4 TRANSLATIONS IN SENTENTIAL LOGIC

1. INTRODUCTION

In the present chapter, we discuss how to translate a variety of English statements into the language of sentential logic.

From the viewpoint of sentential logic, there are five *standard connectives* – ‘and’, ‘or’, ‘if...then’, ‘if and only if’, and ‘not’. In addition to these standard connectives, there are in English numerous non-standard connectives, including ‘unless’, ‘only if’, ‘neither...nor’, among others. There is nothing linguistically special about the five "standard" connectives; rather, they are the connectives that logicians have found most useful in doing symbolic logic.

The translation process is primarily a process of *paraphrase* – saying the same thing using different words, or expressing the same proposition using different sentences. Paraphrase is translation from English into English, which is presumably easier than translating English into, say, Japanese.

In the present chapter, we are interested chiefly in two aspects of paraphrase. The first aspect is paraphrasing statements involving various non-standard connectives into equivalent statements involving only standard connectives.

The second aspect is paraphrasing simple statements into straightforwardly equivalent compound statements. For example, the statement ‘it is *not* raining’ is straightforwardly equivalent to the more verbose ‘*it is not true that* it is raining’. Similarly, ‘Jay *and* Kay are Sophomores’ is straightforwardly equivalent to the more verbose ‘Jay is a Sophomore, *and* Kay is a Sophomore’.

An English statement is said to be in *standard form*, or to be *standard*, if all its connectives are standard and it contains no simple statement that is straightforwardly equivalent to a compound statement; otherwise, it is said to be *non-standard*.

Once a statement is paraphrased into standard form, the only remaining task is to symbolize it, which consists of symbolizing the simple (atomic) statements and symbolizing the connectives. Simple statements are symbolized by upper case Roman letters, and the standard connectives are symbolized by the already familiar symbols – ampersand, wedge, tilde, arrow, and double-arrow.

In translating simple statements, the particular letter one chooses is not terribly important, although it is usually helpful to choose a letter that is suggestive of the English statement. For example, ‘R’ can symbolize either ‘it is raining’ or ‘I am running’; however, if both of these statements appear together, then they must be symbolized by different letters. In general, in any particular context, different letters must be used to symbolize non-equivalent statements, and the same letter must be used to symbolize equivalent statements.

2. THE GRAMMAR OF SENTENTIAL LOGIC; A REVIEW

Before proceeding, let us review the grammar of sentential logic. First, recall that statements may be divided into *simple* statements and *compound* statements. Whereas the latter are constructed from smaller statements using statement connectives, the former are not so constructed.

The grammar of sentential logic reflects this grammatical aspect of English. In particular, formulas of sentential logic are divided into *atomic* formulas and *molecular* formulas. Whereas molecular formulas are constructed from other formulas using connectives, atomic formulas are structureless, they are simply upper case letters (of the Roman alphabet).

Formulas are strings of symbols. In sentential logic, the symbols include all the upper case letters, the five connective symbols, as well as left and right parentheses. Certain strings of symbols count as formulas of sentential logic, and others do not, as determined by the following definition.

Definition of Formula in Sentential Logic:

(1) every upper case letter is a formula;
(2) if $\mathcal{A}$ is a formula, then so is $\sim\mathcal{A}$;
(3) if $\mathcal{A}$ and $\mathcal{B}$ are formulas, then so is $(\mathcal{A}\ \&\ \mathcal{B})$;
(4) if $\mathcal{A}$ and $\mathcal{B}$ are formulas, then so is $(\mathcal{A} \vee \mathcal{B})$;
(5) if $\mathcal{A}$ and $\mathcal{B}$ are formulas, then so is $(\mathcal{A} \rightarrow \mathcal{B})$;
(6) if $\mathcal{A}$ and $\mathcal{B}$ are formulas, then so is $(\mathcal{A} \leftrightarrow \mathcal{B})$;
(7) nothing else is a formula.

In the above definition, the script letters stand for arbitrary strings of symbols. So for example, clause (2) says that if you have a string $\mathcal{A}$ of symbols, then provided $\mathcal{A}$ is a formula, the result of prefixing a tilde sign in front of $\mathcal{A}$ is also a formula. Also, clause (3) says that if you have a pair of strings, $\mathcal{A}$ and $\mathcal{B}$, then provided both strings are formulas, the result of infixing an ampersand and surrounding the resulting expression by parentheses is also a formula.

As noted earlier, in addition to formulas in the strict sense, which are specified by the above definition, we also have formulas in a less strict sense. These are called unofficial formulas, which are defined as follows.

An **unofficial formula** is any string of symbols obtained from an official formula by removing its outermost parentheses, if such exist.

The basic idea is that, although the outermost parentheses of a formula are crucial when it is used to form a larger formula, the outermost parentheses are optional when the formula stands alone. For example, the answers to the exercises, at the back of the chapter, are mostly unofficial formulas.

3. CONJUNCTIONS

The standard English expression for conjunction is ‘and’, but there are numerous other conjunction-like expressions, including the following.

(c1) but
(c2) yet
(c3) although
(c4) though
(c5) even though
(c6) moreover
(c7) furthermore
(c8) however
(c9) whereas

Although these expressions have different connotations, they are all truth-functionally equivalent to one another. For example, consider the following statements.

(s1) it is raining, *but* I am happy
(s2) *although* it is raining, I am happy
(s3) it is raining, *yet* I am happy
(s4) it is raining *and* I am happy

For example, under what conditions is (s1) true? Answer: (s1) is true precisely when ‘it is raining’ and ‘I am happy’ are both true, which is to say precisely when (s4) is true. In other words, (s1) and (s4) are true under precisely the same circumstances, which is to say that they are truth-functionally equivalent.

When we utter (s1)-(s3), we intend to emphasize a *contrast* that is not emphasized in the standard conjunction (s4), or we intend to convey (a certain degree of) *surprise*. The difference, however, pertains to appropriate usage rather than semantic content.

Although they connote differently, (s1)-(s4) have the same truth conditions, and are accordingly symbolized the same:

R & H

4. DISGUISED CONJUNCTIONS

As noted earlier, certain simple statements are straightforwardly equivalent to compound statements. For example,

(e1) Jay and Kay are Sophomores

is equivalent to

(p1) Jay is a Sophomore, *and* Kay is a Sophomore

which is symbolized:

(s1) J & K

Other examples of disguised conjunctions involve relative pronouns (‘who’, ‘which’, ‘that’). For example,

(e2) Jones is a former player *who* coaches basketball

is equivalent to

(p2) Jones is a former (basketball) player, *and* Jones coaches basketball,

which may be symbolized:

(s2) F & C

Further examples do not use relative pronouns, but are easily paraphrased using relative pronouns. For example,

(e3) Pele is a Brazilian soccer player

may be paraphrased as

(p3) Pele is a Brazilian who is a soccer player

which is equivalent to

(p3') Pele is a Brazilian, and Pele is a soccer player,

which may be symbolized:

(s3) B & S

Notice, of course, that

(e4) Jones is a *former* basketball player

is *not* a conjunction, such as the following absurdity.

(??) Jones is a former, and Jones is a basketball player

Sentence (e4) is rather symbolized as a simple (atomic) formula.

5. THE RELATIONAL USE OF 'AND'

As noted in the previous section, the statement,

(c) Jay and Kay are Sophomores,

is equivalent to the conjunction,

Jay is a Sophomore, *and* Kay is a Sophomore,

and is accordingly symbolized:

J & K

Other statements look very much like (c), but are not equivalent to conjunctions. Consider the following statements.

(r1) Jay and Kay are cousins
(r2) Jay and Kay are siblings
(r3) Jay and Kay are neighbors
(r4) Jay and Kay are roommates
(r5) Jay and Kay are lovers

These are definitely not symbolized as conjunctions. The following is an incorrect translation.

(?) J & K WRONG!!!

For example, consider (r1), the standard reading of which is

(r1') Jay and Kay are cousins *of each other*.

In proposing J&K as the analysis of (r1'), we must specify which particular atomic statement each letter stands for. The following is the only plausible choice.

J: Jay is a cousin

K: Kay is a cousin

Accordingly, the formula J&K is read

Jay is a cousin, *and* Kay is a cousin.

But to say that Jay is a cousin is to say that he is a cousin of someone, but not necessarily Kay. Similarly, to say that Kay is a cousin is to say that she a cousin of someone, but not necessarily Jay. In other words, J&K does *not* say that Jay and Kay are cousins *of each other*.

The resemblance between statements like (r1)-(r5) and statements like

(c1) Jay and Kay are Sophomores
(c2) Jay and Kay are Republicans
(c3) Jay and Kay are basketball players

is grammatically superficial. Each of (c1)-(c3) states something about Jay *independently* of Kay, and something about Kay *independently* of Jay.

By contrast, each of (r1)-(r5) states that a particular *relationship* holds between Jay and Kay. The relational quality of (r1)-(r5) may be emphasized by restating them in either of the following ways.

(r1') Jay is a cousin of Kay
(r2') Jay is a sibling of Kay
(r3') Jay is a neighbor of Kay
(r4') Jay is a roommate of Kay
(r5') Jay is a lover of Kay

(r1) Jay and Kay are cousins of each other
(r2) Jay and Kay are siblings of each other
(r3) Jay and Kay are neighbors of each other
(r4) Jay and Kay are roommates of each other
(r5) Jay and Kay are lovers of each other

On the other hand, notice that one cannot paraphrase (c1) as

(??) Jay is a Sophomore of Kay
(??) Jay and Kay are Sophomores of each other

Relational statements like (r1)-(r5) are *not* correctly paraphrased as conjunctions. In fact, they are not correctly paraphrased by any compound statement. From the viewpoint of sentential logic, these statements are *simple*; they have no internal structure, and are accordingly symbolized by atomic formulas.

[NOTE: Later, in predicate logic, we will see how to uncover the internal structure of relational statements such as (r1)-(r5), internal structure that is inaccessible to sentential logic.]

