

&I	$\begin{array}{c} \mathcal{A} \\ \mathcal{B} \\ \hline \mathcal{A} \& \mathcal{B} \end{array}$	&O	$\begin{array}{c} \mathcal{A} \& \mathcal{B} \\ \hline \mathcal{A} \end{array}$ $\begin{array}{c} \mathcal{A} \& \mathcal{B} \\ \hline \mathcal{B} \end{array}$	~&O	$\begin{array}{c} \sim(\mathcal{A} \& \mathcal{B}) \\ \hline \mathcal{A} \rightarrow \sim \mathcal{B} \end{array}$	&D	SHOW: $\mathcal{A} \& \mathcal{B}$ SHOW: \mathcal{A} SHOW: \mathcal{B}
vI	$\begin{array}{c} \mathcal{A} \\ \hline \mathcal{A} \vee \mathcal{B} \end{array}$	vO	$\begin{array}{c} \mathcal{A} \vee \mathcal{B} \\ \sim \mathcal{A} \\ \hline \mathcal{B} \end{array}$ $\begin{array}{c} \mathcal{A} \vee \mathcal{B} \\ \sim \mathcal{B} \\ \hline \mathcal{A} \end{array}$	~vO	$\begin{array}{c} \sim(\mathcal{A} \vee \mathcal{B}) \\ \hline \sim \mathcal{A} \\ \sim \mathcal{B} \end{array}$	vD	(ID) SHOW: $\mathcal{A} \vee \mathcal{B}$ $\sim(\mathcal{A} \vee \mathcal{B})$ SHOW: \mathbb{X}
\leftrightarrow I	$\begin{array}{c} \mathcal{A} \rightarrow \mathcal{B} \quad \mathcal{A} \rightarrow \mathcal{B} \\ \mathcal{B} \rightarrow \mathcal{A} \quad \mathcal{B} \rightarrow \mathcal{A} \\ \hline \mathcal{A} \leftrightarrow \mathcal{B} \quad \mathcal{B} \leftrightarrow \mathcal{A} \end{array}$	\leftrightarrow O	$\begin{array}{c} \mathcal{A} \leftrightarrow \mathcal{B} \\ \hline \mathcal{A} \rightarrow \mathcal{B} \end{array}$ $\begin{array}{c} \mathcal{A} \leftrightarrow \mathcal{B} \\ \hline \mathcal{B} \rightarrow \mathcal{A} \end{array}$	~ \leftrightarrow O	$\begin{array}{c} \sim(\mathcal{A} \leftrightarrow \mathcal{B}) \\ \hline \sim \mathcal{A} \leftrightarrow \mathcal{B} \end{array}$	\leftrightarrow D	SHOW: $\mathcal{A} \leftrightarrow \mathcal{B}$ SHOW: $\mathcal{A} \rightarrow \mathcal{B}$ SHOW: $\mathcal{B} \rightarrow \mathcal{A}$
\rightarrow I	see CD	\rightarrow O	$\begin{array}{c} \mathcal{A} \rightarrow \mathcal{C} \quad \mathcal{A} \rightarrow \mathcal{C} \\ \mathcal{A} \quad \sim \mathcal{C} \\ \hline C \quad \sim \mathcal{A} \end{array}$	~ \rightarrow O	$\begin{array}{c} \sim(\mathcal{A} \rightarrow \mathcal{C}) \\ \hline \mathcal{A} \& \sim \mathcal{C} \end{array}$	CD	SHOW: $\mathcal{A} \rightarrow \mathcal{C}$ \mathcal{A} SHOW: \mathcal{C}
DN	$\begin{array}{c} \mathcal{A} \\ \hline \sim \sim \mathcal{A} \end{array}$	DN	$\begin{array}{c} \sim \sim \mathcal{A} \\ \hline \mathcal{A} \end{array}$	Rep	$\begin{array}{c} \mathcal{A} \\ \hline \mathcal{A} \end{array}$	\sim D	SHOW: $\sim \mathcal{A}$ \mathcal{A} SHOW: \mathbb{X}
\mathbb{X} I	$\begin{array}{c} \mathcal{A} \\ \sim \mathcal{A} \\ \hline \mathbb{X} \end{array}$	\mathbb{X} O	$\begin{array}{c} \mathbb{X} \\ \hline \mathcal{A} \end{array}$	DD	$\begin{array}{c} \text{SHOW: } \mathcal{A} \\ \hline \mathcal{A} \end{array}$	ID	SHOW: \mathcal{A} $\sim \mathcal{A}$ SHOW: \mathbb{X}

PREDICATE LOGIC

In the following, v is any variable; $\mathbb{F}[v]$ is any formula in which v occurs free;
 $\mathbb{F}[\mathbf{o}]$ results by substituting \mathbf{o} for every free occurrence of v , where \mathbf{o} is any **old name**.
 $\mathbb{F}[\mathbf{n}]$ results by substituting \mathbf{n} for every free occurrence of v , where \mathbf{n} is any **new name**.
A name counts as **old** if it occurs in a line that is neither boxed nor cancelled;
otherwise it counts as **new**.

\forall I	see UD	\forall O	$\begin{array}{c} \forall v \mathbb{F}[v] \\ \hline \mathbb{F}[\mathbf{o}] \quad \text{old} \end{array}$	$\sim \forall$ O	$\begin{array}{c} \sim \forall v \Phi \\ \hline \exists v \sim \Phi \end{array}$	UD	SHOW: $\forall v \mathbb{F}[v]$ SHOW: $\mathbb{F}[\mathbf{n}]$
\exists I	$\mathbb{F}[\mathbf{o}] \quad \text{old}$	\exists O	$\begin{array}{c} \exists v \mathbb{F}[v] \\ \hline \mathbb{F}[\mathbf{n}] \quad \text{new} \end{array}$	$\sim \exists$ O	$\begin{array}{c} \sim \exists v \Phi \\ \hline \forall v \sim \Phi \end{array}$	\exists D	(ID) SHOW: $\exists v \mathbb{F}[v]$ $\sim \exists v \mathbb{F}[v]$ SHOW: \mathbb{X}