7 TRANSLATIONS IN POLYADIC PREDICATE LOGIC

1. INTRODUCTION

Recall that predicate logic can be conveniently divided into monadic predicate logic, on the one hand, and polyadic predicate logic, on the other. Whereas the former deals exclusively with 1-place (monadic) predicates, the latter deals with all predicates (1-place, 2-place, etc.). In the present chapter, we turn to quantification in the context of polyadic predicate logic.

The reason for being interested in polyadic logic is simple: although monadic predicate logic reveals much more logical structure in English sentences than does sentential logic, monadic logic often does not reveal enough logical structure.

Consider the following argument.

(A) Every Freshman is a student
/Anyone who respects every student respects every Freshman

If we symbolize this in monadic logic, we obtain the following.

$\forall x(Fx \rightarrow Sx)$	[every F is S]
/ $\forall x(Kx \rightarrow Lx)$	[every K is L]

The following is the translation scheme:

Fx:	x is a Freshman
Sx:	x is a student
Kx:	x respects every student
Lx:	x respects every Freshman

The trouble with this analysis, which is the best we can do in monadic predicate logic, is that the resulting argument form is invalid. Yet, the original concrete argument is valid. This means that our analysis of the logical form of (A) is inadequate.

In order to provide an adequate analysis, we need to provide a deeper analysis of the formulas,

Kx:	x respects every student
Lx:	x respects every Freshman

These formulas are logically analyzed into the following items:

student:	Sy:	y is a student
Freshman:	Fy:	y is a Freshman
respects	Rxy:	x respects y
every:	$\forall$y:	for any person y

Thus, the formulas are symbolized as follows

Kx: x respects every student

$\forall y(Sy \rightarrow Rxy)$

for any person y:
if y is a student, then x respects y

Lx: x respects every Freshman;

$\forall y(Fy \rightarrow Rxy)$

for any person y:
if y is a Freshman, then x respects y

Thus, the argument form, according to our new analysis is:

$\forall x(Fx \rightarrow Sx)$
$/ \forall x(\forall y(Sy \rightarrow Rxy) \rightarrow \forall y(Fy \rightarrow Rxy))$

This argument form is valid, as we will be able to demonstrate in a later chapter. It is a fairly complex example, so it may not be entirely clear at the moment. Don't worry just yet! The important point right now is to realize that many sentences and arguments have further logical structure whose proper elucidation requires polyadic predicate logic. The example above is fairly complex. In the next section, we start with more basic examples of polyadic quantification.

2. SIMPLE POLYADIC QUANTIFICATION

In the present section, we examine the simplest class of examples of polyadic quantification – those involving an atomic formula constructed from a two-place predicate. First, recall that a two-place predicate is an expression that forms a formula (open or closed) when combined with two singular terms.

For example, consider the two-place predicate '...respects...', abbreviated R. With this predicate, we can form various formulas, including the following.

(1)	Jay respects Kay	Rjk
(2)	he respects Kay	Rxk
(3)	Jay respects her	Rjy
(4)	he respects her	Rxy
(5)	she respects herself	Rxx

The particular pronouns used above are completely arbitrary (any third person singular pronoun will do).

Now, the grammar of predicate logic has the following feature: if we have a formula, we can prefix it with a quantifier, and the resulting expression is also a formula. This merely restates the idea that quantifiers are one-place connectives.

Occasionally, however, quantifying a formula is trivial or pointless; for example,

∀xRjk   everyone is such that Jay respects Kay

says exactly the same thing as

Rjk   Jay respects Kay

This is an example of trivial (or vacuous) quantification.

In other cases, quantification is significant. For example, beginning with formulas (2)-(5), we can construct the following formulas, which are accompanied by English paraphrases.

(2a)	∀xRxk	everyone respects Kay
(2b)	∃xRxk	someone respects Kay
(3a)	∀yRjy	Jay respects everyone
(3b)	∃yRjy	Jay respects someone
(4a)	∀xRxy	everyone respects her
(4b)	∃xRxy	someone respects her
(4c)	∀yRxy	he respects everyone
(4d)	∃yRxy	he respects someone
(5a)	∀xRxx	everyone respects him(her)self
(5b)	∃xRxx	someone respects him(her)self

Now, (4a)-(4d) have variables that can be further quantified in a significant way. So prefixing (4a)-(5b) yields the following formulas.

(4a1)	∀y∀xRxy
(4a2)	∃y∀xRxy
(4b1)	∀y∃xRxy
(4b2)	∃y∃xRxy
(4c1)	∀x∀yRxy
(4c2)	∃x∀yRxy
(4d1)	∀x∃yRxy
(4d2)	∃x∃yRxy

How do we translate such formulas into English. As it turns out, there is a handy step-by-step procedure for translating formulas (4a1)-(4d2) into colloquial English – supposing that we are discussing people exclusively, and supposing that the predicate is '...respects...' This procedure is given as follows.

Step 1: Look at the first quantifier, and read it as follows:

(a)	universal (∀)	everyone
(b)	existential (∃)	there is someone who

Step 2: Look to see which variable is quantified (is it 'x' or 'y'?), then check where that variable appears in the quantified formula; does it appear in the first (active) position, or does it appear in the second (passive) position? If it appears in the first (active) position, then read the verb in the active voice as 'respects'. If it appears in the second (passive)

position, then read the verb in the passive voice as 'is respected by' (passive voice).

(a)	active	respects
(b)	passive	is respected by

Step 3: Look at the second quantifier, and read it as follows:

(a)	universal (∀)	everyone
(b)	existential (∃)	someone or other

Step 4: String together the components obtained in steps (1)-(3) to produce the colloquial English sentence.

With this procedure in mind, let us do a few examples.

Example 1: ∀x∃yRxy

(1) the first quantifier is universal, so we read it as: **everyone**

(2) the variable x appears in the active position, so we read the verb in the active voice: **respects**

(3) the second quantifier is existential, so we read it as: **someone (or other)**

(4) altogether: **everyone respects someone (or other)**

Example 2: ∃x∀yRyx

(1) the first quantifier is existential, so we read it as: **there is someone who**

(2) the variable x appears in the passive position, so we read the verb in the passive voice: **is respected by**

(3) the second quantifier is universal, so we read it as: **everyone**

(4) altogether: **there is someone who is respected by everyone**

By following the above procedure, we can translate all the above formulas in colloquial English as follows.

(4a1) ∀y∀xRxy: everyone is respected by everyone

(4a2) ∃y∀xRxy: there is someone who is respected by everyone

(4b1) ∀y∃xRxy: everyone is respected by someone or other

(4b2) $\exists y \exists x Rxy$: there is someone who is respected by someone or other

(4c1) $\forall x \forall y Rxy$: everyone respects everyone

(4c2) $\exists x \forall y Rxy$: there is someone who respects everyone

(4d1) $\forall x \exists y Rxy$: everyone respects someone or other

(4d2) $\exists x \exists y Rxy$: there is someone who respects someone or other

Before continuing, it is important to understand the significance of the expression ‘or other’. In Example 1, the final translation is

everyone respects someone or other

Dropping ‘or other’ yields

everyone respects someone.

This is fine so long as we are completely clear what is meant by the last sentence – namely, that everyone respects someone, not necessarily the same person in each case.

A familiar grammatical transformation converts active sentences into passive ones; for example,

Jay respects Kay

can be transformed into

Kay is respected by Jay.

Both are symbolized the same way.