We have seen so far that 'and' is used both *conjunctively*, as in

Jay and Kay are Sophomores,

and *relationally*, as in

Jay and Kay are cousins (of each other).

In other cases, it is not obvious whether 'and' is used conjunctively or relationally. Consider the following.

(s2) Jay and Kay are married

There are two plausible interpretations of this statement. On the one hand, we can interpret it as

(i1) Jay and Kay are married *to each other*,

in which case it expresses a relation, and is symbolized as an atomic formula, say: M. On the other hand, we can interpret it as

(i2) Jay is married, and Kay is married,
(perhaps, but *not necessarily*, to each other),

in which case it is symbolized by a conjunction, say: J&K. The latter simply reports the marital status of Jay, independently of Kay, and the marital status of Kay, independently of Jay.

We can also say things like the following.

(s3) Jay and Kay are married, but not to each other.

This is equivalent to

(p3) Jay is married, and Kay is married,
but Jay and Kay are not married to each other,

which is symbolized:

(J & K) & ~M

[Note: This latter formula does not uncover all the logical structure of the English sentence; it only uncovers its connective structure, but that is all sentential logic is concerned with.]

6. CONNECTIVE-USES OF ‘AND’ DIFFERENT FROM AMPERSAND

As seen in the previous section, ‘and’ is used both as a connective and as a separator in relation-statements.

In the present section, we consider how ‘and’ is occasionally used as a connective different in meaning from the ampersand connective (&). There are two cases of this use.

First, sentences that have the form ‘P and Q’ sometimes mean ‘P *and then* Q’. For example, consider the following statements.

(s1) I went home and went to bed
(s2) I went to bed and went home

As they are colloquially understood at least, these two statements do not express the same proposition, since ‘and’ here means ‘and then’.

Note, in particular, that the above use of ‘and’ to mean ‘and then’ is *not truth-functional*. Merely knowing that P is true, and merely knowing that Q is true, one does not automatically know the *order* of the two events, and hence one does not know the truth-value of the compound ‘P and then Q’.

Sometimes ‘and’ does not have exactly the same meaning as the ampersand connective. Other times, ‘and’ has a quite different meaning from ampersand.

(e1) keep trying, and you will succeed

(e2) keep it up buster, and I will clobber you

(e3) give him an inch, and he will take a mile (e4) give me a place to stand, and I will move the world (Archimedes, in reference to the power of levers)

(e5) give us the tools of war, and we will finish the job (Churchill, in reference to WW2)

Consider (e1) paraphrased as a conjunction, for example:

(?) K & S

In proposing (?) as an analysis of (e1), we must specify what particular statements K and S abbreviate. The only plausible answer is:

K: you will keep trying

S: you will succeed

Accordingly, the conjunction K&S reads:

you will keep trying, *and* you will succeed

But the original,

keep trying, and you will succeed,

does not say this at all. It does not say the addressee will keep trying, nor does it say that the addressee will succeed. Rather, it merely says (promises, predicts) that the addressee will succeed *if* he/she keeps trying.

Similarly, in the last example, it should be obvious that Churchill was not predicting that the addressee (i.e., Roosevelt) would in fact give him military aid *and* Churchill would in fact finish the job (of course, that was what Churchill was hoping!). Rather, Churchill was saying that he would finish the job *if* Roosevelt were to give him military aid. (As it turned out, of course, Roosevelt eventually gave substantial direct military aid.)

Thus, under very special circumstances, involving requests, promises, threats, warnings, etc., the word 'and' can be used to state conditionals. The appropriate paraphrases are given as follows.

(p1) *if* you keep trying, *then* you will succeed
(p2) *if* you keep it up buster, *then* I will clobber you
(p3) *if* you give him an inch, *then* he will take a mile
(p4) *if* you give me a place to stand, *then* I will move the world
(p5) *if* you give us the tools of war, *then* we will finish the job

The treatment of conditionals is discussed in a later section.

7. NEGATIONS, STANDARD AND IDIOMATIC

The standard form of the negation connective is

it is not true that _____

The following expressions are standard variants.

it is not the case that _____

it is false that _____

Given any statement, we can form its *standard negation* by placing 'it is not the case that' (or a variant) in front of it.

As noted earlier, standard negations seldom appear in colloquial-idiomatic English. Rather, the usual colloquial-idiomatic way to negate a statement is to place the modifier 'not' in a strategic place within the statement, usually immediately after the verb. The following is a simple example.

statement: it is raining
idiomatic negation: it is *not* raining
standard negation: *it is not true that* it is raining

Idiomatic negations are symbolized in sentential logic exactly like standard negations, according to the following simple principle.

> If sentence S is symbolized by the formula $\mathcal{A}$, then the negation of S (standard or idiomatic) is symbolized by the formula $\sim\mathcal{A}$.

Note carefully that this principle applies whether S is simple or compound. As an example of a compound statement, consider the following statement.

(e1) Jay is a Freshman basketball player.

As noted in Section 2, this may be paraphrased as a conjunction:

(p1) Jay is a Freshman, *and* Jay is a basketball player.

Now, there is no simple idiomatic negation of the latter, although there is a standard negation, namely

(n1) it is not true that (Jay is a Freshman and Jay is a basketball player)

The parentheses indicate the scope of the negation modifier.

However, there is a simple idiomatic negation of the former, namely,

(n1′) Jay is *not* a Freshman basketball player.

We consider (n1) and (n1′) further in the next section.

8. NEGATIONS OF CONJUNCTIONS

As noted earlier, the sentence

(s1) Jay is a Freshman basketball player,

may be paraphrased as a conjunction,

(p1) Jay is a Freshman, *and* Jay is a basketball player,

which is symbolized:

(f1) F & B

Also, as noted earlier, the idiomatic negation of (p1) is

(n1) Jay is *not* a Freshman basketball player.

Although there is no simple idiomatic negation of (p1), its standard negation is:

(n2) *it is not true that* (Jay is a Freshman, *and* Jay is a Basketball player),

which is symbolized:

~(F & B)

Notice carefully that, when the conjunction stands by itself, the outer parentheses may be dropped, as in (f2), but when the formula is negated, the outer parentheses *must* be restored before prefixing the negation sign. Otherwise, we obtain:

~F & B,

which is reads:

Jay is *not* a Freshman, *and* Jay is a Basketball player,

which is *not* equivalent to ~(F&B), as may be shown using truth tables.

How do we read the negation

~(F & B)?

Many students suggest the following *erroneous* paraphrase,

Jay is *not* a Freshman,
and
Jay is *not* a basketball player, WRONG!!!

which is symbolized:

~J & ~B.

But this is clearly not equivalent to (n1). To say that Jay isn't a Freshman basketball player is to say that one of the following states of affairs obtains.

(1) Jay is a Freshman who does not play Basketball;

(2) Jay is a Basketball player who is not a Freshman;

(3) Jay is neither a Freshman nor a Basketball player.

On the other hand, to say that Jay is not a Freshman and not a Basketball player is to say precisely that the last state of affairs (3) obtains.

We have already seen the following, in a previous chapter (voodoo logic notwithstanding!)

$\sim(\mathcal{A} \,\&\, \mathcal{B})$ is ***NOT*** logically equivalent to $(\sim\mathcal{A} \,\&\, \sim\mathcal{B})$

This is easily demonstrated using truth-tables. Whereas the latter entails the former, the former does not entail the latter.

The correct logical equivalence is rather:

$\sim(\mathcal{A} \,\&\, \mathcal{B})$ is logically equivalent to $(\sim\mathcal{A} \vee \sim\mathcal{B})$

The disjunction may be read as follows.

Jay is *not* a Freshman *and/or* Jay is *not* a Basketball player.

One more example might be useful. The colloquial negation of the sentence

Jay and Kay are *both* Republicans   J & K

is

Jay and Kay are *not both* Republicans   ~(J & K)

This is definitely not the same as

Jay and Kay are both non-Republicans,

which is symbolized:

~J & ~K.

The latter says that *neither* of them is a Republican (see later section concerning 'neither'), whereas the former says less – that at least one of them isn't a Republican, *perhaps* neither of them is a Republican.

9. DISJUNCTIONS

The standard English expression for disjunction is ‘or’, a variant of which is ‘either...or’. As noted in a previous chapter, ‘or’ has two senses – an inclusive sense and an exclusive sense.

The legal profession has invented an expression to circumvent this ambiguity – ‘and/or’. Similarly, Latin uses two different words: one, ‘vel’, expresses the inclusive sense of ‘or’; the other, ‘aut’, expresses the exclusive sense.

The standard connective of sentential logic for disjunction is the wedge ‘∨’, which is suggestive of the first letter of ‘vel’. In particular, the wedge connective of sentential logic corresponds to the inclusive sense of ‘or’, which is the sense of ‘and/or’ and ‘vel’.

Consider the following statements, where the inclusive sense is distinguished (parenthetically) from the exclusive sense.

(is) Jones will win *or* Smith will win (possibly both)

(es) Jones will win *or* Smith will win (but not both)

We can imagine a scenario for each. In the first scenario, Jones and Smith, and a third person, Adams, are the only people running in an election in which two people are elected. So Jones or Smith will win, maybe both. In the second scenario, Jones and Smith are the two finalists in an election in which only one person is elected. In this case, one will win, the other will lose.

These two statements may be symbolized as follows.

(f1) J ∨ S

(f2) (J ∨ S) & ~(J & S)

We can read (f1) as saying that Jones will win and/or Smith will win, and we can read (f2) as saying that Jones will win or Smith will win *but* they won't both win (recall previous section on negations of conjunctions).

As with conjunctions, certain simple statements are straightforwardly equivalent to disjunctions, and are accordingly symbolized as such. The following are examples.

(s1)	it is raining or sleeting	
(d1)	it raining, *or* it is sleeting	R ∨ S
(s2)	Jones is a fool or a liar	
(d2)	Jones is a fool, *or* Jones is a liar	F ∨ L

10. ‘NEITHER...NOR’

Having considered disjunctions, we next look at negations of disjunctions. For example, consider the following statement.

