[(Mx \,\&\, Rxx) \rightarrow \sim Axs]$

Some Additional Examples

Example 1:
All men love Fergie.
Becomes $\forall x(Mx \rightarrow Lxf)$

Example 2:
No beautiful women love Diddy.
Becomes $\forall x[(Bx \,\&\, Wx) \rightarrow \sim Lxd]$
OR: $\sim\exists x[(Bx \,\&\, Wx) \,\&\, Lxd]$

Example 3:
All men love themselves.
Becomes $\forall x(Mx \rightarrow Lxx)$

Example 4:
Only morons love Diddy.
Becomes $\forall x(\sim Mx \rightarrow \sim Lxd)$

Example 5:
Something Diddy loves loves itself.
Becomes $\exists x(Ldx \,\&\, Lxx)$

Quantifier Movement

When the quantifier word in the English occurs in the middle of the statement, try rewording it with the quantifier out in front.

Usually, this will result in a switch from active to passive voice

Example 1:
Diddy loves all guns.
This is the same as:
All guns are loved by Diddy.
Becomes $\forall x(Gx \rightarrow Ldx)$

Example 2:
Angelina loves nothing that loves Brad.
This is the same as:
Nothing that loves Brad is loved by Angelina.
Becomes $\forall x(Lxb \rightarrow \sim Lax)$
OR: $\sim\exists x(Lxb \,\&\, Lax)$

Multiple Quantification with Polyadic Predicates

Often, when polyadic predicates are involved, both sides of the relation can be quantified.

Here, because one quantifier can appear within the scope of another, use different variables (e.g., 'x' amd 'y') for each one.

English	Translations
Something loves something.	$\exists x\exists yLxy$
Everything loves everything.	$\forall x\forall yLxy$
Something loves everything.	$\exists x\forall yLxy$
*Everything loves something.**	$\forall x\exists yLxy$

Everything is loved by everything.	$\forall x \forall y Lyx$
Something is loved by something.	$\exists x \exists y Lyx$
*Something is loved by everything.**	$\exists x \forall y Lyx$
*Everything is loved by something.**	$\forall x \exists y Lyx$
Something loves nothing.	$\exists x \sim \exists y Lxy$
	$\exists x \forall y \sim Lxy$
Nothing loves everything.	$\forall x \sim \forall y Lxy$
	$\sim \exists x \forall y Lxy$
Something does not love everything.	
	$\exists x \sim \forall y Lxy$
Nothing loves anything.	$\forall x \forall y \sim Lxy$
	$\sim \exists x \exists y Lxy$

etc.

<u>More Complicated Forms of Multiple Quantification</u>

These are perhaps best seen by example.

Example 1:
Every rockstar loves something.
Becomes $\forall x(Rx \rightarrow \exists y Lxy)$

Example 2:
Something is loved by every rockstar.
Becomes $\exists x \forall y(Ry \rightarrow Lyx)$

Example 3:
Every man loves every woman.
Becomes $\forall x[Mx \rightarrow \forall y(Wy \rightarrow Lxy)]$

Example 4:
Every beautiful woman loves some man (or other).
Becomes $\forall x[(Bx \& Wx) \rightarrow \exists y(My \& Lxy)]$

Example 5:
Every beautiful woman is loved by some man (or other).
Becomes $\forall x[(Bx \& Wx) \rightarrow \exists y(My \& Lyx)]$

Example 6:
Some men are not loved by every woman.
Becomes $\exists x[Mx \& \sim \forall y(Wy \rightarrow Lyx)]$

Example 7:
No beautiful women love every man.
Becomes $\forall x[(Bx \& Wx) \rightarrow \sim \forall y(My \rightarrow Lxy)]$
OR: $\sim \exists x[(Bx \& Wx) \& \forall y(My \rightarrow Lxy)]$

<u>Restrictive Clauses</u>

The words *that*, *which*, *who*, and *whom* are used to introduce **restrictive clauses**.

These operate much like complex predicates.

Example 1:
All rockstars that dance are sexy.
Same as:
All dancing rockstars are sexy.
Becomes $\forall x[(Rx \& Dx) \rightarrow Sx]$

Example 2:
Some rockstars that love Diddy dance.
Same as:
Some Diddy-loving rockstars dance.
Becomes $\exists x[(Rx \& Lxd) \& Dx]$

Such clauses are usually translated with '&', conjoined together with the predicate that they restrict.

An exception occurs when such clauses follow words like 'anything' and 'everything', in which case they simply take the place of the predicate they would otherwise restrict.