Rjk

If we perform the same grammatical transformation on

everyone respects someone,

we obtain:

someone is respected by everyone,

which *might* be thought to be equivalent to

there is someone who is respected by everyone.

The following lists the various sentences.

(1) everyone respects someone or other
(2) everyone respects someone
(3) someone is respected by everyone
(4) there is someone who is respected by everyone

The problem we face is simple: (1) and (4) are *not equivalent*; although (4) implies (1), (1) does not imply (4).

In order to see this, consider a very small world with only three persons in it: Adam (a), Eve (e), and Cain (c). For the sake of argument, suppose that Cain respects Adam (but not vice versa), Adam respects Eve (but not vice versa), and Eve respects Cain (but not vice versa). Also, suppose that no one respects him(her)self (although the argument does not depend upon this). Thus, we have the following state of affairs.

Rae	Adam respects Eve
Rec	Eve respects Cain
Rca	Cain respects Adam
~Rea	Eve doesn't respect Adam
~Rce	Cain doesn't respect Eve
~Rac	Adam doesn't respect Cain
~Rcc	Cain doesn't respect himself
~Raa	Adam doesn't respect himself
~Ree	Eve doesn't respect herself

Now, to say that everyone respects someone *or other* is to say everyone respects someone, but not necessarily the same person in each case. In particular, it is to say *all* of the following:

Adam respects someone	∃xRax
Eve respects someone	∃xRex
Cain respects someone	∃xRcx

The first is true, since Adam respects Eve; the second is true, since Eve respects Cain; finally, the third is true, since Cain respects Adam. Thus, in the very small world we are imagining, everyone respects someone *or other*, but not necessarily the same person in each case.

They all respect someone, but there is no single person they all respect. To say that there is someone who is respected by everyone is to say that *at least one* of the following is true.

Adam is respected by everyone	∀xRxa
Eve is respected by everyone	∀xRxe
Cain is respected by everyone	∀xRxc

But the first is false, since Eve doesn't respect Adam; the second is false, since Cain doesn't respect Eve, and the third is false, since Adam doesn't respect Cain. Also, in this world, no one respects him(her)self, but that doesn't make any difference. Thus, in this world, it is not true that there is someone who is respected by everyone, although it is true that everyone respects someone or other.

Thus, sentences (1) and (4) are not equivalent. It follows that the following can't all be true:

(1) is equivalent to (2)
(2) is equivalent to (3)
(3) is equivalent to (4)

For then we would have that (1) and (4) are equivalent, which we have just shown is not the case.

The problem is that (2) and (3) are ambiguous. *Usually*, (2) means the same thing as (1), so that the 'or other' is not necessary. But, *sometimes*, (2) means the same thing as (4), so that the 'or other' is definitely necessary to distinguish (1) and (2). It is best to avoid (2) in favor of (1), if that is what is meant. On the other hand, (3) *usually* means the same thing as (4), but occasionally it is equivalent to (1).

In other words, it is best to avoid (2) and (3) altogether, and say either (1) or (4), depending on what is meant.

3. NEGATIONS OF SIMPLE POLYADIC QUANTIFIERS

What happens when we take the formulas considered in Section 2 and introduce a negation (~) at any of the three possible positions? That is what we consider in the present section.

The quantified formulas obtainable from the atomic formulas 'Rxy' and 'Ryx' are the following.

(1)	∀x∀yRxy	∀y∀xRyx	everyone respects everyone
(2)	∀y∀xRxy	∀x∀yRyx	everyone is respected by everyone
(3)	∃x∃yRxy	∃y∃xRyx	someone respects someone
(4)	∃y∃xRxy	∃x∃yRyx	someone is respected by someone
(5)	∀x∃yRxy	∀y∃xRyx	everyone respects someone
(6)	∃y∀xRxy	∃x∀yRyx	someone is respected by everyone
(7)	∀y∃xRxy	∀x∃yRyx	everyone is respected by someone or other
(8)	∃x∀yRxy	∃y∀xRyx	someone respects everyone

Now, at any stage in the construction of these formulas, we could interpolate a negation connective. That gives us not just 8 formulas but 64 distinct formulas (plus alphabetic variants). The basic form is the following.

SIGN..QUANTIFIER..SIGN..QUANTIFIER..SIGN..FORMULA

Each sign is either negative or positive (i.e., negated or not negated); each quantifier is either universal or existential; finally, the formula has the first quantified variable in active or passive position. All told, there are 64 (2×2×2×2×2×2) combinations!

Let us consider two examples.

(e1) ~∀x~∃y~Rxy

In this formula, all the signs are negative, the first quantifier is universal, the second quantifier is existential, the first quantified variable (‘x’) is in active position.

(e2) $\sim\exists x \forall y \sim Ryx$

In this formula the first and third signs are negative, the second sign is positive, the first quantifier is existential, the second quantifier is universal, the first quantified variable (‘x’) is in passive position.

There are 54 more combinations! We have seen the latter two combinations, not to mention the original eight, which are the combinations in which every sign is positive.

But how does one translate formulas with negations into colloquial English? This is considerably trickier than before. The problem concerns where to place the negation operator in the colloquial sentence. Consider the following sentences.

(1) j dislikes k;
(2) j doesn't like k;
(3) it is not true that j likes k.

The problem is that sentence (2) is actually ambiguous in meaning between the sentence (1) and sentence (3). Furthermore, this is not a harmless ambiguity, since (1) and (3) are *not* equivalent. In particular, the following is not valid in ordinary English.

it is not true that Jay likes Kay;
therefore, Jay dislikes Kay.

The premise may be true simply because Jay doesn't even know Kay, so he can't like her. But he doesn't dislike her either, for the same reason – he doesn't know her.

Now, the problem is that, when someone utters the following,

I don't like spinach,

he or she *usually* means,

I *dis*like spinach,

although he/she *might* go on to say,

but I don't dislike spinach, either (since I've never tried it),

Given that ordinary English seldom provides us with simple negations, we need some scheme for expressing them. Toward this end, let us employ the somewhat awkward expression ‘fails to...’ to construct simple negations. In particular, let us adopt the following translation.

x fails to Respect y :: not(x Respects y)

With this in mind, let us proceed. Recall that a simple double-quantified formula has the following form.

SIGN..QUANTIFIER..SIGN..QUANTIFIER..SIGN..FORMULA

Let us further parse this construction as follows.

[SIGN-QUANTIFIER]..[SIGN-QUANTIFIER]..[SIGN-FORMULA]

In particular, let us use the word quantifier to refer to the combination sign-quantifier. In this case, there are four quantifiers (plus alphabetic variants):

$\forall$x, ~$\forall$x, $\exists$x, ~$\exists$x

We are now, finally, in a position to offer a systematic translation scheme, given as follows.

Step 1: Look at the first quantifier, and read it as follows:

(a)	universal ($\forall$)	everyone
(b)	existential ($\exists$)	there is someone who
(c)	negation universal (~$\forall$)	not everyone
(d)	negation existential (~$\exists$)	there is no one who

Step 2: Check the quantified formula, and check whether the first quantified variable occurs in the active or passive position, and read the verb as follows:

(a)	positive active	respects
(b)	positive passive	is respected by
(c)	negative active	fails to respect
(d)	negative passive	fails to be respected by

Step 3: Look at the second quantifier, and read it as follows:

(a)	universal ($\forall$)	everyone
(b)	existential ($\exists$)	someone or other
(c)	negation universal (~$\forall$)	not...everyone*
(d)	negation existential (~$\exists$)	no one

*Here, it is understood that 'not' goes in front of the verb phrase.