(e1) Kay *isn't either* a Freshman *or* a Sophomore

This may be paraphrased in the following, non-idiomatic, way.

(p1) it is not true that (Kay is either a Freshman or a Sophomore)

This is a negation of a disjunction, and is accordingly symbolized as follows.

(s1) $\sim(F \vee S)$

Now, an alternative, idiomatic, paraphrase of (e1) uses the expression 'neither...nor', as follows.

(p1') Kay is *neither* a Freshman *nor* a Sophomore

Comparing (p1') with the original statement (e1), we can discern the following principle.

> 'neither...nor'
> is the negation of
> 'either...or'

This suggests introducing a non-standard connective, neither-nor with the following defining property.

> neither $\mathcal{A}$ nor $\mathcal{B}$
> is logically equivalent to
> $\sim(\mathcal{A} \vee \mathcal{B})$

Note carefully that neither-nor in its connective guise is highly non-idiomatic. In particular, in order to obtain a grammatically *general* reading of it, we must read it as follows.

> neither $\mathcal{A}$ nor $\mathcal{B}$
> is officially read:
> *neither is it true that* $\mathcal{A}$
> *nor is it true that* $\mathcal{B}$

This is completely analogous to the standard (grammatically general) reading of 'not P' as 'it is not the case that P'.

For example, if R stands for 'it is raining' and S stands for 'it is sleeting', then 'neither R nor S' is read

neither is it true that it is raining
nor is it true that it is sleeting

This awkward reading of neither-nor is required in order to insure that 'neither P nor Q' is grammatical irrespective of the actual sentences P and Q. Of course, as with simple negation, one can usually transform the sentence into a more colloquial form. For example, the above sentence is more naturally read neither is it raining nor is it sleeting,

or more naturally still,

it is neither raining nor sleeting.

We have suggested that neither-nor is the negation of either-or. Other uses of the word 'neither' suggest another, equally natural, paraphrase of neither-nor. Consider the following sentences.

neither Jay nor Kay is a Sophomore

Jay is not a Sophomore, and neither is Kay

A bit of linguistic reflection reveals that these two sentences are equivalent to one another. Further reflection reveals that the latter sentence is simply a stylistic variant of the more monotonous sentence

Jay is *not* a Sophomore, *and* Kay is *not* a Sophomore

The latter is a conjunction of two negations, and is accordingly symbolized:

$\sim J \ \& \sim K$

Thus, we see that a neither-nor sentence can be symbolized as a conjunction of two negations. This is entirely consistent with the truth-functional behavior of 'and', 'or', and 'not', since the following pair are logically equivalent, as is easily demonstrated using truth-tables.

> $\sim(\mathcal{A} \vee \mathcal{B})$ is logically equivalent to $(\sim\mathcal{A} \ \& \sim\mathcal{B})$

We accordingly have two equally natural paraphrases of sentences involving neither-nor, given by the following principle.

> neither $\mathcal{A}$ nor $\mathcal{B}$
> may be paraphrased
> $\sim(\mathcal{A} \vee \mathcal{B})$
> or equivalently
> $\sim\mathcal{A} \ \& \sim\mathcal{B}$

11. CONDITIONALS

The standard English expression for the conditional connective is 'if...then'. A standard conditional (statement) is a statement of the form

if $\mathcal{A}$, then $\mathcal{C}$,

where $\mathcal{A}$ and $\mathcal{B}$ are any statements (simple or compound), and is symbolized:

$\mathcal{A} \rightarrow \mathcal{C}$

Whereas $\mathcal{A}$ is called the *antecedent* of the conditional, C is called the *consequent* of the conditional. Note that, unlike conjunction and disjunction, the constituents of a conditional do not play symmetric roles.

There are a number of idiomatic variants of 'if...then'. In particular, all of the following statement forms are equivalent ($\mathcal{A}$ and C being any statements whatsoever).

(c1) if $\mathcal{A}$, then C

(c2) if $\mathcal{A}$, C
(c2') C if $\mathcal{A}$

(c3) provided (that) $\mathcal{A}$, C
(c3') C provided (that) $\mathcal{A}$

(c4) in case $\mathcal{A}$, C
(c4') C in case $\mathcal{A}$

(c5) on the condition that $\mathcal{A}$, C
(c5') C on the condition that $\mathcal{A}$

In particular, all of the above statement forms are symbolized in the same manner:

$$\mathcal{A}\to C$$

As the reader will observe, the order of antecedent and consequent is not fixed: in idiomatic English usage, sometimes the antecedent goes first, sometimes the consequent goes first. The following principles, however, should enable one systematically to identify the antecedent and consequent.

'if' always introduces the antecedent

'then' always introduces the consequent

'provided (that)',
'in case', and
'on the condition that'
are variants of 'if'

12. 'EVEN IF'

The word 'if' frequently appears in combination with other words, the most common being 'even' and 'only', which give rise to the expressions 'even if', 'only if'.

In the present section, we deal very briefly with 'even if', leaving 'only if' to the next section.

The expression 'even if' is actually quite tricky. Consider the following examples.

(e1) the Allies would have won *even if* the U.S. had *not* entered the war (in reference to WW2)

(i1) the Allies would have won *if* the U.S. had *not* entered the war

These two statements *suggest* quite different things. Whereas (e1) suggests that the Allies did win, (i1) suggests that the Allies didn't win. A more apt use of 'if' would be:

(i2) the Axis powers would have won *if* the U.S. had not entered the war.

Notwithstanding the pragmatic matters of appropriate, sincere usage, it seems that the *pure semantic content* of 'even if' is the same as the pure semantic content of 'if'. The difference is not one of meaning but of *presupposition*, on the part of the speaker. In such examples, we tend to use 'even if' when we presuppose that the consequent is true, and we tend to use 'if' when we presuppose that the consequent is false. This is summarized as follows.

it would have been the case that $\mathcal{B}$ **if** it had been the case that $\mathcal{A}$ pragmatically presupposes $\sim\mathcal{B}$

it would have been the case that $\mathcal{B}$ **even if** it had been the case that $\mathcal{A}$ pragmatically presupposes $\mathcal{B}$

To say that one statement $\mathcal{A}$ *pragmatically presupposes* another statement $\mathcal{B}$ is to say that when one (sincerely) asserts $\mathcal{A}$, one takes for granted the truth of $\mathcal{B}$.

Given the subtleties of content versus presupposition, we will not consider 'even if' any further in this text.

13. 'ONLY IF'

The word 'if' frequently appears in combination with other words, the most common being 'even' and 'only', which give rise to the expressions 'even if', 'only if'.

The expression 'even if' is very complex, and somewhat beyond the scope of intro logic, so we do not consider it any further. So, let us turn to the other expression, 'only if', which involves its own subtleties, but subtleties that can be dealt with in intro logic.

First, we note that 'only if' is definitely *not* equivalent to 'if'. Consider the following statements involving 'only if'.

(o1) I will get an A in logic *only if* I take all the exams
(o2) I will get into law school *only if* I take the LSAT

Now consider the corresponding statements obtained by replacing 'only if' by 'if'.

(i1) I will get an A in logic *if* I take all the exams
(i2) I will get into law school *if* I take the LSAT

Whereas the 'only if' statements are true, the corresponding 'if' statements are false. It follows that 'only if' is not equivalent to 'if'.

The above considerations show that an 'only if' statement does not imply the corresponding 'if' statement. One can also produce examples of 'if' statements that do not imply the corresponding 'only if' statements. Consider the following examples.

(i3) I will pass logic *if* I score 100 on every exam
(i4) I am guilty of a felony *if* I murder someone

(o3) I will pass logic *only if* I score 100 on every exam
(o4) I am guilty of a felony *only if* I murder someone

Whereas both 'if' statements are true, both 'only if' statements are false. Thus, 'A if B' does not imply 'A only if B', and 'A only if B' does not imply 'A if B'.

So how do we paraphrase 'only if' statements using the standard connectives? The answer is fairly straightforward, being related to the general way in which the word 'only' operates in English – as a special dual-negative modifier.

As an example of 'only' in ordinary discourse, a sign that reads 'employees only' means to *exclude* anyone who is *not* an employee. Also, if I say 'Jay loves only Kay', I mean that he does *not* love anyone *except* Kay.

In the case of the connective 'only if', 'only' modifies 'if' by introducing two negations; in particular, the statement

$\mathcal{A}$ <u>only</u> if $\mathcal{B}$

is paraphrased

<u>not</u> $\mathcal{A}$ if <u>not</u> $\mathcal{B}$

In other words, the 'if' stays put, and in particular continues to introduce the antecedent, but the 'only' becomes two negations, one in front of the antecedent (introduced by 'if'), the other in front of the consequent.

With this in mind, let us go back to original examples, and paraphrase them in accordance with this principle. In each case, we use a colloquial form of negation.

(p1) I will *not* get an A in logic *if* I do *not* take all the exams
(p2) I will *not* get into law school *if* I do *not* take the LSAT

Now, (p1) and (p2) are not in standard form, the problem being the relative position of antecedent and consequent. Recalling that '$\mathcal{A}$ if $\mathcal{B}$' is an idiomatic variant of 'if $\mathcal{B}$, then $\mathcal{A}$', we further paraphrase (p1) and (p2) as follows.

(p1') *if* I do *not* take all the exams, *then* I will *not* get an A in logic
(p2') *if* I do *not* take the LSAT, *then* I will *not* get into law school

These are symbolized, respectively, as follows.

(s1) $\sim T \rightarrow \sim A$
(s2) $\sim T \rightarrow \sim L$

Combining the paraphrases of 'only if' and 'if', we obtain the following principle.

