PICTORIAL SUMMARY OF RULES OF DERIVATION

SENTENTIAL LOGIC

&I

$$\frac{\mathcal{A}\quad \mathcal{B}}{\mathcal{A}\ \&\ \mathcal{B}} \qquad \frac{\mathcal{A}\quad \mathcal{B}}{\mathcal{B}\ \&\ \mathcal{A}}$$

&O

$$\frac{\mathcal{A}\ \&\ \mathcal{B}}{\mathcal{A}} \qquad \frac{\mathcal{A}\ \&\ \mathcal{B}}{\mathcal{B}}$$

~&O

$$\frac{\sim(\mathcal{A}\ \&\ \mathcal{B})}{\mathcal{A} \rightarrow \sim\mathcal{B}}$$

∨I

$$\frac{\mathcal{A}}{\mathcal{A} \vee \mathcal{B}} \qquad \frac{\mathcal{A}}{\mathcal{B} \vee \mathcal{A}}$$

∨O

$$\frac{\mathcal{A} \vee \mathcal{B}\quad \sim\mathcal{A}}{\mathcal{B}} \qquad \frac{\mathcal{A} \vee \mathcal{B}\quad \sim\mathcal{B}}{\mathcal{A}}$$

~∨O

$$\frac{\sim(\mathcal{A} \vee \mathcal{B})}{\sim\mathcal{A}\quad \sim\mathcal{B}}$$

↔I

$$\frac{\mathcal{A} \rightarrow \mathcal{B}\quad \mathcal{B} \rightarrow \mathcal{A}}{\mathcal{A} \leftrightarrow \mathcal{B}} \qquad \frac{\mathcal{A} \rightarrow \mathcal{B}\quad \mathcal{B} \rightarrow \mathcal{A}}{\mathcal{B} \leftrightarrow \mathcal{A}}$$

↔O

$$\frac{\mathcal{A} \leftrightarrow \mathcal{B}}{\mathcal{A} \rightarrow \mathcal{B}} \qquad \frac{\mathcal{A} \leftrightarrow \mathcal{B}}{\mathcal{B} \rightarrow \mathcal{A}}$$

~↔O

$$\frac{\sim(\mathcal{A} \leftrightarrow \mathcal{B})}{\sim\mathcal{A} \leftrightarrow \mathcal{B}}$$

DN

$$\frac{\mathcal{A}}{\sim\sim\mathcal{A}} \qquad \frac{\sim\sim\mathcal{A}}{\mathcal{A}}$$

→O

$$\frac{\mathcal{A} \rightarrow C\quad \mathcal{A}}{C} \qquad \frac{\mathcal{A} \rightarrow C\quad \sim C}{\sim\mathcal{A}}$$

~→O

$$\frac{\sim(\mathcal{A} \rightarrow C)}{\mathcal{A}\ \&\ \sim C}$$

×I

$$\frac{\mathcal{A}\quad \sim\mathcal{A}}{\times}$$

×O

$$\frac{\times}{\mathcal{A}}$$

R

$$\frac{\mathcal{A}}{\mathcal{A}}$$

DD

SHOW: $\mathcal{A}$

$\mathcal{A}$

CD

SHOW: $\mathcal{A} \rightarrow C$

$\mathcal{A}$

SHOW: C

ID

SHOW: $\sim\mathcal{A}$

$\mathcal{A}$

SHOW: ×

ID

SHOW: $\mathcal{A}$

$\sim\mathcal{A}$

SHOW: ×

PREDICATE LOGIC

In the following, v is any variable, a and n are names, and $\mathbb{F}$ is a formula. Furthermore, $\mathbb{F}$[a/v] is the formula that results when a is substituted for v at all its *free* occurrences in $\mathbb{F}$, and similarly, $\mathbb{F}$[n/v] is the formula that results when n is so substituted.

∀O

$\forall v\mathbb{F}$

$\mathbb{F}$[a/v] — a can be **any** name

∃I

$\mathbb{F}$[a/v] — a can be **any** name

$\exists v\mathbb{F}$

∃O

$\exists v\mathbb{F}$

$\mathbb{F}$[n/v] — n must be a **new** name

UD

~~SHOW~~: $\forall v\mathbb{F}$

| ~~SHOW~~: $\mathbb{F}$[n/v] — n must be a **new** name

~∀O

$\sim\forall v\mathbb{F}$

$\exists v\sim\mathbb{F}$

~∃O

$\sim\exists v\mathbb{F}$

$\forall v\sim\mathbb{F}$