Step 4: String together the components obtained in steps (1)-(3) to produce the colloquial English sentence.

With this procedure in mind, let us do a few examples.

Example 1: $\exists$x~$\exists$yRyx

(1) the first quantifier is existential,
so we read it as: **there is someone who**

(2) the quantified formula is positive,
and the first quantified variable ‘x’
is in the passive position,
so we read the verb as: **is respected by**

(3) the second quantifier is negation-existential,
so we read it as: **no one**

(4) altogether: **there is someone who is respected by no one**

Example 2: ~∀x∃y~Rxy

(1) the first quantifier is negation-universal,
so we read it as: **not everyone**

(2) the quantified formula is negative,
and the first quantified variable ‘x’
is in the active position,
so we read the verb as: **fails to respect**

(3) the second quantifier is existential,
so we read it as: **someone (or other)**

(4) altogether: **not everyone fails to respect someone (or other)**

Example 3: ∃x~∀yRxy

(1) the first quantifier is existential,
so we read it as: **there is someone who**

(2) the quantified formula is positive,
and the first quantified variable ‘x’
is in active position, so we read the verb as: **respects**

(4) the second quantifier is negation-universal,
so we read it as: **not...everyone**

(5) altogether: **there is someone who respects not...everyone**

(5*) or, more properly: **there is someone who does not respect everyone**

4. THE UNIVERSE OF DISCOURSE

The reader has probably noticed a small discrepancy in the manner in which the quantifiers are read. On the one hand, the usual readings are the following.

∀x: every*thing* x is such that...
for any*thing* x...

∃x: some*thing* x is such that...
there is at least one *thing* x such that...

On the other hand, in the previous sections in particular, the following readings are used.

∀x: every *person* x is such that...
for any *person* x...

∃x: some *person* x is such that...
there is at least one *person* x such that...

In other words, depending on the specific example, the various quantifiers are read differently. If we are talking *exclusively* about persons, then it is convenient to read '∀x' as 'everyone' and '∃x' as 'someone', rather than the more general 'everything' and 'something'. If, on the other hand, we are talking *exclusively* about *numbers* (as in arithmetic), then it is equally convenient to read '∀x' as 'every number' and '∃x' as 'some number'.

The reason that this is allowed is that, for any symbolic context (formula or argument), we can agree to specify the associated *universe of discourse*. The universe of discourse is, in any given context, the set of all the possible things that the constants and variables refer to.

Thus, depending upon the particular universe of discourse, U, we read the various quantifiers differently.

In symbolizing English sentences, one must first establish exactly what U is. For sake of simplifying our choices, in the exercises, we allow only two possible choices for U, namely:

U = things (in general)

U = persons

In particular, if the sentence uses 'everyone' or 'someone', then the student is allowed to set U=persons, but if the sentence uses 'every **person**' or 'some **person**', then the student must set U=things.

In some cases (but never in the exercises) both 'every(some)one' and 'every(some)thing' appear in the same sentence. In such cases, one must explicitly supply the predicate '...is a person' in order to symbolize the sentence.

Consider the following example.

there is some*one* who hates every*thing*,

which means

there is some *person* who hates every *thing*.

The following is **not** a correct translation.

∃x∀yHxy WRONG!!!

In translating this back into English, we first must specify the reading of the quantifiers, which is to say we must specify the universe of discourse. In the present context at least, there are only two choices; either U=persons or U=things. So the two possible readings are:

there is some *person* who hates every *person*

there is some *thing* that hates every *thing*

Neither of these corresponds to the original sentence. In particular, the following is *not* an admissible reading of the above formula.

there is some *person* who hates every *thing* WRONG!!!

The principle at work here may be stated as follows.

One cannot change the universe of discourse in the middle of a sentence. All the quantifiers in a sentence must have a uniform reading

5. QUANTIFIER SPECIFICATION

So, how do we symbolize

there is some*one* (some *person*) who hates every*thing*.

First, we must choose a universe of discourse that is large enough to encompass everything that we are talking about. In the context of intro logic, if we are talking about anything whatsoever that is not a person, then we must set U=things. In that case, we have to specify which things in the sentence are persons by employing the predicate '...is a person'. The following paraphrase makes significant headway.

there is something such that
    it is a person who hates everything

Now we have a sentence with uniform quantifiers. Continuing the translation yields the following sequence.

there is something such that	∃x
it is a person and	(Px &
it hates everything	∀yHxy)
	∃x(Px & ∀yHxy)

Let's do another example much like the previous one.

everyone hates something (or other)

This means

every person hates something (or other)

which can be paraphrased pretty much like every other sentence of the form 'every A is B':

everything is such that	$\forall x$
if it is a person,	$(Px \rightarrow$
then it hates something (or other)	$\exists yHxy)$
	$\forall x(Px \rightarrow \exists yHxy)$

At this point, let us compare the sentences.

there is something that hates everything	$\exists x\forall yHxy$
there is some person who hates everything	$\exists x(Px \,\&\, \forall yHxy)$
everything hates something (or other)	$\forall x\exists yHxy$
every person hates something (or other)	$\forall x(Px \rightarrow \exists yHxy)$

The general forms of the above may be formulated as follows.

there is something that is K	$\exists xKx$
there is some person who is K	$\exists x(Px \,\&\, Kx)$
everything is K	$\forall xKx$
every person is K	$\forall x(Px \rightarrow Kx)$

We have already seen this particular transition – from completely general claims to more specialized claims. This maneuver, which might be called *quantifier specification*, still works.

everything is B:	$\forall x.........Bx$
every A is B:	$\forall x(Ax \rightarrow Bx)$
something is B:	$\exists x.........Bx$
some A is B:	$\exists x(Ax \,\&\, Bx)$

Quantifier specification is the process of modifying quantifiers by further specifying (or delimiting) the domain of discussion. The following are simple examples of quantifier specification.

converting 'everything' into 'every physical object'
converting 'everyone' into 'every student'

converting 'something' into 'some physical object'
converting 'someone' into 'some student'

The general process (in the special case of a simple predicate P) is described as follows.

SIMPLE QUANTIFIER SPECIFICATION:

Where **v** is any variable, **P** is any one-place predicate, and **F** is any formula, quantifier specification involves the following substitutions.

substitute ∀**v**(**Pv** → **F**) for ∀**vF**

substitute ∃**v**(**Pv** & **F**) for ∃**vF**

Note carefully the use of '→' in one and '&' in the other.

Examples

something is evil	∃xEx
some physical thing is evil	∃x(Px & Ex)
everything is evil	∀xEx
every physical thing is evil	∀x(Px → Ex)
someone respects everyone	∃x∀yRxy
some student respects everyone	∃x(Sx & ∀yRxy)
everyone respects someone	∀x∃yRxy
every student respects someone	∀x(Sx → ∃yRxy)

So far we have dealt exclusively with the outermost quantifier. However, we can apply quantifier specification to any quantifier in a formula. Consider the following example:

everyone respects someone (or other) ∀x∃yRxy

versus

everyone respects some student (or other) ???

In applying quantifier specification, we note the following.

overall formula:	∃yRxy
specified quantifier:	∃y
specifying predicate:	Sy
modified formula:	Rxy

So applying the procedure, we obtain:

resulting formula: ∃y(Sy & Rxy)

So plugging this back into our original formula, we obtain

everyone respects some student (or other)
∀x∃y(Sy & Rxy).