$\mathcal{A}$ only if $\mathcal{B}$

is paraphrased

not $\mathcal{A}$ if not $\mathcal{B}$

which is further paraphrased

if not $\mathcal{B}$, then not $\mathcal{A}$

which is symbolized

$\sim\mathcal{B} \rightarrow \sim\mathcal{A}$

14. A PROBLEM WITH THE TRUTH-FUNCTIONAL IF-THEN

The reader will recall that the truth-functional version of 'if...then' is characterized by the truth-function that makes '$\mathcal{A}\rightarrow\mathcal{B}$' false precisely when $\mathcal{A}$ is true and $\mathcal{B}$ is false. As noted already, this is not a wholly satisfactory analysis of English 'if...then'; rather, it is simply the best we can do by way of a truth-functional version of 'if...then'. Whereas the truth-functional analysis of 'if...then' is well suited to the timeless, causeless, eventless realm of mathematics, it is not so well suited to the realm of ordinary objects and events.

In the present section, we examine one of the problems resulting from the truth-functional analysis of 'if...then', a problem specifically having to do with the expression 'only if'.

We have paraphrased '$\mathcal{A}$ only if $\mathcal{B}$' as 'not $\mathcal{A}$ if not $\mathcal{B}$', which is paraphrased 'if not $\mathcal{B}$, then not $\mathcal{A}$', which is symbolized '$\sim\mathcal{B}\rightarrow\sim\mathcal{A}$'. The reader may recall that, using truth tables, one can show the following.

$$\sim\mathcal{B} \rightarrow \sim\mathcal{A}$$

is equivalent to

$$\mathcal{A} \rightarrow \mathcal{B}$$

Now, $\sim\mathcal{B}\rightarrow\sim\mathcal{A}$ is the translation of '$\mathcal{A}$ only if $\mathcal{B}$', whereas $\mathcal{A}\rightarrow\mathcal{B}$ is the translation of 'if $\mathcal{A}$, then $\mathcal{B}$'. Therefore, since $\sim\mathcal{B}\rightarrow\sim\mathcal{A}$ is truth-functionally equivalent to $\mathcal{A}\rightarrow\mathcal{B}$, we are led to conclude that '$\mathcal{A}$ only if $\mathcal{B}$' is truth-functionally equivalent to 'if $\mathcal{A}$, then $\mathcal{B}$'.

This means, in particular that our original examples,

(o1) I will get an A in logic *only if* I take the exams
(o2) I will get into law school *only if* I take the LSAT

are truth-functionally equivalent to the following, respectively:

(e1) *if* I get an A in logic, *then* I will take the exams
(e2) *if* I get into law school, *then* I will take the LSAT

Compared with the original statements, these sound odd indeed. Consider the last one. My response is that, if you get into law school, why bother taking the LSAT!

The oddity we have just discovered further underscores the shortcomings of the truth-functional if-then connective. The particular difficulty is summarized as follows.

$\mathcal{A}$ only if $\mathcal{B}$
is equivalent (in English) to
not $\mathcal{A}$ if not $\mathcal{B}$
which is equivalent (in English) to
if not $\mathcal{B}$, then not $\mathcal{A}$
which is symbolized

$$\sim\mathcal{B} \rightarrow \sim\mathcal{A}$$

which is equivalent (by truth tables) to

$$\mathcal{A} \rightarrow \mathcal{B}$$

which is the symbolization of
if $\mathcal{A}$ then $\mathcal{B}$.

To paraphrase '$\mathcal{A}$ only if $\mathcal{B}$' as 'if $\mathcal{A}$ then $\mathcal{B}$' is at the very least misleading in cases involving temporal or causal factors. Consider the following example.

(o3) my tree will grow *only if* it receives adequate light

is best paraphrased

(p3) my tree will *not* grow *if* it does *not* receive adequate light

which is quite different from

(e3) *if* my tree grows, *then* it will receive adequate light.

The latter statement may indeed be true, but it suggests that the growing leads to, and precedes, getting adequate light (as often happens with trees competing with one another for available light). By contrast, the former suggests that getting adequate light is required, and hence precedes, growing (as happens with all photosynthetic organisms).

A major problem with (e1)-(e3) is with the tense in the consequents. The word 'then' makes it natural to use future tense, probably because 'then' is used both in a *logical* sense and in a *temporal* sense (for example, recall 'and then').

If we insist on translating 'only if' statements into 'if... then' statements, following the method above, then we must adjust the tenses appropriately. So, for example, getting adequate light precedes growing, so the appropriate tense is not simple future but future perfect. Adjusting the tenses in this manner, we obtain the following re-paraphrases of (e1)-(e3).

(p1') *if* I get an A in logic, *then* I *will have* taken the exams
(p2') *if* I get into law school, *then* I *will have* taken the LSAT
(p3') *if* my tree grows, *then* it *will have* received adequate light

Unlike the corresponding statements using simple future, these statements, which use future perfect tense, are more plausible paraphrases of the original 'only if' statements.

Nonetheless, 'not $\mathcal{A}$ if not $\mathcal{B}$' remains the generally most accurate paraphrase of '$\mathcal{A}$ only if B'.

15. 'IF AND ONLY IF'

Having examined 'if', and having examined 'only if', we next consider their natural conjunction, which is 'if and only if'. Consider the following sentence.

(e) you will pass *if and only if* you average at least fifty

This is naturally thought of as dividing into two halves, a promise-half and a threat-half. The promise is

(p) you will pass *if* you average at least fifty,

and the threat is

(t) you will pass *only if* you average at least fifty,

which we saw in the previous section may be paraphrased:

(t') you will *not* pass *if* you do *not* average at least fifty.

So (e) may be paraphrased as a conjunction:

(t'') you will pass *if* you average at least fifty,
and
you will *not* pass *if* you do *not* average at least fifty.

The first conjunct is symbolized:

$$A \rightarrow P$$

and the second conjunct is symbolized:

$$\sim A \rightarrow \sim P$$

so the conjunction is symbolized:

$$(A \rightarrow P) \& (\sim A \rightarrow \sim P)$$

The reader may recall that our analysis of the biconditional connective $\leftrightarrow$ is such that the above formula is truth-functionally equivalent to

$$P \leftrightarrow A$$

So $P\leftrightarrow A$ also counts as an acceptable symbolization of 'P if and only if A', although it does not do full justice to the internal logical structure of 'if and only if' statements, which are more naturally thought of as conjunctions of 'if' statements and 'only if' statements.

16. 'UNLESS'

There are numerous ways to express conditionals in English. We have already seen several conditional-forming expressions, including 'if', 'provided', 'only if'. In the present section, we consider a further conditional-forming expression – 'unless'.

'Unless' is very similar to 'only if', in the sense that it has a built-in negation. The difference is that, whereas 'only if' incorporates two negations, 'unless' incorporates only one. This means, in particular, that in order to paraphrase 'only if' statements using 'unless', one must add one explicit negation to the sentence. The following are examples of 'only if' statements, followed by their respective paraphrases using 'unless'.

(o1) I will graduate *only if* I pass logic
(u1) I will *not* graduate *unless* I pass logic
(u1') *unless* I pass logic, I will *not* graduate

(o2) I will pass logic *only if* I study
(u2) I will *not* pass logic *unless* I study
(u2') *unless* I study, I will *not* pass logic

Let us concentrate on the first one. We already know how to paraphrase and symbolize (o1), as follows.

(p1) I will *not* graduate *if* I do *not* pass logic
(p1') *if* I do *not* pass logic, *then* I will *not* graduate
(s1) $\sim P \rightarrow \sim G$

Now, comparing (u1) and (u1') with the last three items, we discern the following principle concerning 'unless'.

'unless' is equivalent to 'if not'

Here, 'if not' is short for 'if it is not true that'. Notice that this principle applies when 'unless' appears at the beginning of the statement, as well as when it appears in the middle of the statement.

The above principle may be restated as follows.

$\mathcal{A}$ unless $\mathcal{B}$ is equivalent to $\mathcal{A}$ if not $\mathcal{B}$ which is symbolized $\sim\mathcal{B} \rightarrow \mathcal{A}$	unless $\mathcal{A}$, $\mathcal{B}$ is equivalent to if not $\mathcal{A}$, then $\mathcal{B}$ which is symbolized $\sim\mathcal{A} \rightarrow \mathcal{B}$

17. THE STRONG SENSE OF 'UNLESS'

As with many words in English, the word 'unless' is occasionally used in a way different from its "official" meaning. As with the word 'or', which has both a weak (inclusive) sense and a strong (exclusive) sense, the word 'unless' also has both a weak and strong sense.

Just as we opt for the weak (inclusive) sense of 'or' in logic, we also opt for the weak sense of 'unless', which is summarized in the following principle.

the weak sense of 'unless' is equivalent to 'if not'

Unfortunately, 'unless' is not always intended in the weak sense. In addition to the meaning 'if not', various Webster Dictionaries give 'except when' and 'except on the condition that' as further meanings.

First, let us consider the meaning of 'except'; for example, consider the following fairly ordinary 'except' statement, which is taken from a grocery store sign.

(e1) open 24 hours a day *except* Sundays

It is plausible to suppose that (e1) means that the store is open 24 hours Monday-Saturday, and is *not* open 24 hours on Sunday (on Sunday, it may not be open at all, or it may only be open 8 hours). Thus, there are two implicit conditionals, as follows, where we let 'open' abbreviate 'open 24 hours'.

(c1) *if* it is *not* Sunday, *then* the store is open
(c2) *if* it is Sunday, *then* the store is *not* open

These two can be combined into the following biconditional.

(b) the store is open *if and only if* it is *not* Sunday

which is symbolized:

(s) $O \leftrightarrow \sim S$

Now, similar statements can be made using 'unless'. Consider the following statement from a sign on a swimming pool.

(u1) the pool may *not* be used *unless* a lifeguard is on duty

Following the dictionary definition, this is equivalent to:

(u1') the pool may *not* be used *except when* a lifeguard is on duty

which amounts to the conjunction,

(c) the pool may *not* be used *if* a lifeguard is *not* on duty, *and* the pool may be used *if* a lifeguard is on duty.

which, as noted earlier, is equivalent to the following biconditional,

(b) the pool may be used *if and only if* a lifeguard is on duty

By comparing (b) with the original statement (u1), we can discern the following principle about the strong sense of 'unless'.