Example 3:
Everything that loves Fergie dances.
Same as:
Every Fergie-lover dances.
Becomes $\forall x(Lxf \rightarrow Dx)$

Example 4:
Something that Diddy owns is not a gun.
Same as:
Some Diddy-owned-thing is not a gun.
Becomes $\exists x(Odx \& \sim Gx)$

<u>*Quantified Restrictive Clauses*</u>

Restrictive clauses can get very complex, and even include quantified phrases.

In such cases, it is usually best to break the problem into steps.

Example 1:
Everything that loves all women loves Fergie.

Overall, this is an *all*-statement, which gets translated with '∀' along with '→'.

Mixing English and symbolic logic, we can do our first step as:

∀x[x loves all women → x loves Fergie]

Now becomes: *x loves all women*
∀y(Wy → Lxy)

And becomes: *x loves Fergie*
Lxf

Together we get: ∀x[∀y(Wy → Lxy) → Lxf]

Example 2:
Some rockstars whom all women love dance.

STEP 1: ∃x{x is a rockstar whom all women love & x dances}
STEP 2: ∃x{[Rx & all women love x] & Dx}
STEP 3: ∃x{[Rx & ∀y(Wy → Lyx)] & Dx}

Example 3.
Some rockstars who own some guns are loved by no women.

STEP 1: ∃x{x is rockstar who owns some guns & x is loved by no women}
STEP 2: ∃x{(x is rockstar & some guns are owned by x) & no women love x}
STEP 3: ∃x{[Rx & some guns are owned by x] & ∀y(Wy → ~Lyx)}
STEP 4: ∃x{[Rx & ∃y(Gy & Oxy)] & ∀y(Wy → ~Lyx)}

Example 4.
Every jabberwock that only outgribes those things that every bandernatch outgribes gimbles no toves that gimble themselves.

STEP 1: ∀x{x is a jabberwock that only outgribes those things that every bandernatch outgribes → x gimbles no toves that gimble themselves}

STEP 2: ∀x{[Jx & the only things that x outgribes are things that every bandersnatch outgribes] → no toves that gimble themselves are gimbled by x}

STEP 3: ∀x{[Jx & ∀y(x outgribes y → every bandernatch outgribes y)] → [∀y(y is a tove that gimbles itself → ~(y is gimbled by x)]}

STEP 4: ∀x{[Jx & ∀y(Oxy → ∀z(z is a bandersnatch → z outgribes y))] → [∀y((Ty & Gyy) → ~Gxy)]}

STEP 5: ∀x{[Jx & ∀y(Oxy → ∀z(Bz → Ozy))] → [∀y((Ty & Gyy) → ~Gxy)]}

F. REVIEW

Predicate logic uses lower case letters to stand for *specific individual* persons, places or things.

Warning: Do not use them for plural noun phrases ("professors", "planets", "all countries"), or noun phrases that refer to some undetermined things ("some country", "a professor").

Predicate logic uses uppercase letters to stand for properties, traits, characteristics or things that might be said about the things above.

We write them before the lowercase letter(s) standing for the individual(s) we are saying something about.

Monadic predicates represent a property that a thing has on its own.

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Polyadic predicates represent relationships between two or more things.

Tk	for	Kevin is tall.
Pn	for	Neptune is a planet.
Df	for	France is democratic.
Lfa	for	France is larger than Amherst.
Lnf	for	Neptune is larger than France.
Age	for	Gabrielle admires (X)ena.

[Do not use 'x', 'y' or 'z' as names. These are used as variables.]

When two proper names appear in a statement joined together with 'and', 'or', etc., sometimes the statement should be broken up into two simple statements joined by a logical sign:

France and Britain are democratic.
Df & Db
Either Orion or Callisto is a constellation.
Co v Cc

Something similar can happen with predicates:

Sanjukta is intelligent and talented.
Is & Ts

However, these statements cannot be broken up when a polyadic predicate is involved.

Mercury are Pluto are similar.
Smp
France and Britain are allies.
Afb

Both can happen in the same statement.

Consider:
Callisto orbits either Jupiter or Neptune.
This is: Ocj v Ocn

Warning: Logical connectives '&', 'v', etc. must always be flanked by things that can be true or false. Never use them simply between two subjects or predicates.

The first example cannot be written as D(f & b), etc.

<u>Quantifiers</u>
Quantifiers in English are words like 'all', 'some', 'only', etc.

In predicate logic, we have two quantifiers.
∀x… for all x . . . (everything is…)
∃x… there is an x… (something is…)

The simplest use of the quantifiers is to translate statements about 'everything', 'something', 'nothing', etc.

