

Introduction to Logic

Practice Exam Answers for Unit 4 (Exams 4 & 6):

(Keep in mind that there are always multiple ways to do a problem. For each, I have given only one possible solution, usually the shortest.)

Problem #1

(1)	$\forall x(Gx \leftrightarrow Hx)$	Pr
(2)	$\exists xHx$	Pr
(3)	$\sim Ha$	Pr
(4)	SHOW: $\exists xGx \ \& \ \sim \forall xGx$	DD
(5)	Hb	2 $\exists O$
(6)	$Gb \leftrightarrow Hb$	1 $\forall O$
(7)	$Hb \rightarrow Gb$	6 $\leftrightarrow O$
(8)	Gb	5,7 $\rightarrow O$
(9)	$\exists xGx$	4 $\exists I$
(10)	SHOW: $\sim \forall xGx$	ID
(11)	$\forall xGx$	Ass
(12)	SHOW: \times	DD
(13)	Ga	11 $\forall O$
(14)	$Ga \leftrightarrow Ha$	1 $\forall O$
(15)	$Ga \rightarrow Ha$	14 $\leftrightarrow O$
(16)	Ha	13,15 $\rightarrow O$
(17)	\times	3,16 $\times I$
(18)	$\exists xGx \ \& \ \sim \forall xGx$	9,10 $\& I$

Problem #2

(1)	$\forall x[(Fx \vee Gx) \rightarrow (Hx \ \& \ Ix)]$	Pr
(2)	$\sim \forall xHx$	Pr
(3)	SHOW: $\exists x \sim Fx$	ID
(4)	$\sim \exists x \sim Fx$	Ass
(5)	SHOW: \times	DD
(6)	$\forall x \sim \sim Fx$	4 $\sim E O$
(7)	$\exists x \sim Hx$	2 $\sim \forall O$
(8)	$\sim Ha$	7 $\exists O$
(9)	$\sim \sim Fa$	6 $\forall O$
(10)	Fa	9 DN
(11)	$Fa \vee Ga$	10 $\vee I$
(12)	$(Fa \vee Ga) \rightarrow (Ha \ \& \ Ia)$	1 $\forall O$

(13)	Ha & Ia	11,12 $\rightarrow O$
(14)	Ha	13 $\& O$
(15)	\times	8,14 $\times I$

Problem #3

(1)	$\forall x(\sim Hx \rightarrow \sim Fx)$	Pr
(2)	$\sim \exists x(Gx \ \& \ Hx)$	Pr
(3)	SHOW: $\forall x[(Fx \ \& \ Rxa) \rightarrow \sim Gx]$	UD
(4)	SHOW: $(Fb \ \& \ Rba) \rightarrow \sim Gb$	CD
(5)	Fb & Rba	Ass
(6)	SHOW: $\sim Gb$	DD
(7)	$\forall x \sim(Gx \ \& \ Hx)$	2 $\sim E O$
(8)	$\sim(Gb \ \& \ Hb)$	7 $\forall O$
(9)	$Gb \rightarrow \sim Hb$	8 $\sim \& O$
(10)	$\sim Hb \rightarrow \sim Fb$	1 $\forall O$
(11)	Fb	5 $\& O$
(12)	$\sim \sim Fb$	11 DN
(13)	$\sim \sim Hb$	10,12 $\rightarrow O$
(14)	$\sim Gb$	9,13 $\rightarrow O$

Problem #4

(1)	$\forall xFx \vee \sim \exists xGx$	Pr
(2)	$\forall x(Fx \rightarrow \sim Hx)$	Pr
(3)	SHOW: $\forall x(Hx \rightarrow Gx) \rightarrow \sim \exists xHx$	CD
(4)	$\forall x(Hx \rightarrow Gx)$	Ass
(5)	SHOW: $\sim \exists xHx$	ID
(6)	$\exists xHx$	Ass
(7)	SHOW: \times	DD
(8)	Ha	6 $\exists O$
(9)	$Fa \rightarrow \sim Ha$	2 $\forall O$
(10)	$Ha \rightarrow Ga$	4 $\forall O$
(11)	Ga	8,10 $\rightarrow O$
(12)	$\exists xGx$	11 $\exists I$
(13)	$\sim \sim \exists xGx$	12 DN
(14)	$\forall xFx$	1,13 $\vee O$
(15)	Fa	14 $\forall O$
(16)	$\sim Ha$	9,15 $\rightarrow O$
(17)	\times	8,16 $\times I$