> the strong sense of
> 'unless'
> is equivalent to
> 'if and only if not'

Or stating it using our symbols, we may state the principle as follows.

> $\mathcal{A}$ unless $\mathcal{B}$
> (in the strong sense of unless)
> is equivalent to
> $\mathcal{A} \leftrightarrow {\sim}\mathcal{B}$

It is not always clear whether 'unless' is intended in the strong or in the weak sense. Most often, the overall context is important for determining this. The following rules of thumb may be of some use.

> Usually, if it is intended in the strong sense, 'unless' is placed in the middle of a sentence; (the converse, however, is not true).

> Usually, if 'unless' is at the beginning of a statement, then it is intended in the weak sense.

> If it is not obvious that 'unless' is intended in the strong sense, you should assume that it is intended in the weak sense.

Note carefully: Although 'unless' is occasionally used in the strong sense, you may assume that every exercise uses 'unless' in the weak sense.

Exercise (an interesting coincidence): show that, whereas the weak sense of 'unless' is truth-functionally equivalent to the weak (inclusive) sense of 'or', the strong sense of 'unless' is truth-functionally equivalent to the strong (exclusive) sense of 'or'.

18. NECESSARY CONDITIONS

There are still other words used in English to express conditionals, most importantly the words ‘necessary’ and ‘sufficient’. In the present section, we examine conditional statements that involve ‘necessary’, and in the next section, we do the same thing with ‘sufficient’.

The following expressions are some of the common ways in which ‘necessary’ is used.

(n1) in order that...it is necessary that...
(n2) in order for...it is necessary for...
(n3) in order to...it is necessary to...
(n4) ...is a necessary condition for...
(n5) ...is necessary for...

The following are examples of mutually equivalent statements using ‘necessary’.

(N1) *in order that* I get an A, *it is necessary that* I take all the exams
(N2) *in order for* me to get an A, *it is necessary for* me to take all the exams
(N3) *in order to* get an A, *it is necessary to* take all the exams
(N4) taking all the exams *is a necessary condition for* getting an A
(N5) taking all the exams *is necessary for* getting an A

Statements involving ‘necessary’ can all be paraphrased using ‘only if’. A more direct approach, however, is first to paraphrase the sentence into the simplest form, which is:

(f) $\mathcal{A}$ is necessary for $\mathcal{B}$

Now, to say that one state of affairs (event) $\mathcal{A}$ is necessary for another state of affairs (event) $\mathcal{B}$ is just to say that *if* the first thing does *not* obtain (happen), *then neither* does the second. Thus, for example, to say

taking all the exams *is necessary for* getting an A

is just to say that if E (i.e., taking-the-exams) doesn't obtain then neither does A (i.e., getting-an-A). The sentence is accordingly paraphrased and symbolized as follows.

if not E, then not A $[\sim E \rightarrow \sim A]$

The general paraphrase principle is as follows.

$\mathcal{A}$ is necessary for $\mathcal{B}$
is paraphrased
if not $\mathcal{A}$, then not $\mathcal{B}$

19. SUFFICIENT CONDITIONS

The natural logical counterpart of 'necessary' is 'sufficient', which is used in the following ways, completely analogous to 'necessary'.

(s1) in order that...it is sufficient that...
(s2) in order for....it is sufficient for...
(s3) in order to....it is sufficient to....
(s4) ...is a sufficient condition for...
(s5) ...is sufficient for...

The following are examples of mutually equivalent statements using these different forms.

(S1) *in order that* I get an A *it is sufficient that* I get a 100 on every exam

(S2) *in order for* me to get an A *it is sufficient for* me to get a 100 on every exam

(S3) *in order to* get an A *it is sufficient* to get a 100 on every exam

(S4) getting a 100 on every exam *is a sufficient condition* for getting an A

(S5) getting a 100 on every exam *is sufficient for* getting an A

Just as necessity statements can be paraphrased like 'only if' statements, sufficiency statements can be paraphrased like 'if' statements. The direct approach is first to paraphrase the sufficiency statement in the following form.

(f) $\mathcal{A}$ is sufficient for $\mathcal{B}$

Now, to say that one state of affairs (event) $\mathcal{A}$ is sufficient for another state of affairs (event) $\mathcal{B}$ is just to say that $\mathcal{B}$ obtains (happens) *provided* (*if*) $\mathcal{A}$ obtains (happens). So for example, to say that

getting a 100 on every exam *is sufficient for* getting an A

is to say that

getting-an-A happens *provided* (*if*) getting-a-100 happens

which may be symbolized quite simply as:

$H \rightarrow A$

The general principle is as follows.

$\mathcal{A}$ is sufficient for $\mathcal{B}$ is paraphrased if $\mathcal{A}$, then $\mathcal{B}$

20. NEGATIONS OF NECESSITY AND SUFFICIENCY

First, note carefully that necessary conditions are quite different from sufficient conditions. For example,

> taking all the exams *is necessary for* getting an A,
> but
> taking all the exams *is not sufficient for* getting an A.

Similarly,

> getting a 100 *is sufficient for* getting an A,
> but
> getting a 100 *is not necessary for* getting an A.

This suggests that we can combine necessity and sufficiency in a number of ways to obtain various statements about the relation between two events (states of affairs). For example, we can say all the following, with respect to $\mathcal{A}$ and $\mathcal{B}$.

(c1) $\mathcal{A}$ is necessary for $\mathcal{B}$
(c2) $\mathcal{A}$ is sufficient for $\mathcal{B}$
(c3) $\mathcal{A}$ is not necessary for $\mathcal{B}$
(c4) $\mathcal{A}$ is not sufficient for $\mathcal{B}$

(c5) $\mathcal{A}$ is both necessary and sufficient for $\mathcal{B}$
(c6) $\mathcal{A}$ is necessary but not sufficient for $\mathcal{B}$
(c7) $\mathcal{A}$ is sufficient but not necessary for $\mathcal{B}$
(c8) $\mathcal{A}$ is neither necessary nor sufficient for $\mathcal{B}$

We have already discussed how to paraphrase (c1)-(c2). In the present section, we consider how to paraphrase (c3)-(c4), leaving (c5)-(c8) to a later section.

We start with the following example involving ‘not necessary’.

(1) attendance *is not necessary* for passing logic

This may be regarded as the negation of

(2) attendance *is necessary* for passing logic

As seen earlier, the latter may be paraphrased and symbolized as follows.

(p2) if I do *not* attend class, *then* I will *not* pass logic

(s2) $\sim A \rightarrow \sim P$

So the negation of (2), which is (1), may be paraphrased and symbolized as follows.

(p1) *it is not true that if* I do *not* attend class, *then* I will *not* pass logic;

(s1) $\sim(\sim A \rightarrow \sim P)$

Notice, once again, that voodoo does not prevail in logic; there is no obvious simplification of the three negations in the formula. The negations do not simply cancel each other out. In particular, the latter is *not* equivalent to the following.

(voodoo) $A \rightarrow P$

The latter says (roughly) that attendance will ensure passing; this is, of course, not true. Your dog can attend every class, if you like, but it won't pass the course. The former says that attendance is not necessary for passing; this is true, in the sense that attendance is not an official requirement.

Next, consider the following example involving 'not sufficient'.

(3) taking all the exams is *not sufficient* for passing logic

This may be regarded as the negation of

(4) taking all the exams *is sufficient* for passing logic.

The latter is paraphrased and symbolized as follows.

(p4) *if* I take all the exams, *then* I will pass logic

(s4) $E \rightarrow P$

So the negation of (4), which is (3), may be paraphrased and symbolized as follows.

(p3) *it is not true that if* I take all the exams, *then* I will pass logic

(s4) $\sim(E \rightarrow P)$

As usual, there is no simple-minded (voodoo) transformation of the negation. The negation of an English conditional does not have a straightforward simplification. In particular, it is *not* equivalent to the following

(voodoo) $\sim E \rightarrow \sim P$

The former says (roughly) that taking all the exams does *not* ensure passing; this is true; after all, you can fail all the exams. On the other hand, the latter says that if you don't take all the exams, then you won't pass. This is not true, a mere 70 on each of the first three exams will guarantee a pass, in which case you don't have to take all the exams in order to pass.

21. YET ANOTHER PROBLEM WITH THE TRUTH-FUNCTIONAL IF-THEN

According to our analysis, to say that one state of affairs (event) $\mathcal{A}$ is *not sufficient* for another state of affairs (event) $\mathcal{B}$ is to say that it is not true that if the first obtains (happens), then so will the second. In other words,

> $\mathcal{A}$ is not sufficient for $\mathcal{B}$
> is paraphrased:
> it is not true that if $\mathcal{A}$ then $\mathcal{B}$,
> which is symbolized:
> $\sim(\mathcal{A} \to \mathcal{B})$

As noted in the previous section, there is no obvious simple transformation of the latter formula. On the other hand, the latter formula can be simplified in accordance with the following truth-functional equivalence, which can be verified using truth tables.

> $\sim(\mathcal{A} \to \mathcal{B})$
>
> is truth-functionally equivalent to
>
> $\mathcal{A}$ & $\sim\mathcal{B}$

Consider our earlier example,

(1) taking all the exams is *not sufficient* for passing logic

Our proposed paraphrase and symbolization is:

(p1) *it is not true that if* I take all the exams *then* I will pass logic
(s1) $\sim$(E $\to$ P)

But this is truth-functionally equivalent to:

(s2) E & $\sim$P
(p2) I will take all the exams, *and* I will *not* pass

However, to say that taking the exams is *not sufficient* for passing logic is not to say you *will* take all the exams yet you won't pass; rather, it says that it is *possible* (in some sense) for you to take the exams and yet not pass.