The more or less literal reading of the latter formula is:

for any person x,
  there is a person y such that,
    y is a student
      and x respects y.

More colloquially,

for any person, there is a person such that
the latter is a student and the former respects the latter.

Still more colloquially,

for any person, there is a person such that
the latter is a student whom the former respects.

We can deal with the following in the same way.

there is someone who respects every student

This results from

there is someone who respects everyone
$\exists x \forall y Rxy$,

by specifying the second quantifier, as follows:

overall formula:	$\forall y Rxy$
specified quantifier:	$\forall y$
specifying predicate:	Sy
modified formula:	Rxy

So applying the procedure, we obtain:

resulting formula:	$\forall y(Sy \rightarrow Rxy)$

So plugging this back into our original formula, we obtain

there is someone who respects every student
$\exists x \forall y(Sy \rightarrow Rxy)$

The more or less literal reading of the latter formula is:

there is a person x such that,
  for any person y,
    if y is a student,
      then x respects y.

More colloquially,

there is a person such that,
  for any person,
    if the latter is a student
      then the former respects the latter.

Still more colloquially,

there is a person such that,
for any student,
the former respects the latter.

So far, we have only done examples in which a single quantifier is specified by a predicate. We can also do examples in which both quantifiers are specified, and by different predicates. The principles remain the same; they are simply applied more generally. Consider the following examples.

(1) there is someone who respects everyone
(1a) there is a student who respects every professor
(1b) there is a professor who respects every student

(2) there is someone who is respected by everyone
(2a) there is a student who is respected by every professor
(2b) there is a professor who is respected by every student

(3) everyone respects someone or other
(3a) every student respects some professor or other
(3b) every professor respects some student or other

(4) everyone is respected by someone or other
(4a) every student is respected by some professor or other
(4b) every professor is respected by some student or other

The following are the corresponding formulas; in each case, the latter two are obtained from the first one by specifying the quantifiers appropriately.

(1) ∃x.........∀y........Rxy
(1a) ∃x(Sx & ∀y(Py → Rxy))
(1b) ∃x(Px & ∀y(Sy → Rxy))

(2) ∃x.........∀y........Ryx
(2a) ∃x(Sx & ∀y(Py → Ryx))
(2b) ∃x(Px & ∀y(Sy → Ryx))

(3) ∀x........∃y.........Rxy
(3a) ∀x(Sx → ∃y(Py & Rxy))
(3b) ∀x(Px → ∃y(Sy & Rxy))

(4) ∀x........∃y.........Ryx
(4a) ∀x(Sx → ∃y(Py & Ryx))
(4b) ∀x(Px → ∃y(Sy & Ryx))

6. COMPLEX PREDICATES

In order to further understand the translations that appear in the previous sections, and in order to be prepared for more complex translations still, we now examine the notion of *complex predicate*.

Roughly, complex predicates stand to simple (ordinary) predicates as complex (molecular) formulas stand to simple (atomic) formulas. Like ordinary predicates, complex predicates have places; there are one-place, two-place, etc., complex predicates. However, we are going to concentrate exclusively on one-place complex predicates.

The notion of a complex one-place predicate depends on the notion of a free occurrence of a variable. This is discussed in detail in an appendix. Briefly, an occurrence of a variable in a formula is *bound* if it falls inside the scope of a quantifier governing that variable; otherwise, the occurrence is *free*.

Examples

(1)	Fx	the one and only occurrence of 'x' is free.
(2)	∀x(Fx → Gx)	all three occurrences of 'x' are bound by '∀x'.
(3)	∀xRxy	every occurrence of 'x' is bound; the one and only occurrence of 'y' is free.

Next, to say that a variable (say, 'x') is *free* in a formula F is to say that at least one occurrence of 'x' is free in F; on the other hand, to say that 'x' is *bound* in F is to say that no occurrence of 'x' is free in F. For example, in the following formulas, 'x' is free, but 'y' is bound.

(f1) ∀yRxy
(f2) ∃yRxy
(f3) ∀yRyx
(f4) ∃yRyx

Any formula with exactly one free variable (perhaps with many occurrences) may be thought of as a complex one-place predicate. To see how this works, let us translate formulas (1)-(4) into nearly colloquial English.

(e1) x (he/she) respects everyone
(e2) x (he/she) respects someone
(e3) x (he/she) is respected by everyone
(e4) x (he/she) is respected by someone

Now, if we say of someone that he(she) respects everyone, then we are attributing a complex predicate to that person. We can abbreviate this complex predicate '$\mathcal{A}$x', which stands for 'x respects everyone'. Similarly with all the other formulas above; each one corresponds to a complex predicate, which can be abbreviated by a single letter. These abbreviations may be summarized by the following schemes.

$\mathcal{A}$x :: ∀yRxy
$\mathcal{B}$x :: ∃yRxy
$\mathcal{C}$x :: ∀yRyx
$\mathcal{D}$x :: ∃yRyx

Here, '::' basically means '...is short for...'.

Now, complex predicates can be used in sentences just like ordinary predicates. For example, we can say the following:

some Freshman is $\mathcal{A}$
every Freshman is $\mathcal{B}$
no Freshman is $\mathcal{C}$
some Freshman is not $\mathcal{D}$

Recalling what '$\mathcal{A}$', '$\mathcal{B}$', '$\mathcal{C}$', and '$\mathcal{D}$' are short for, these are read colloquially as follows.

some Freshman respects everyone
every Freshman respects someone or other
no Freshman is respected by everyone
some Freshman is not respected by someone (or other)

These have the following as overall symbolizations.

∃x(Fx & $\mathcal{A}$x)
∀x(Fx → $\mathcal{B}$x)
~∃x(Fx & $\mathcal{C}$x)
∃x(Fx & ~$\mathcal{D}$x)

But '$\mathcal{A}$x', '$\mathcal{B}$x', '$\mathcal{C}$x', and '$\mathcal{D}$x' are short for more complex formulas, which when substituted yield the following formulas.

∃x(Fx & ∀yRxy)
∀x(Fx → ∃yRxy)
~∃x(Fx & ∀yRyx)
∃x(Fx & ~∃yRyx)

We can also make the following claims.

every $\mathcal{A}$ is $\mathcal{B}$
every $\mathcal{A}$ is $\mathcal{C}$
every $\mathcal{A}$ is $\mathcal{D}$

Given what '$\mathcal{A}$', '$\mathcal{B}$', '$\mathcal{C}$', and '$\mathcal{D}$' are short for, these read colloquially as follows.

every one who respects everyone respects someone
every one who respects everyone is respected by everyone
every one who respects everyone is respected by someone

The overall symbolizations of these sentences are given as follows.

$\forall x(\mathcal{A}x \rightarrow \mathcal{B}x)$
$\forall x(\mathcal{A}x \rightarrow \mathcal{C}x)$
$\forall x(\mathcal{D}x \rightarrow \mathcal{D}x)$

But '$\mathcal{A}x$', '$\mathcal{B}x$', '$\mathcal{C}x$', and '$\mathcal{D}x$' are short for more complex formulas, which when substituted yield the following formulas.

$\forall x(\forall yRxy \rightarrow \exists yRxy)$
$\forall x(\forall yRxy \rightarrow \forall yRyx)$
$\forall x(\forall yRxy \rightarrow \exists yRyx)$

Let's now consider somewhat more complicated complex predicates, given as follows.

