

Introduction to Logic

Practice Exam Answers for Unit/Exam 2:

(Keep in mind that there are always multiple ways of doing a problem. For each, I have given only one possible solution, usually the shortest.)

Problem #1

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|------|--|-----------------------|
| (1) | $(P \rightarrow \sim Q) \& (Q \vee R)$ | Pr |
| (2) | $T \rightarrow (R \rightarrow H)$ | Pr |
| (3) | $T \& \sim H$ | Pr |
| (4) | SHOW: $S \vee \sim P$ | DD |
| (5) | T | 3 &O |
| (6) | $\sim H$ | 3 &O |
| (7) | $R \rightarrow H$ | 2,5 \rightarrow O |
| (8) | $\sim R$ | 6,7 \rightarrow O |
| (9) | $Q \vee R$ | 1 &O |
| (10) | Q | 8,9 vO |
| (11) | $P \rightarrow \sim Q$ | 1 &O |
| (12) | $\sim \sim Q$ | 10 DN |
| (13) | $\sim P$ | 11,12 \rightarrow O |
| (14) | $S \vee \sim P$ | 13 vl |

Problem #2

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|------|--|-----------------------|
| (1) | $(S \vee P) \leftrightarrow (\sim T \rightarrow \sim R)$ | Pr |
| (2) | SHOW: $(R \& S) \rightarrow T$ | CD |
| (3) | R & S | Ass |
| (4) | SHOW: T | DD |
| (5) | R | 3 &O |
| (6) | S | 3 &O |
| (7) | $S \vee P$ | 6 vl |
| (8) | $(S \vee P) \rightarrow (\sim T \rightarrow \sim R)$ | 1 \leftrightarrow O |
| (9) | $\sim T \rightarrow \sim R$ | 7,8 \rightarrow O |
| (10) | $\sim \sim R$ | 5 DN |
| (11) | $\sim \sim T$ | 9,10 \rightarrow O |
| (12) | T | 11 DN |

Problem #3

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|-----|--|----|
| (1) | $(A \vee B) \vee C$ | Pr |
| (2) | $A \rightarrow C$ | Pr |
| (3) | $(\sim C \vee A) \rightarrow \sim(B \vee C)$ | Pr |

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|------|------------------|----------------------|
| (4) | SHOW: C | ID |
| (5) | $\sim C$ | Ass |
| (6) | SHOW: \times | DD |
| (7) | $A \vee B$ | 1,5 vO |
| (8) | $\sim A$ | 2,5 \rightarrow O |
| (9) | B | 7,8 vO |
| (10) | $\sim C \vee A$ | 5 vl |
| (11) | $\sim(B \vee C)$ | 3,10 \rightarrow O |
| (12) | $B \vee C$ | 9 vl |
| (13) | \times | 11,12 \times I |

Problem #4

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|------|--|-----------------------|
| (1) | $D \vee \sim C$ | Pr |
| (2) | $\sim D \rightarrow (B \rightarrow C)$ | Pr |
| (3) | SHOW: $[A \rightarrow (B \vee C)] \rightarrow (A \rightarrow D)$ | CD |
| (4) | $A \rightarrow (B \vee C)$ | Ass |
| (5) | SHOW: $A \rightarrow D$ | CD |
| (6) | A | Ass |
| (7) | SHOW: D | ID |
| (8) | $\sim D$ | Ass |
| (9) | SHOW: \times | DD |
| (10) | $\sim C$ | 1,8 vO |
| (11) | $B \rightarrow C$ | 2,8 \rightarrow O |
| (12) | $\sim B$ | 10,11 \rightarrow O |
| (13) | $B \vee C$ | 4,6 \rightarrow O |
| (14) | C | 12,13 \rightarrow O |
| (15) | \times | 10,14 \times I |

Problem #5

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|------|-----------------------------------|---------------------|
| (1) | $(P \vee Q) \rightarrow \sim R$ | Pr |
| (2) | $P \vee [S \& \sim(S \& \sim R)]$ | Pr |
| (3) | SHOW: $R \leftrightarrow \sim P$ | DD |
| (4) | SHOW: $R \rightarrow \sim P$ | CD |
| (5) | R | Ass |
| (6) | SHOW: $\sim P$ | DD |
| (7) | $\sim \sim R$ | 5 DN |
| (8) | $\sim(P \vee Q)$ | 1,7 \rightarrow O |
| (9) | $\sim P$ | 8 \sim vO |
| (10) | SHOW: $\sim P \rightarrow R$ | CD |
| (11) | $\sim P$ | Ass |
| (12) | SHOW: R | DD |