However, possibility is not a truth-functional concept; some falsehoods are possible; some falsehoods are impossible. Thus, possibility cannot be analyzed in truth-functional logic.

We have dealt with negations of conditionals, which lead to difficulties with the truth-functional analysis of necessity and sufficiency. Nevertheless, our paraphrase technique involving 'if...then' is not impugned, only the truth-functional analysis of 'if...then'.

22. COMBINATIONS OF NECESSITY AND SUFFICIENCY

Recall that the possible combinations of statements about necessity and sufficiency are as follows.

(c1) $\mathcal{A}$ is necessary for $\mathcal{B}$
(c2) $\mathcal{A}$ is sufficient for $\mathcal{B}$
(c3) $\mathcal{A}$ is not necessary for $\mathcal{B}$
(c4) $\mathcal{A}$ is not sufficient for $\mathcal{B}$

(c5) $\mathcal{A}$ is both necessary and sufficient for $\mathcal{B}$
(c6) $\mathcal{A}$ is necessary, but not sufficient, for $\mathcal{B}$
(c7) $\mathcal{A}$ is sufficient, but not necessary, for $\mathcal{B}$
(c8) $\mathcal{A}$ is neither necessary nor sufficient for $\mathcal{B}$

We have already dealt with (c1)-(c4). We now turn to (c5)-(c8).

First, notice carefully that (c1)-(c4) are less informative than (c5)-(c8). For example, if I say $\mathcal{A}$ is necessary for $\mathcal{B}$, and leave it at that, I am not saying whether $\mathcal{A}$ is sufficient for $\mathcal{B}$, one way or the other. Similarly, if I say that Jay is a Sophomore, and leave it at that, I have said nothing concerning whether Kay is a Sophomore, one way or the other.

Consider the following example of combination (c5).

(e5) averaging at least 50 *is both necessary and sufficient for* passing

This is quite clearly the conjunction of a necessity statement and a sufficiency statement, as follows.

averaging at least fifty *is necessary for* passing,
and
averaging at least fifty *is sufficient for* passing

The latter is symbolized:

$(\sim F \rightarrow \sim P) \;\&\; (F \rightarrow P)$

Reading this back into English, we obtain

if I do *not* average at least fifty, *then* I will *not* pass,
and
if do average at least fifty, *then* I will pass

Next, consider the following example of combination (c6).

(e6) taking all the exams *is necessary, but not sufficient, for* getting an A

This is a somewhat more complex conjunction:

taking all the exams *is necessary for* getting an A,
but
taking all the exams *is not sufficient for* getting an A

which is symbolized:

(~T → ~A) & ~(T → A)

Reading this back into English, we obtain

if I do *not* take all the exams, *then* I will *not* get an A, *but it is not true that if* I do take all the exams *then* I will get an A

Next, consider the following example of combination (c7).

(e7) getting 100 on every exam *is sufficient,*
but not necessary, for getting an A

This too is a conjunction:

getting 100 on every exam *is sufficient for* getting an A,
but
getting 100 on every exam *is not necessary for* getting an A

which is symbolized:

(H → A) & ~(~H → ~A)

Reading this back into English, we obtain

if I get a 100 on every exam, *then* I will get an A,
but it is not true that
if I do *not* get a 100 on every exam *then* I will *not* get an A

Finally, consider the following example of combination (c8).

(e8) attending class is neither necessary nor sufficient for passing

which may be paraphrased as a complex conjunction:

attending class *is not necessary for* passing,
and
attending class *is not sufficient for* passing

which is symbolized:

~(~A → ~P) & ~(A → P)

Reading this back into English, we obtain

it is not true that
if I do *not* attend class
then I will *not* pass,
nor is it true that
if I do attend class
then I will pass

23. ‘OTHERWISE’

In the present section, we consider two three-place connective expressions that are used to express conditionals in English. The key words are ‘otherwise’ and ‘in which case’.

First, the general forms for ‘otherwise’ statements are the following:

(o1) if $\mathcal{A}$, then $\mathcal{B}$; otherwise $\mathcal{C}$
(o2) if $\mathcal{A}$, $\mathcal{B}$; otherwise $\mathcal{C}$
(o3) $\mathcal{B}$ if $\mathcal{A}$; otherwise $\mathcal{C}$

The following is a typical example.

(e1) *if* it is sunny, I'll play tennis
otherwise, I'll play racquetball

This statement asserts what the speaker will do *if* it is sunny, and it further asserts what the speaker will do *otherwise*, i.e., *if* it is *not* sunny. In other words, (e1) can be paraphrased as a conjunction, as follows.

(p1) *if* it is sunny, *then* I'll play tennis,
and
if it is *not* sunny, *then* I'll play racquetball

The latter statement is symbolized:

(s1) $(S \rightarrow T) \,\&\, ({\sim}S \rightarrow R)$

The general principle governing the paraphrase of ‘otherwise’ statements is as follows.

if $\mathcal{A}$, then $\mathcal{B}$; otherwise $\mathcal{C}$ is paraphrased if $\mathcal{A}$, then $\mathcal{B}$, and if not $\mathcal{A}$, then $\mathcal{C}$, which is symbolized $(\mathcal{A} \rightarrow \mathcal{B}) \,\&\, ({\sim}\mathcal{A} \rightarrow \mathcal{C})$

A simple variant of ‘otherwise’ is ‘else’, which is largely interchangeable with ‘otherwise’. In a number of high level programming languages, including BASIC and PASCAL, ‘else’ is used in conjunction with ‘if...then’ to issue commands. For example, the following is a typical BASIC command.

(c) if X<=100 then goto 300 else goto 400

This is equivalent to two commands in succession:

```
if X<=100 then goto 300
if not(X<=100) then goto 400
```

In a computer language, such as BASIC, there is always a "default" ‘else’ command, namely to go to the next line and follow that command. So, for example, the command line

if X<=100 then goto 400

standing alone means

if X<=100 then goto 400 else goto next line

Unlike ‘if...then’ statements in computer languages, English ‘if...then’ statements do not incorporate default ‘else’ clauses. For example, the statement

(e2) I'll go to the doctor *if* I break my arm

says nothing about what the speaker will or won't do if he/she does *not* break an arm. Similarly, if I say I won't play tennis if it is raining, and leave it at that, I am not committing myself to anything in case it is not raining; I leave that case open, or undetermined.

That brings us to an expression that is very similar to ‘otherwise’ – namely, ‘in which case’. Consider the following example.

(e2) I'll play tennis *unless* it is raining, *in which case* I'll play squash

Recall that ‘unless’ is equivalent to ‘if not’. So, as with ‘otherwise’ statements, there are two cases considered – it rains; it doesn't rain. Statement (e2) asserts what the speaker will do in each case – *in case* it is *not* raining, and *in case* it is raining. Recall ‘in case’ is a variant of ‘if’.

The paraphrase of (e2) is similar to that of (e1).

(p) *if* it is *not* raining, *then* I'll play tennis,
and
if it is raining, *then* I'll play squash

The latter is symbolized:

(s) $(\sim R \rightarrow T) \& (R \rightarrow S)$

The overall paraphrase pattern is given by the following principle.

$\mathcal{A}$ unless $\mathcal{B}$, in which case $\mathcal{C}$
is paraphrased
if not $\mathcal{B}$, then $\mathcal{A}$, and if $\mathcal{B}$ then $\mathcal{C}$
which is symbolized
$(\sim\mathcal{B} \rightarrow \mathcal{A}) \& (\mathcal{B} \rightarrow \mathcal{C})$

24. PARAPHRASING COMPLEX STATEMENTS

As noted earlier, compound statements may be built up from statements which are themselves compound statements. There are no theoretical limits to the complexity of compound statements, although there are practical limits, based on human linguistic capabilities.

We have already dealt with a number of complex statements in connection with the various non-standard connectives. We now systematically consider complex statements that involve various combinations of non-standard connectives. For example, we are interested in what happens when both 'unless' and 'only if' appear in the same sentence.

In paraphrasing and symbolizing complex statements, it is best to proceed systematically, in small steps. As one gets better, many intermediate steps can be done in one's head. On the easy ones, perhaps all the intermediate steps can be done in one's head. Still, it is a good idea to reason through the easy ones systematically, in order to provide practice in advance of doing the hard ones.

The first step in paraphrasing statements is:

Step 1:	Identify the simple (atomic) statements, and abbreviate them by upper case letters.

In most of the exercises, certain words are entirely capitalized in order to suggest to the student what the atomic statements are. For example, in the statement 'JAY and KAY are Sophomores' the atomic formulas are 'J' and 'K'.

At this stage of analysis, it is important to be clear concerning what each atomic formula stands for; it is especially important to be clear that each letter abbreviates a *complete sentence*. For example, in the above statement, 'J' does not stand for 'Jay', since this is not a sentence. Rather, it stands for 'Jay is a Sophomore'. Similarly, 'K' does not stand for 'Kay', but rather 'Kay is a Sophomore'.

Having identified the simple statements, and having established their abbreviations, the next step is:

Step 2:	Identify all the connectives, noting which ones are standard, and which ones are not standard.

Having identified the atomic statements and the connectives, the next step is:

Step 3:	Write down the first hybrid formula, making sure to retain internal punctuation.

The first hybrid formula is obtained from the original statement by replacing the simple statements by their abbreviations. A hybrid formula is so called because it contains both English words and symbols from sentential logic. Punctuation provides important clues about the logical structure of the sentence.

The first three steps may be better understood by illustration. Consider the following example.

Example 1

(e1) if neither Jay nor Kay is working, then we will go on vacation.

In this example, the simple statements are:

J: Jay is working
K: Kay is working
V: we go on vacation

and the connectives are:

if...then	(standard)
neither...nor	(non-standard)

Thus, our first hybrid formula is:

(h1) if neither J nor K, then V

Having obtained the first hybrid formula, the next step is to

Step 4:	Identify the major connective.

Here, the commas are important clues. In (h1), the placement of the comma indicates that the major connective is 'if...then', the structure being:

if neither J nor K,
then V

Having identified the major connective, we go on to the next step.