$\mathcal{A}x$: x respects every professor
$\mathcal{B}x$: x is respected by every student
$\mathcal{C}x$: x respects at least one professor
$\mathcal{D}x$: x is respected by at least one student

Given the symbolizations of the formulas to the right, we have the following abbreviations.

$\mathcal{A}x$ :: $\forall y(Py \rightarrow Rxy)$
$\mathcal{B}x$ :: $\forall y(Sy \rightarrow Ryx)$
$\mathcal{C}x$ :: $\exists y(Py \ \& \ Rxy)$
$\mathcal{D}x$ :: $\exists y(Sy \ \& \ Ryx)$

We can combine these complex predicates with simple predicates or with each other. The following are examples.

(1) some S is $\mathcal{A}$
(2) some P is $\mathcal{B}$
(3) every S is $\mathcal{C}$
(4) every P is $\mathcal{D}$

The colloquial readings are:

(r1) there is a student who respects every professor
(r2) there is a professor who is respected by every student
(r3) every student respects at least one professor
(some professor or other)
(r4) every professor is respected by at least one student
(some student or other)

And the overall symbolizations are given as follows.

(o1) $\exists x(Sx \ \& \ \mathcal{A}x)$
(o2) $\exists x(Px \ \& \ \mathcal{B}x)$
(o3) $\forall x(Sx \rightarrow \mathcal{C}x)$
(o4) $\forall x(Px \rightarrow \mathcal{D}x)$

But '$\mathcal{A}$x', '$\mathcal{B}$x', '$\mathcal{C}$x', '$\mathcal{D}$x' are short for more complex formulas, which when substituted yield the following formulas.

(f1) $\exists x(Sx \& \forall y(Py \to Rxy))$
(f2) $\exists x(Px \& \forall y(Sy \to Ryx))$
(f3) $\forall x(Sx \to \exists y(Py \& Rxy))$
(f4) $\forall x(Px \to \exists y(Sy \& Ryx))$

These correspond to the formulas obtained by the technique of quantifier specification, presented in the previous section.

The advantage of understanding complex predicates is that it allows us to combine the complex predicates into the same formula. The following are examples.

$\mathcal{A}$x: x respects every professor
$\mathcal{B}$x: x is respected by every student
$\mathcal{C}$x: x is respected by at least one professor

no $\mathcal{A}$ is $\mathcal{B}$
every $\mathcal{B}$ is $\mathcal{C}$

These may be read colloquially as

no one who respects every professor is respected by every student

everyone who is respected by every student is respected by at least one professor

The overall symbolizations are, respectively,

$\sim\exists x(\mathcal{A}x \& \mathcal{B}x)$
$\forall x(\mathcal{B}x \to \mathcal{C}x)$

but '$\mathcal{A}$x', '$\mathcal{B}$x', and '$\mathcal{C}$x' stand for more complex formulas, which when substituted yield the following formulas.

$\sim\exists x(\forall y(Py \to Rxy) \& \forall y(Sy \to Ryx))$
$\forall x(\forall y(Sy \to Ryx) \to \exists y(Py \& Ryx))$

7. THREE-PLACE PREDICATES

So far, we have concentrated on two-place predicates. In the present section, we look at examples that involve quantification over formulas based on three-place predicates.

As mentioned in the previous chapter, there are numerous three place predicate expressions in English. The most common, perhaps, are constructed from verbs that take a subject, a direct object, and an indirect object. For example, in the sentence

Kay loaned her car to Jay

may be grammatically analyzed thus:

subject:	Kay
verb:	loaned
direct object:	her car
indirect object:	Jay

The remaining word, 'to', marks 'Jay' as the indirect object of the verb. In general, prepositions such as 'to' and 'from', as well as others, are used to mark indirect objects. The following sentence uses 'from' to mark the indirect object.

Jay borrowed Kay's car (from Kay)

Letting 'c' name the particular individual car in question, the above sentences can be symbolized as follows.

Lkcj
Bjck

The convention is to write subject first, direct object second, and indirect object last.

As usual, variables (pronouns) may replace one or more of the constants (proper nouns) in above formulas, and as usual, the resulting formulas can be quantified, either universally or existentially. The following are examples.

Kay loaned her car to him(her)	Lkcx
Kay loaned her car to someone	∃xLkcx
Kay loaned her car to everyone	∀xLkcx
Jay borrowed it from Kay	Bjxk
Jay borrowed something from Kay	∃xBjxk
Jay borrowed everything from Kay	∀xBjxk

As before, we can also further specify the quantifiers. Rather than saying 'someone' or 'everyone', we can say 'some student' or 'every student'; rather than saying 'something' or 'everything', we can say 'some car' or 'every car'. Quantifier specification works the same as before.

Kay loaned her car to some student	∃x(Sx & Lkcx)
Kay loaned her car to every student	∀x(Sx → Lkcx)
Jay borrowed some car from Kay	∃x(Cx & Bjxk)
Jay borrowed every car from Kay	∀x(Cx → Bjxk)

These are examples of single-quantification; we can quantify over every place in a predicate, so in the predicates we are considering, we can quantify over three places.

Two quantifiers first; let's change our example slightly. First note the following:

x rents y to z ↔ z rents y from x

For example,

Avis rents this car to Jay iff Jay rents this car from Avis.

Letting 'Rxyz' stand for 'x rents y **to** z', consider the following.

Example 1

every student has rented a car from Avis

∀x(Sx → ∃y(Cy & Rayx))

for any x,
    if x is a student,
        then there is a y such that,
            y is a car
                and Avis has rented y to x

Example 2

there is at least one car that Avis has rented to every student

∃x(Cx & ∀y(Sy → Raxy))

there is an x such that,
    x is a car
        and for any y,
            if y is a student,
                then Avis has rented x to y

8. 'ANY' REVISITED

Recall that certain quantifier expressions of English are wide-scope universal quantifiers. The most prominent wide-scope quantifier is 'any', whose standard derivatives are 'anything' and 'anyone'. Also recall that other words are also occasionally used as wide-scope universal quantifiers – including 'a' and 'some'; these are discussed in the next section.

To say that 'any' is a wide-scope universal quantifier is to say that, when it is attached to another logical expression, the scope of 'any' is wider than the scope of the attached expression.

In the context of monadic predicate logic, 'any' most frequently attaches to 'if' to produce the 'if any' locution. In particular, statements of the form:

if anything is A, then $\mathcal{B}$

appears to have the form:

if $\mathcal{A}$, then $\mathcal{B}$,

but because of the wide-scope of 'any', the sentence *really* has the form:

for anything (if it is A, then $\mathcal{B}$)

which is symbolized:

$\forall x(Ax \rightarrow \mathcal{B})$

In monadic logic, 'any' usually attaches to 'if'. In polyadic logic, 'any' often attaches to other words as well, most particularly 'no' and 'not', as in the following examples.

no one respects *any* one
Jay does *not* respect *any* one

Let us consider the second example, since it is easier. One way to understand this sentence is to itemize its content, which might go as follows.

Jay does not respect Adams	~Rja
Jay does not respect Brown	~Rjb
Jay does not respect Carter	~Rjc
Jay does not respect Dickens	~Rjd
Jay does not respect Evans	~Rje
Jay does not respect Field	~Rjf
etc.	

in short:

Jay does not respect *anyone*.