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|------|-----------------------------|--------------------------|
| (13) | $S \& \sim(S \& \sim R)$ | 2,11 vO |
| (14) | S | 13 &O |
| (15) | $\sim(S \& \sim R)$ | 13 &O |
| (16) | $S \rightarrow \sim \sim R$ | 15 \sim &O |
| (17) | $\sim \sim R$ | 14,16 \rightarrow O |
| (18) | R | 17 DN |
| (19) | $R \leftrightarrow \sim P$ | 4,10 \leftrightarrow I |

Problem #6

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|------|-----------------------------------|----------------------|
| (1) | $T \rightarrow S$ | Pr |
| (2) | $T \vee (\sim S \rightarrow T)$ | Pr |
| (3) | $(Q \rightarrow R) \rightarrow P$ | Pr |
| (4) | SHOW: $S \& (P \vee Q)$ | DD |
| (5) | SHOW: S | ID |
| (6) | $\sim S$ | Ass |
| (7) | SHOW: \times | DD |
| (8) | $\sim T$ | 1,6 \rightarrow O |
| (9) | $\sim S \rightarrow T$ | 2,8 vO |
| (10) | T | 6,9 \rightarrow O |
| (11) | \times | 8,10 \times I |
| (12) | SHOW: $P \vee Q$ | ID |
| (13) | $\sim(P \vee Q)$ | Ass |
| (14) | SHOW: \times | DD |
| (15) | $\sim P$ | 13 \sim vO |
| (16) | $\sim Q$ | 13 \sim vO |
| (17) | $\sim(Q \rightarrow R)$ | 3,15 \rightarrow O |
| (18) | $Q \& \sim R$ | 17 \sim &O |
| (19) | Q | 18 &O |
| (20) | \times | 16,19 \times I |
| (21) | $S \& (P \vee Q)$ | 5,12 &I |

Problem #7

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|------|------------------------------------|----------------|
| (1) | $(M \& N) \vee (\sim M \& \sim N)$ | Pr |
| (2) | SHOW: $M \leftrightarrow N$ | DD |
| (3) | SHOW: $M \rightarrow N$ | CD |
| (4) | M | Ass |
| (5) | SHOW: N | DD |
| (6) | SHOW: $\sim(\sim M \& \sim N)$ | ID |
| (7) | $\sim M \& \sim N$ | Ass |
| (8) | SHOW: \times | DD |
| (9) | $\sim M$ | 7 &O |
| (10) | \times | 4,9 \times I |

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|------|--------------------------------|--------------------------|
| (11) | M & N | 1,6 vO |
| (12) | N | 11 &O |
| (13) | SHOW: $N \rightarrow M$ | CD |
| (14) | N | Ass |
| (15) | SHOW: M | DD |
| (16) | SHOW: $\sim(\sim M \& \sim N)$ | ID |
| (17) | $\sim M \& \sim N$ | Ass |
| (18) | SHOW: \times | DD |
| (19) | $\sim N$ | 17 &O |
| (20) | \times | 14,19 \times I |
| (21) | M & N | 1,16 vO |
| (22) | M | 21 &O |
| (23) | $M \leftrightarrow N$ | 3,13 \leftrightarrow I |

Problem #8

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|------|--------------------------------------|-----------------------|
| (1) | $P \rightarrow R$ | Pr |
| (2) | $\sim R \vee (R \rightarrow \sim R)$ | Pr |
| (3) | SHOW: $\sim(P \vee R)$ | ID |
| (4) | $P \vee R$ | Ass |
| (5) | SHOW: \times | DD |
| (6) | SHOW: $\sim P$ | ID |
| (7) | P | Ass |
| (8) | SHOW: \times | DD |
| (9) | R | 1,7 \rightarrow O |
| (10) | $\sim \sim R$ | 9 DN |
| (11) | $R \rightarrow \sim R$ | 2,10 vO |
| (12) | $\sim R$ | 9,11 \rightarrow O |
| (13) | \times | 9,12 \times I |
| (14) | R | 4,6 vO |
| (15) | $\sim \sim R$ | 14 DN |
| (16) | $R \rightarrow \sim R$ | 2,15 vO |
| (17) | $\sim R$ | 14,16 \rightarrow O |
| (18) | \times | 14,17 \times I |