Step 5:	Symbolize the major connective if it is standard; otherwise, paraphrase it into standard form, and go back to step 4, and work on the resulting (hybrid) formula.

In (h1), the major connective is 'if...then', which is standard, so we symbolize it, which yields the following hybrid formula.

(h2) (neither J nor K) $\rightarrow$ V

Notice that, as we symbolize the connectives, we must provide the necessary logical punctuation (i.e., parentheses).

At this point, the next step is:

Step 6: Work on the constituent formulas separately.

In (h2), the constituent formulas are:

(c1) neither J nor K
(c2) V

The latter formula is fully symbolic, so we are through with it. The former is not fully symbolic, so we must work on it further. It has only one connective, 'neither...nor', which is therefore the major connective. It is not standard, so we must paraphrase it, which is done as follows.

(c1) neither J nor K
(p1) not J and not K

The latter formula is in standard form, so we symbolize it as follows.

(s1) ~J & ~K

Having dealt with the constituent formulas, the next step is:

Step 7: Substitute symbolizations of constituents back into (original) hybrid formula.

In our first example, this yields:

(s2) (~J & ~K) → V

Once you have a purely symbolic formula, the final step is:

Step 8: Translate the formula back into English and compare with the original statement.

This is to make sure the final formula says the same thing as the original statement. In our example, translating yields the following.

(t1) *if* Jay is *not* working and Kay is *not* working, *then* we will go on vacation.

Comparing this with the original,

(e1) if neither Jay nor Kay is working, then we will go on vacation

we see they are equivalent, so we are through.

Our first example is simple insofar as the major connective is standard. In many statements, all the connectives are non-standard, and so they have to be paraphrased in accordance with the principles discussed in previous sections. Consider the following example.

Example 2

(e2) you will pass unless you goof off, provided that you are intelligent.

In this statement, the simple statements are:

I: you are intelligent
P: you pass
G: you goof off

and the connectives are:

unless	(non-standard)
provided that	(non-standard)

Thus, the first stage of the symbolization yields the following hybrid formula.

(h1) P unless G, provided that I

Next, we identity the major connective. Once again, the placement of the comma tells us that 'provided that' is the major connective, the overall structure being:

P unless G,
provided that I

We cannot directly symbolize 'provided that', since it is non-standard. We must first paraphrase it. At this point, we recall that 'provided that' is equivalent to 'if', which is a simple variant of 'if...then'. This yields the following successive paraphrases.

(h2) P unless G, if I
(h3) if I, then P unless G

In (h3), the major connective is 'if...then', which is standard, so we symbolize it, which yields:

(h4) I → (P unless G)

We next work on the parts. The antecedent is finished, so we more to the consequent.

(c) P unless G

This has one connective, 'unless', which is non-standard, so we paraphrase and symbolize it as follows.

(c) P unless G
(p) P if not G,

(p') if not G, then P,
(s) ~G → P

Substituting the parts back into the whole, we obtain the final formula.

(f) I → (~G → P)

Finally, we translate (f) back into English, which yields:

(t) *if* you are intelligent, *then* if you do *not* goof off *then* you will pass

Although this is not the exact same sentence as the original, it should be clear that they are equivalent in meaning.

Let us consider an example similar to Example 2.

Example 3

(e3) unless the exam is very easy, I will make a hundred only if I study

In this example, the simple statements are:

E: the exam is very easy
H: I make a hundred
S: I study

and the connectives are:

unless	(non-standard)
only if	(non-standard)

Having identified the logical parts, we write down the first hybrid formula.

(h1) unless E, H only if S

Next, we observe that 'unless' is the principal connective. Since it is non-standard, we cannot symbolize it directly, so we paraphrase it, as follows.

(h2) if not E, then H only if S

We now work on the new hybrid formula (h2). We first observe that the major connective is 'if...then'; since it is standard, we symbolize it, which yields:

(h3) not E $\rightarrow$ (H only if S)

Next, we work on the separate parts. The antecedent is simple, and is standard form, being symbolized:

(a) $\sim$E

The consequent has just one connective 'only if', which is non-standard, so we paraphrase and symbolize it as follows.

(c) H only if S
(p) not H if not S
(p') if not S, then not H
(s) $\sim S \rightarrow \sim H$

Next, we substitute the parts back into (h3), which yields:

(f) $\sim E \rightarrow (\sim S \rightarrow \sim H)$

Finally, we translate (f) back into English, which yields:

(t) *if* the exam is *not* very easy,
  then if I do *not* study
    then I will *not* get a hundred

Comparing this statement with the original statement, we see that they say the same thing.

The next example is slightly more complicated, being a conditional in which both constituents are conditionals.

Example 4

(e4) *if* Jones will work *only if* Smith is fired, *then* we should fire Smith *if* we want the job finished

In (e4), the simple statements are:

J: Jones works
F: we do fire Smith
S: we should fire Smith
W: we want the job finished

and the connectives are:

if...then	(standard)
only if	(non-standard)
if	(non-standard)

Next, we write down the first hybrid formula, which is:

(h1) if J only if F, then S if W

The comma placement indicates that the principal connective is 'if...then'. It is standard, so we symbolize it, which yields:

(h2) (J only if F) $\rightarrow$ (S if W)

Next, we work on the constituents separately. The antecedent is paraphrased and symbolized as follows.

(a) J only if F
(p) not J if not F
(p') if not F, then not J
(s) $\sim F \rightarrow \sim J$

The consequent is paraphrased and symbolized as follows.

(c) S if W
(p) if W, then S
(s) $W \rightarrow S$

Substituting the constituent formulas back into (h2) yields:

(f) $(\sim F \rightarrow \sim J) \rightarrow (W \rightarrow S)$

The direct translation of (f) into English reads as follows.

(t) *if if* we do *not* fire Smith *then* Jones does *not* work,
then if we want the job finished
then we should fire Smith

The complexity of the conditional structure of this sentence renders a direct translation difficult to understand. The major problem is the "stuttering" at the beginning of the sentence. The best way to avoid this problem is to opt for a more idiomatic translation (just as we do with negations); specifically, we replace some if-then's by simple variant forms. The following is an example of a more natural, idiomatic translation.

(t') *if* Jones will *not* work *if* Smith is *not* fired,
then if we want the job finished we should fire Smith

Comparing this paraphrase, in more idiomatic English, with the original statement, we see that they are equivalent in meaning.

Our last example involves the notion of necessary condition.

Example 5

(e5) in order to put on the show it will be necessary to find a substitute, if neither the leading lady nor her understudy recovers from the flu

In (e5), the simple statements are:

P: we put on the show
S: we find a substitute
L: the leading lady recovers from the flu
U: the understudy recovers from the flu

and the connectives are:

in order to... it is necessary to	(non-standard)
if	(non-standard)
neither...nor	(non-standard)

The first hybrid formula is:

(h1) in order that P it is necessary that S, if neither L nor U

Next, the principal connective is ‘if’, which is not in standard form; converting it into standard form yields:

(h2) if neither L nor U, then in order that P it is necessary that S

Here, the principal connective is ‘if...then’, which is standard, so we symbolize it as follows.

(h3) (neither L nor U) $\rightarrow$ (in order that P it is necessary that S)

We next attack the constituents. The antecedent is paraphrased as follows.

(a) neither L nor U
(p) not L and not U
(s) ~L & ~U

The consequent is paraphrased as follows.

(c) in order that P it is necessary that S
(p) S is necessary for P
(p') if not S, then not P
(s) ~S → ~P

Substituting the parts back into (h3), we obtain:

(f) (~L & ~U) → (~S → ~P)

Translating (f) back into English, we obtain:

(t) *if* the leading lady does *not* recover from the flu *and* her understudy does *not* recover from the flu, *then if* we do *not* find a substitute *then* we do *not* put on the show

Comparing (t) with the original statement, we see that they are equivalent in meaning.

By way of concluding this chapter, let us review the basic steps involved in symbolizing complex statements.

25. GUIDELINES FOR TRANSLATING COMPLEX STATEMENTS

Step 1:	Identify the simple (atomic) statements, and abbreviate them by upper case letters. What complete sentence does each letter stand for?

Step 2:	Identify all the connectives, noting which ones are standard, and which ones are non-standard.

Step 3:	Write down the first hybrid formula, making sure to retain internal punctuation.

Step 4:	Identify the major connective.

Step 5:	Symbolize the major connective if it is standard, introducing parentheses as necessary; otherwise, paraphrase it into standard form, and go back to step 4, and work on the resulting (hybrid) formula.

Step 6:	Work on the constituent formulas separately, which means applying steps 4-5 to each constituent formula.

Step 7:	Substitute symbolizations of constituents back into (original) hybrid formula.

Step 8:	Translate the formula back into English and compare with the original statement.

26. EXERCISES FOR CHAPTER 4

Directions: Translate each of the following statements into the language of sentential logic. Use the suggested abbreviations (capitalized words), if provided; otherwise, devise an abbreviation scheme of your own. In each case, write down what atomic statement each letter stands for, making sure it is a *complete sentence*. Letters should stand for positively stated sentences, not negatively stated ones; for example, the negative sentence 'I am not hungry' should be symbolized as '~H' using 'H' to stand for 'I am hungry'.

EXERCISE SET A

1. Although it is RAINING, I plan to go JOGGING this afternoon.
2. It is not RAINING, but it is still too WET to play.
3. JAY and KAY are Sophomores.
4. It is DINNER time, but I am not HUNGRY.
5. Although I am TIRED, I am not QUITTING.
6. Jay and Kay are roommates, but they hate one another.
7. Jay and Kay are Republicans, but they both hate Nixon.
8. KEEP trying, and the answer will APPEAR.
9. GIVE him an inch, and he will TAKE a mile.
10. Either I am CRAZY or I just SAW a flying saucer.
11. Either Jones is a FOOL or he is DISHONEST.
12. JAY and KAY won't both be present at graduation.
13. JAY will win, or KAY will win, but not both.
14. Either it is RAINING, or it is SUNNY and COLD.
15. It is RAINING or OVERCAST, but in any case it is not SUNNY.
16. If JONES is honest, then so is SMITH.
17. If JONES isn't a crook, then neither is SMITH.
18. Provided that I CONCENTRATE, I will not FAIL.
19. I will GRADUATE, provided I pass both LOGIC and HISTORY.
20. I will not GRADUATE if I don't pass both LOGIC and HISTORY.