Given that 'Jay does not respect anyone' summarizes the list,

~Rja
~Rjb
~Rjc
~Rjd
~Rje
~Rjf
etc.

it is natural to regard 'Jay does not respect anyone' as a universally quantified statement, namely,

$\forall x{\sim}Rjx$.

Notice that the main logical operator is '$\forall x$'; the formula is a universally quantified formula.

Another way to symbolize the above 'any' statement employs the following series of paraphrases.

Jay does not respect anyone
Jay does not respect x, for any x
for any x, Jay does not respect x

$\forall x{\sim}Rjx$

Before considering more complex examples, let us contrast the any-sentence with the corresponding every-sentence.

Jay does not respect *any*one

versus

Jay does not respect *every*one

The latter certainly does not entail the former; 'any' and 'every' are not interchangeable, but we already know that. Also, we already know how to paraphrase and symbolize the latter sentence:

Jay does not respect *every*one
not(Jay does respect everyone)
it is not true that Jay respects everyone
not everyone is respected by Jay

~∀xRjx

Notice carefully that, although both 'any' and 'every' are universal quantifiers, they are quite different in meaning. The difference pertains to their respective scopes, which is summarized as follows, in respect to 'not'.

'not' has wider scope than 'every';

'any' has wider scope than 'not'.

not everyone	~∀x
not anyone	∀x~

Having considered the basic 'not any' form, let us next consider quantifier specification. For example, consider the following pair.

Jay does not respect *every* Freshman;
Jay does not respect *any* Freshman.

We already know how to paraphrase and symbolize the first one, as follows.

Jay does not respect every Freshman
not(Jay does respect every Freshman)
it is not true that Jay respects every Freshman
not every Freshman is respected by Jay

~∀x(Fx → Rjx)

The corresponding 'any' statement is more subtle. One approach involves the following series of paraphrases.

Jay does not respect any Freshman
Jay does not respect x, for any Freshman x
for any Freshman x, Jay does not respect x

$\forall x(Fx \rightarrow \sim Rjx)$

Notice that this is obtained from

Jay does not respect anyone
$\forall x \sim Rjx$

by quantifier specification, as described in an earlier section.

9. COMBINATIONS OF 'NO' AND 'ANY'

As mentioned in the previous section, 'any' attaches to 'if', 'not', and 'no' to form special compounds. We have already seen how 'any' interacts with 'if' and 'not'; in the present section, we examine how 'any' interacts with 'no'. Consider the following example.

(a) no Senior respects any Freshman

First we observe that 'any' and 'every' are not interchangeable. In particular, (a) is *not* equivalent to the following formula, which results by replacing 'any' by 'every'.

(e) no Senior respects every Freshman

The latter is equivalent to the following.

(e') there is no Senior who Respects every Freshman

The latter is symbolized, in parts, as follows.

(1) there is no S who R's every F
(2) there is no S who is $\mathcal{A}$
(3) no S is $\mathcal{A}$
(4) $\sim\exists x(Sx \,\&\, \mathcal{A}x)$
$\mathcal{A}x$ :: x R's every F :: $\forall y(Fy \rightarrow Rxy)$
(5) $\sim\exists x(Sx \,\&\, \forall y(Fy \rightarrow Rxy))$

Now let us go back and do the 'any' example (a); if we symbolize it in parts, we might proceed as follows.

(1) no S R's any F
(2) no S is $\mathcal{A}$
(3) $\sim\exists x(Sx \,\&\, Kx)$
$\mathcal{A}x$ :: x R's any F
???

The problem is that the complex predicate '$\mathcal{A}$' involves 'any', which cannot be straightforwardly symbolized in isolation; 'any' requires a *correlative* word to which it attaches.

At this point, it might be useful to recall (previous chapter) that 'no A is B' may be plausibly symbolized in either of the following ways.

(s1) $\sim\exists x(Ax \& Bx)$
(s2) $\forall x(Ax \rightarrow \sim Bx)$

These are logically equivalent, as we will demonstrate in the following chapter, so either counts as a correct symbolization. Each symbolization has its advantages; the first one shows the relation between 'no A is B' and 'some A is B' – they are negations of one another. The second one shows the relation between 'no A is B' and 'every A is unB' – they are equivalent.

In choosing a standard symbolization for 'no A is B' we settled on (s1) because it uses a single logical operator – namely $\sim\exists x$ – to represent 'no'. However, there are a few sentences of English that are more profitably symbolized using the second scheme, especially sentences involving 'any'.

So let us approach sentence (a) using the alternative symbolization of 'no'.

(a) no Senior respects any Freshman
(1) no S R's any F
(2) no S is $\mathcal{A}$
(3) $\forall x(Sx \rightarrow \sim\mathcal{A}x)$
$\mathcal{A}x$ :: x R's any F
???

Once again, we get stuck, because we can't symbolize '$\mathcal{A}x$' in isolation. However, we can rephrase (3) by treating '$\sim\mathcal{A}x$' as a unit, '$\mathcal{B}x$', in which the negation gets attached to 'any'.

(4) $\forall x(Sx \rightarrow \mathcal{B}x)$
$\mathcal{B}x$ :: x does not R any F
$\forall y(Fy \rightarrow \sim Rxy)$

Substituting the symbolization of '$\mathcal{B}x$' into (4), we obtain the following formula.

(5) $\forall x(Sx \rightarrow \forall y(Fy \rightarrow \sim Rxy))$

The latter formula reads

for any x,
if x is a Senior,
then for any y,
if y is a Freshman,
then x does not respect y

The latter may be read more colloquially as follows.

for any Senior, for any Freshman,
the Senior does not respect the Freshman

On the other hand, if we follow the suggested translation scheme from earlier in the chapter, (5) is read colloquially as follows.

every Senior fails to respect every Freshman

The following is a somewhat more complex example.

no woman respects any man who does not respect her

We attack this in parts, but we note that one of the parts is a no-any combination. So the overall form is:

(1) no W R's any $\mathcal{A}$

As we already saw, this may be symbolized:

(2) $\forall x(Wx \rightarrow \forall y(\mathcal{A}y \rightarrow {\sim}Rxy))$

$\mathcal{A}y$ :: y is a man who does not respect her (x) :: $(My \ \& \ {\sim}Ryx)$

Substituting the symbolization of '$\mathcal{A}y$' into (2), we obtain:

(3) $\forall x(Wx \rightarrow \forall y([My \ \& \ {\sim}Ryx] \rightarrow {\sim}Rxy))$

for any x,
  if x is a woman,
    then for any y,
      if y is a man
        and y does not respect x,
          then x does not respect y

Because of our wish to symbolize 'any' as a wide-scope universal quantifier, our symbolization of 'no A R's any B' is different from our symbolization of 'no A is $\mathcal{B}$'. Specifically, we have the following symbolization.

(1) no A R's any B
$\forall x(Ax \rightarrow \forall y(By \rightarrow {\sim}Rxy))$

(2) no A is $\mathcal{B}$
${\sim}\exists x(Ax \ \& \ \mathcal{B}x)$

We conclude with an alternative symbolization which preserves 'no' but sacrifices the universal quantifier reading of 'any'. We start with (2) and perform two logical transformations, both based on the following equivalence.

(e) $\forall x(\mathcal{A} \rightarrow {\sim}\mathcal{B})$ :: ${\sim}\exists x(\mathcal{A} \ \& \ \mathcal{B})$.