Problem #5

(1)	$\forall x[Fx \rightarrow \forall y(Gy \rightarrow \sim Ryx)]$	Pr
(2)	SHOW: $\exists xFx \rightarrow$	
	$\forall x(\forall yRxy \rightarrow \sim Gx)$	CD
(3)	$\exists xFx$	Ass
(4)	SHOW: $\forall x(\forall yRxy \rightarrow \sim Gx)$	UD
(5)	SHOW: $\forall yRay \rightarrow \sim Ga$	CD
(6)	$\forall yRay$	Ass
(7)	SHOW: $\sim Ga$	DD
(8)	Fb	3 $\exists O$
(9)	$Fb \rightarrow \forall y(Gy \rightarrow \sim Ryb)$	1 $\forall O$
(10)	$\forall y(Gy \rightarrow \sim Ryb)$	8,9 $\rightarrow O$
(11)	$Ga \rightarrow \sim Rab$	10 $\forall O$
(12)	Rab	6 $\forall O$
(13)	$\sim \sim Rab$	12 DN
(14)	$\sim Ga$	11,13 $\rightarrow O$

Problem #6

(1)	$\forall xFx \vee \exists x(Gx \ \& \ Hx)$	Pr
(2)	$\exists x \sim Fx$	Pr
(3)	SHOW: $\exists xHx$	ID
(4)	$\sim \exists xHx$	Ass
(5)	SHOW: \times	DD
(6)	$\forall x \sim Hx$	4 $\sim E O$
(7)	$\sim Fa$	2 $\exists O$
(8)	SHOW: $\sim \forall xFx$	ID
(9)	$\forall xFx$	Ass
(10)	SHOW: \times	DD
(11)	Fa	9 $\forall O$
(12)	\times	7,11 $\times I$
(13)	$\exists x(Gx \ \& \ Hx)$	1,8 $\vee O$
(14)	Gb & Hb	13 $\exists O$
(15)	Hb	14 $\& O$
(16)	$\sim Hb$	6 $\forall O$
(17)	\times	15,16 $\times I$

Problem #7

(1)	$\exists x \exists y Lxy$	Pr
(2)	$\exists x \forall y (\exists z Lyz \rightarrow Lxy)$	Pr
(3)	SHOW: $\exists x Lxx$	DD
(4)	$\exists y Lya$	1 $\exists O$
(5)	Lba	4 $\exists O$
(6)	$\forall y (\exists z Lyz \rightarrow Lcy)$	2 $\forall O$
(7)	$\exists z Lbz \rightarrow Lcb$	6 $\forall O$
(8)	$\exists z Lbz$	5 $\exists I$
(9)	Lcb	7,8 $\rightarrow O$
(10)	$\exists z Lcz \rightarrow Lcc$	6 $\forall O$
(11)	$\exists z Lcz$	9 $\exists I$
(12)	Lcc	10,11 $\rightarrow O$
(13)	$\exists x Lxx$	12 $\exists I$

Problem #8

(1)	Rab	Pr
(2)	$\exists z Raz \rightarrow \forall y (Rya \rightarrow \forall z Rza)$	Pr
(3)	SHOW: $\exists y \forall x Rxy \vee \exists y \forall x \sim Rxy$	ID
(4)	$\sim (\exists y \forall x Rxy \vee \exists y \forall x \sim Rxy)$	Ass
(5)	SHOW: \times	DD
(6)	$\sim \exists y \forall x Rxy$	4 $\sim \vee O$
(7)	$\sim \exists y \forall x \sim Rxy$	4 $\sim \vee O$
(8)	$\forall y \sim \forall x Rxy$	6 $\sim E O$
(9)	$\forall y \sim \forall x \sim Rxy$	7 $\sim E O$
(10)	$\exists z Raz$	1 $\exists I$
(11)	$\forall y (Rya \rightarrow \forall z Rza)$	2,10 $\rightarrow O$
(12)	$\sim \forall x Rxa$	8 $\forall O$
(13)	$\sim \forall x \sim Rxa$	9 $\forall O$
(14)	$\exists x \sim Rxa$	12 $\sim \forall O$
(15)	$\exists x \sim \sim Rxa$	13 $\sim \forall O$
(16)	$\sim Rca$	14 $\exists O$
(17)	$\sim \sim Rda$	15 $\exists O$
(18)	Rda	17 DN
(19)	$Rda \rightarrow \forall z Rza$	11 $\forall O$
(20)	$\forall z Rza$	18,19 $\rightarrow O$
(21)	Rca	20 $\forall O$
(22)	\times	16,21 $\times I$