Everything is good.	∀xGx
Something is lost.	∃xLx
Nothing is free.	∀x~Fx OR ~∃xFx
Something is not free.	∃x~Fx OR ~∀xFx
Not everything is good.	∃x~Gx OR ~∀xGx
Everything is either good or bad.	∃x(Gx v Bx)

Quantifiers can also be used to make statements about some, all or no members of a group.

These make use of '→' or '&' signs as well.

All ... are ___.	No ... are ____.
∀x(…x… → __x__)	∀x(…x… → ~__x__) ~∃x(…x… & __x__)
Some ... are ____.	Some ... are not ___.
∃x(…x… & __x__)	∃x(…x… & ~__x__) ~∀x(…x… → …x…)

Warning: when your translation involves '∀', you'll almost always use '→', and
when your translation involves '∃', you'll almost always use '&'.

Be sure you don't use the wrong logical sign with your quantifier.

All birds fly
becomes $\forall x(Bx \rightarrow Fx)$
NOT
$\forall x(Bx \& Fx)$ = everything is a bird and flies.

Sometimes we talk not simply about all countries, but all *democratic countries*… etc.

This means the same as all things that are *both* democratic *and* countries…

Some democratic countries are socialist.
$\exists x[(Dx \& Cx) \& Sx]$
No local restaurants are open.
$\forall x[(Lx \& Rx) \rightarrow {\sim}Ox]$
Some countries are not socialist democracies.
$\exists x[Cx \& {\sim}(Dx \& Sx)]$

That, *which*, *who* and *whom* clauses operate like combined predicates.

No planets that rotate are inhabitable.
$\forall x[(Px \& Rx) \rightarrow {\sim}Ix]$

Sometimes, there will be disjunctive combinations of predicates that use ‘v’ instead of ‘&’.

All countries are either democratic are autocratic.
$\forall x[Cx \rightarrow (Dx \vee Ax)]$

All trucks and motorcycles are prohibited.
$\forall x[(Tx \vee Mx) \rightarrow Px]$
NOT: $\forall x[(Tx \& Mx) \rightarrow Px]$

Review Translations

No country is larger than China.
$\forall x(Cx \rightarrow {\sim}Lxc)$
All island countries border no countries.
$\forall x[(Cx \& Ix) \rightarrow \forall y(Cy \rightarrow {\sim}Bxy)]$
Only island countries border no countries.
1: $\forall x\{[Cx \& \forall y(Cy \rightarrow {\sim}Bxy)] \rightarrow Ix\}$
2: $\forall x[\forall y(Cy \rightarrow {\sim}Bxy)] \rightarrow (Cx \& Ix)]$
No country borders every country.
$\forall x[Cx \rightarrow {\sim}\forall y(Cy \rightarrow Bxy)]$

Every democracy borders some socialist country.
$\forall x\{Dx \rightarrow \exists y[(Sy \& Cy) \& Bxy]\}$
All countries that are larger than Japan are larger than Korea.
$\forall x[(Cx \& Lxj) \rightarrow Lxk]$
No countries that border Ethiopia are larger than India.
$\forall x[(Cx \& Bxe) \rightarrow {\sim}Lxi]$
All countries that are larger than every country that borders Ghana are larger than some country that borders France.
$\forall x\{(Cx \& \forall y[(Cy \& Byg) \rightarrow Lxy]) \rightarrow \exists y[(Cy \& Byf) \& Lxy]\}$
If Norway does not border Sweden, then it borders nothing.
${\sim}Bns \rightarrow \forall x{\sim}Bnx$
Either all countries that border Norway are democratic, or none are.
$\forall x[(Cx \& Bxn) \rightarrow Dx] \vee \forall x[(Cx \& Bxn) \rightarrow {\sim}Dx]$
If Russia is socialist, then all countries larger than the U.S. are socialist.
$Sr \rightarrow \forall x[(Cx \& Lxu) \rightarrow Sx]$
If some countries are socialist and all countries are democracies, then some countries are socialist democracies.
$[\exists x(Cx \& Sx) \& \forall x(Cx \rightarrow Dx)] \rightarrow \exists x[Cx \& (Sx \& Dx)]$
There is something that is orbited by every planet if all planets orbit the sun.
$\forall x(Px \rightarrow Oxs) \rightarrow \exists x\forall y(Py \rightarrow Oyx)$
Unless everything is socialist, some democratic countries are not socialist.
${\sim}\forall xSx \rightarrow \exists x[(Dx \& Cx) \& {\sim}Sx]$
If Finland borders Norway then every country that borders Sweden borders all countries that Sweden borders.
$Bfn \rightarrow \forall x\{(Cx \& Bxs) \rightarrow \forall y[(Cy \& Bsy) \rightarrow Bxy]\}$