EXERCISE SET B

21. Neither JAY nor KAY is able to attend the meeting.
22. Although I have been here a LONG time, I am neither TIRED nor BORED.
23. I will GRADUATE this semester only if I PASS intro logic.
24. KAY will attend the party only if JAY does not.
25. I will SUCCEED only if I WORK hard and take RISKS.
26. I will go to the BEACH this weekend, unless I am SICK.
27. Unless I GOOF off, I will not FAIL intro logic.
28. I won't GRADUATE unless I pass LOGIC and HISTORY.
29. In order to ACE intro logic, it is sufficient to get a HUNDRED on every exam.
30. In order to PASS, it is necessary to average at least FIFTY.
31. In order to become a PHYSICIAN, it is necessary to RECEIVE an M.D. and do an INTERNSHIP.
32. In order to PASS, it is both necessary and sufficient to average at least FIFTY.
33. Getting a HUNDRED on every exam is sufficient, but not necessary, for ACING intro logic.
34. TAKING all the exams is necessary, but not sufficient, for ACING intro logic.
35. In order to get into MEDICAL school, it is necessary but not sufficient to have GOOD grades and take the ADMISSIONS exam.
36. In order to be a BACHELOR it is both necessary and sufficient to be ELIGIBLE but not MARRIED.
37. In order to be ARRESTED, it is sufficient but not necessary to COMMIT a crime and GET caught.
38. If it is RAINING, I will play BASKETBALL; otherwise, I will go JOGGING.
39. If both JAY and KAY are home this weekend, we will go to the BEACH; otherwise, we will STAY home.
40. JONES will win the championship unless he gets INJURED, in which case SMITH will win.

EXERCISE SET C

41. We will have DINNER and attend the CONCERT, provided that JAY and KAY are home this weekend.
42. If neither JAY nor KAY can make it, we should either POSTPONE or CANCEL the trip.
43. Both Jay and Kay will go to the beach this weekend, provided that neither of them is sick.
44. I'm damned if I do, and I'm damned if I don't.
45. If I STUDY too hard I will not ENJOY college, but at the same time I will not ENJOY college if I FLUNK out.
46. If you NEED a thing, you will have THROWN it away, and if you THROW a thing away, you will NEED it.
47. If you WORK hard only if you are THREATENED, then you will not SUCCEED.
48. If I do not STUDY, then I will not PASS unless the prof ACCEPTS bribes.
49. Provided that the prof doesn't HATE me, I will PASS if I STUDY.
50. Unless logic is very DIFFICULT, I will PASS provided I CONCENTRATE.
51. Unless logic is EASY, I will PASS only if I STUDY.
52. Provided that you are INTELLIGENT, you will FAIL only if you GOOF off.
53. If you do not PAY, Jones will KILL you unless you ESCAPE.
54. If he CATCHES you, Jones will KILL you unless you PAY.
55. Provided that he has made a BET, Jones is HAPPY if and only if his horse WINS.
56. If neither JAY nor KAY comes home this weekend, we shall not stay HOME unless we are SICK.
57. If you MAKE an appointment and do not KEEP it, then I shall be ANGRY unless you have a good EXCUSE.
58. If I am not FEELING well this weekend, I will not GO out unless it is WARM and SUNNY.
59. If JAY will go only if KAY goes, then we will CANCEL the trip unless KAY goes.

EXERCISE SET D

60. If KAY will come to the party only if JAY does not come, then provided we WANT Kay to come we should DISSUADE Jay from coming.
61. If KAY will go only if JAY does not go, then either we will CANCEL the trip or we will not INVITE Jay.
62. If JAY will go only if KAY goes, then we will CANCEL the trip unless KAY goes.
63. If you CONCENTRATE only if you are INSPIRED, then you will not SUCCEED unless you are INSPIRED.
64. If you are HAPPY only if you are DRUNK, then unless you are DRUNK you are not HAPPY.
65. In order to be ADMITTED to law school, it is necessary to have GOOD grades, unless your family makes a large CONTRIBUTION to the law school.
66. I am HAPPY only if my assistant is COMPETENT, but if my assistant is COMPETENT, then he/she is TRANSFERRED to a better job and I am not HAPPY.
67. If you do not CONCENTRATE well unless you are ALERT, then you will FLY an airplane only if you are SOBER; provided that you are not a MANIAC.
68. If you do not CONCENTRATE well unless you are ALERT, then provided that you are not a MANIAC you will FLY an airplane only if you are SOBER.
69. If you CONCENTRATE well only if you are ALERT, then provided that you are WISE you will not FLY an airplane unless you are SOBER.
70. If you CONCENTRATE only if you are THREATENED, then you will not PASS unless you are THREATENED – provided that CONCENTRATING is a necessary condition for PASSING.
71. If neither JAY nor KAY is home this weekend, we will go to the BEACH; otherwise, we will STAY home.

27. ANSWERS TO EXERCISES FOR CHAPTER 4

1. R & J
2. ~R & W
3. J & K
4. D & ~H
5. T & ~Q
6. R & (J & K)
   R: Jay and Kay are roommates
   J: Jay hates Kay
   K: Kay hates Jay
7. (J & K) & (H & N)
   J: Jay is a Republican; K: Kay is a Republican
   H: Jay hates Nixon; N: Kay hates Nixon
8. K → A
9. G → T
10. C ∨ S
11. F ∨ D
12. ~(J & K)
13. (J ∨ K) & ~(J & K)
14. R ∨ (S & C)
15. (R ∨ O) & ~S
16. J → S
17. ~J → ~S
18. C → ~F
19. (L & H) → G
20. ~(L & H) → ~G
21. ~J & ~K [or: ~(J ∨ K)]
22. L & (~T & ~B) [or: L & ~(T ∨ B)]
23. ~P → ~G
24. ~~J → ~K [J → ~K]
25. ~(W & R) → ~S
26. ~S → B
27. ~G → ~F
28. ~(L & H) → ~G
29. H → A
30. ~F → ~P
31. ~(R & I) → ~P
32. (~F → ~P) & (F → P)
33. (H → A) & ~(~H → ~A)
34. (~T → ~A) & ~(T → A)
35. [~(G & A) → ~M] & ~[(G & A) → M]
36. [~(E & ~M) → ~B] & [(E & ~M) → B]
37. [(C & G) → A] & ~[~(C & G) → ~A]
38. (R → B) & (~R → J)
39. [(J & K) → B] & [~(J & K) → S]
40. (~I → J) & (I → S)
41. (J & K) → (D & C)

42. $(\sim J \& \sim K) \rightarrow (P \vee C)$
43. $(\sim S \& \sim T) \rightarrow (J \& K)$
    S: Jay is sick; T: Kay is sick;
    J: Jay will go to the beach; K: Kay will go to the beach.
44. $(A \rightarrow D) \& (\sim A \rightarrow D)$
    A: I do (what ever action is being discussed);
    D: I am damned.
45. $(S \rightarrow \sim E) \& (F \rightarrow \sim E)$
46. $(N \rightarrow T) \& (T \rightarrow N)$
47. $(\sim T \rightarrow \sim W) \rightarrow \sim S$
48. $\sim S \rightarrow (\sim A \rightarrow \sim P)$
49. $\sim H \rightarrow (S \rightarrow P)$
50. $\sim D \rightarrow (C \rightarrow P)$
51. $\sim E \rightarrow (\sim S \rightarrow \sim P)$
52. $I \rightarrow (\sim G \rightarrow \sim F)$
53. $\sim P \rightarrow (\sim E \rightarrow K)$
54. $C \rightarrow (\sim P \rightarrow K)$
55. $B \rightarrow [(W \rightarrow H) \& (\sim W \rightarrow \sim H)]$
56. $(\sim J \& \sim K) \rightarrow (\sim S \rightarrow \sim H)$
57. $(M \& \sim K) \rightarrow (\sim E \rightarrow A)$
58. $\sim F \rightarrow [\sim(W \& S) \rightarrow \sim G]$
59. $(\sim K \rightarrow \sim J) \rightarrow (\sim K \rightarrow C)$
60. $(\sim\sim J \rightarrow \sim K) \rightarrow (W \rightarrow D)$
61. $(\sim\sim J \rightarrow \sim K) \rightarrow (C \vee \sim I)$
62. $(\sim K \rightarrow \sim J) \rightarrow (\sim K \rightarrow C)$
63. $(\sim I \rightarrow \sim C) \rightarrow (\sim I \rightarrow \sim S)$
64. $(\sim D \rightarrow \sim H) \rightarrow (\sim D \rightarrow \sim H)$
65. $\sim C \rightarrow (\sim G \rightarrow \sim A)$
66. $(\sim C \rightarrow \sim H) \& (C \rightarrow [T \& \sim H])$
67. $\sim M \rightarrow [(\sim A \rightarrow \sim C) \rightarrow (\sim S \rightarrow \sim F)]$
68. $(\sim A \rightarrow \sim C) \rightarrow [\sim M \rightarrow (\sim S \rightarrow \sim F)]$
69. $(\sim A \rightarrow \sim C) \rightarrow [W \rightarrow (\sim S \rightarrow \sim F)]$
70. $(\sim C \rightarrow \sim P) \rightarrow [(\sim T \rightarrow \sim C) \rightarrow (\sim T \rightarrow \sim P)]$
71. $[(\sim J \& \sim K) \rightarrow B] \& [\sim(\sim J \& \sim K) \rightarrow S]$