(3) $\forall x(Ax \rightarrow {\sim}\exists y(By \ \& \ Rxy))$
(4) ${\sim}\exists x(Ax \ \& \ \exists y(By \ \& \ Rxy))$

The latter is read:

it is not true that
there is an x such that,
x is A,
and there is a y such that,
y is B,
and x R's y

Following our earlier scheme, we read (4) as

no A R's some B or other
no A R's at least one B

10. MORE WIDE-SCOPE QUANTIFIERS

Recall from the previous chapter the following example.

if anyone can fix your car, then Jones can (fix your car).

for anyone x, if x can fix your car Jones can

$\forall x(Fx \rightarrow Fj)$

Monadic logic does not do full justice to this sentence; in particular, we must symbolize ‘can fix your car’ as a simple one-place predicate, even though it includes a direct object. This expression is more adequately analyzed as a complex one-place predicate derived from the two-place predicate ‘...can fix...’ and the singular term ‘your car’. Then the symbolization goes as follows.

$\forall x(Fxc \rightarrow Fjc)$

This analysis is based on construing the expression ‘your car’ as referring to a particular car, named c in this context. In this reading, the speaker is speaking to a particular person, about a particular car.

This is not entirely accurate, because we now include cars in our domain of discourse, so we need to specify the quantifier to persons, as follows.

$\forall x[Px \rightarrow (Fxc \rightarrow Fjc)]$

However, there is another equally plausible analysis of the original sentence, which construes the ‘you’ in the sentence, not as a particular person to whom the speaker is speaking, but as a universal quantifier. In this case, the following is a more precise paraphrase.

if anyone can fix anyone's car, then Jones can fix it.

The use of ‘you’ as a universal quantifier is actually quite common in English. The following is representative.

you can't win at Las Vegas

can be paraphrased as

no one can win at Las Vegas.

Another example:

if you murder someone, then you are guilty of a capital crime,

can be paraphrased as

if anyone murders someone, then he/she is guilty of a capital crime.

More about this example in a moment. First, let us finish the car example. By saying that ‘your car’ means ‘anyone’s car' we are saying that the formula

$\forall x[Px \rightarrow (Fxc \rightarrow Fjc)]$

is true, not just of c, but of every car, which is to say that the following formula holds.

$\forall y\{Cy \rightarrow \forall x[Px \rightarrow (Fxc \rightarrow Fjc)]\}$

An alternative symbolization puts all the quantifiers in front.

$\forall x\forall y([(Px \,\&\, Cy) \,\&\, Fxy] \rightarrow Fjy)$

Now, let us consider the murder example, which also involves two wide-scope universal quantifiers. First, notice that the word ‘someone’ does not act as an existential quantifier in this sentence. In this sentence, the most plausible reading of ‘someone’ is ‘anyone’.

if anyone murders anyone,
then the former is guilty of a capital crime

Let us treat the predicate ‘...is guilty of a capital crime’ as simple, symbolizing it simply as ‘G’. Simple versus complex predicates is not the issue at the moment. The issue is that there are two occurrences of ‘any’. How do we deal with sentences of the form

if any... any... , then ...

The best way to treat the appearance of two wide-scope quantifiers is to treat them as double-universal quantifiers, thus:

for any x, for any y, if..., then...

So the murder-example is symbolized as follows.

$\forall x\forall y(Mxy \rightarrow Gx)$

Another example that has a similar form is the following.

if someone injures someone, then the latter sues the former

Once again, there are two wide-scope quantifiers, both being occurrences of ‘someone’. This can be paraphrased and symbolized as follows.

if someone injures someone, then the latter sues the former

for any two people, if the former injures the latter,
then the latter sues the former

for any x, for any y, if x injures y, then y sues x

∀x∀y(Ixy → Syx)

Next, let us consider the word ‘a’, which (like ‘some’ and ‘any’) is often used as a wide-scope quantifier. Consider the following two examples, which have the same form.

if a solid object is heated, it expands
if a game is rained out, it is rescheduled

These appear, at first glance, to be conditionals, but the occurrence of ‘a’ with the attached pronoun ‘it’ indicates that they are actually universal statements. The following is a plausible paraphrase of the first one.

if a solid object is heated, it expands

for any solid object, if it is heated, then it expands

for any S, if it is H, then it is E

for any S (x), if x is H, then x is E

for any x, if x is S, then if x is H, then x is E

∀x(Sx → [Hx → Ex])

The word ‘a’ (‘an’) can also appear twice in the antecedent of a conditional, as in the following example.

if a student misses an exam, then he/she retakes that exam

This may be paraphrased and symbolized as follows.

if a S M's an E, then the S R's the E

for any S, for any E, if the S M's the E, then the S R's the E

for any S x, for any E y, if x M's y, then x R's y

∀x(Sx → ∀y[Ey → (Mxy → Rxy)])

Having seen examples involving various wide-scope quantifiers, including ‘any’, ‘some’, and ‘a’, it is important to recognize how they differ from one another. Compare the following sentences.

if a politician isn't respected by a citizen, then the politician is displeased;
if a politician isn't respected by any citizen, then the politician is displeased.

The difference is between ‘a citizen’ and ‘any citizen’. Curiously, ‘any citizen’ attaches to ‘not’ (in the contraction ‘isn't’), whereas ‘a citizen’ attaches to ‘if’. In both cases, the quantifier 'a politician' attaches to 'if'. The former is paraphrased and symbolized as follows.

> if a politician isn't respected by a citizen, then he/she is displeased
> for any P, for any C, if the P isn't R'ed by the C, then the P is D
> for any P x, for any C y, if x isn't R'ed by y, then x is D
> $\forall x(Px \rightarrow \forall y[Cy \rightarrow ({\sim}Ryx \rightarrow Dx)])$

The following example further illustrates the difference between 'a' and 'any'.

> if no one respects a politician, then the politician isn't re-elected;

If we substitute 'any politician' for 'a politician', we obtain a sentence of dubious grammaticality.

> ?? if no one respects any politician, then the politician isn't re-elected;

The reason this is grammatically dubious is that 'any' attaches to 'no', which is closer than 'if', and hence 'any' does not attach to the quasi-pronoun 'the politician'. By contrast, 'a' attaches to 'if' and 'the politician'; it does not attach to 'no'.

The rule of thumb that prevails is the following.

> 'any' attaches to the nearest logical operator
> from the following list:
> 'if', 'no', 'not'

> 'a' attaches to the nearest occurrence of 'if'

By way of concluding this section, we consider how 'a' interacts with 'every', which is a special case of how it interacts with 'if'. Recall that sentences of the form

> everyone who is *A* is *B*

are given an overall paraphrase/symbolization as follows

> for anyone, if he/she is *A*, he/she is *B*
> $\forall x(Ax \rightarrow Bx)$

In particular, many sentences involving 'every' are paraphrased using 'if-then'. Consider the following.

> every person who likes a movie recommends it

Let us simplify matters by treating 'recommends' as a two-place predicate. Then the sentence is paraphrased and symbolized as follows.

> for any person x, if x likes a movie, then x recommends it

for any person x,
for any movie y, if x likes y, then x recommends y
$\forall x(Px \rightarrow \forall y[My \rightarrow (Lxy \rightarrow Rxy)])$

11. EXERCISES FOR CHAPTER 7

Directions: Using the suggested abbreviations (the capitalized words), translate each of the following into the language of predicate logic.

EXERCISE SET A

1. Everyone RESPECTS JAY.
2. JAY RESPECTS everyone.
3. Someone RESPECTS JAY.
4. JAY RESPECTS someone.
5. Someone doesn't RESPECT JAY.
6. There is someone JAY does not RESPECT.
7. No one RESPECTS JAY.
8. JAY RESPECTS no one.
9. JAY doesn't RESPECT everyone.
10. Not everyone RESPECTS JAY.
11. Everyone RESPECTS everyone.
12. Everyone is RESPECTED by everyone.
13. Everyone RESPECTS someone (or other).
14. Everyone is RESPECTED by someone (or other).
15. There is someone who RESPECTS everyone.
16. There is someone who is RESPECTED by everyone.
17. Someone RESPECTS someone.
18. Someone is RESPECTED by someone.
19. Every event is CAUSED by some event or other (U=events).
20. There is some event that CAUSES every event.

EXERCISE SET B

21. There is no one who RESPECTS everyone.
22. There is no one who is RESPECTED by everyone.
23. There is someone who RESPECTS no one.
24. There is someone whom no one RESPECTS.
25. Not everyone RESPECTS everyone.
26. Not everyone is RESPECTED by everyone.
27. Not everyone RESPECTS someone or other.

28. Not everyone is RESPECTED by someone or other.
29. There is no one who doesn't RESPECT someone or other.
30. There is no one who isn't RESPECTED by someone or other.
31. There is no one who doesn't RESPECT everyone.
32. There is no one who isn't RESPECTED by everyone.
33. There is no one who isn't RESPECTED by at least one person.
34. There is no one who RESPECTS no one.
35. There is no one who is RESPECTED by no one.
36. There is no one who doesn't RESPECT at least one person.
37. For any person there is someone he/she doesn't RESPECT.
38. For any person there is someone who doesn't RESPECT him/her.
39. For any event there is an event that doesn't CAUSE it. (U=events)
40. There is no event that is not CAUSED by some event or other.

EXERCISE SET C

41. Every FRESHMAN RESPECTS someone or other.
42. Every FRESHMAN IS RESPECTED BY someone or other.
43. Everyone RESPECTS some FRESHMAN or other.
44. Everyone is RESPECTED by some FRESHMAN or other.
45. There is some FRESHMAN who RESPECTS everyone.
46. There is some FRESHMAN who is RESPECTED by everyone.
47. There is some one who RESPECTS every FRESHMAN.
48. There is some one who is RESPECTED by every FRESHMAN.
49. There is no FRESHMAN who is RESPECTED by everyone.
50. There is no one who RESPECTS every FRESHMAN.

EXERCISE SET D

51. Every PROFESSOR is RESPECTED by some STUDENT or other.
52. Every PROFESSOR RESPECTS some STUDENT or other.
53. Every STUDENT is RESPECTED by some PROFESSOR or other.
54. Every STUDENT RESPECTS some PROFESSOR or other.
55. For every PROFESSOR, there is a STUDENT who doesn't RESPECT that professor.

56. For every STUDENT, there is a PROFESSOR who doesn't RESPECT that student.
57. For every PROFESSOR, there is a STUDENT whom the professor doesn't RESPECT.
58. For every STUDENT, there is a PROFESSOR whom the student doesn't RESPECT.
59. There is a STUDENT who RESPECTS every PROFESSOR.
60. There is a PROFESSOR who RESPECTS every STUDENT.
61. There is a STUDENT who is RESPECTED by every PROFESSOR.
62. There is a PROFESSOR who is RESPECTED by every STUDENT.
63. There is a STUDENT who RESPECTS no PROFESSOR.
64. There is a PROFESSOR who RESPECTS no STUDENT.
65. There is a STUDENT who is RESPECTED by no PROFESSOR.
66. There is a PROFESSOR who is RESPECTED by no STUDENT.
67. There is no STUDENT who RESPECTS every PROFESSOR.
68. There is no PROFESSOR who RESPECTS every STUDENT.
69. There is no STUDENT who is RESPECTED by every PROFESSOR.
70. There is no STUDENT who RESPECTS no PROFESSOR.
71. There is no PROFESSOR who RESPECTS no STUDENT.
72. There is no STUDENT who is RESPECTED by no PROFESSOR.
73. There is no PROFESSOR who is RESPECTED by no STUDENT.
74. There is a STUDENT who doesn’t RESPECT every PROFESSOR.
75. There is a PROFESSOR who doesn’t RESPECT every STUDENT.
76. There is a PROFESSOR who isn’t RESPECTED by every STUDENT.
77. There is a STUDENT who isn’t RESPECTED by every PROFESSOR.
78. There is no STUDENT whom every PROFESSOR RESPECTS.
79. There is no PROFESSOR whom every STUDENT RESPECTS.
80. There is no PROFESSOR who isn't RESPECTED by every STUDENT.

EXERCISE SET E

81. Everyone who RESPECTS him(her)self RESPECTS everyone.
82. Everyone who RESPECTS him(her)self is RESPECTED by everyone.
83. Everyone who RESPECTS everyone is RESPECTED by everyone.
84. Everyone who RESPECTS every FRESHMAN is RESPECTED by every FRESHMAN.
85. Anyone who is SHORTER than every JOCKEY is SHORTER than JAY.
86. Anyone who is TALLER than KAY is TALLER than every STUDENT.
87. Anyone who is TALLER than every BASKETBALL player is TALLER than every JOCKEY.
88. JAY RESPECTS everyone who RESPECTS KAY.
89. JAY RESPECTS no one who RESPECTS KAY.
90. Everyone who KNOWS JAY RESPECTS at least one person who KNOWS KAY.
91. At least one person RESPECTS no one who RESPECTS JAY.
92. There is a GANGSTER who is FEARED by everyone who KNOWS him.
93. There is a PROFESSOR who is RESPECTED by every STUDENT who KNOWS him(her).
94. There is a STUDENT who is RESPECTED by every PROFESSOR who RESPECTS him(her)self.
95. There is a PROFESSOR who RESPECTS every STUDENT who ENROLLS in every COURSE the professor OFFERS.
96. Every STUDENT who KNOWS JAY RESPECTS every PROFESSOR who RESPECTS JAY.
97. There is a PROFESSOR who RESPECTS no STUDENT who doesn't RESPECT him(her)self.
98. There is a PROFESSOR who RESPECTS no STUDENT who doesn't RESPECT every PROFESSOR.
99. There is no PROFESSOR who doesn't RESPECT every STUDENT who ENROLLS in every COURSE he/she TEACHES.
100. Every STUDENT RESPECTS every PROFESSOR who RESPECTS every STUDENT.
101. Only MISANTHROPES HATE everyone.
102. Only SAINTS LOVE everyone.
103. The only MORTALS who are RESPECTED by everyone are movie STARS.
104. MORONS are the only people who IDOLIZE every movie STAR.

105. Only MORONS RESPECT only POLITICIANS.

EXERCISE SET F

106. JAY RECOMMENDS every BOOK he LIKES to KAY.
107. JAY LIKES every BOOK RECOMMENDED to him by KAY.
108. Every MAGAZINE that JAY READS is BORROWED from KAY.
109. Every BOOK that KAY LENDS to JAY she STEALS from CHRIS.
110. For every PROFESSOR, there is a STUDENT who LIKES every BOOK the professor RECOMMENDS to the student.

EXERCISE SET G

111. JAY doesn't RESPECT anyone.
112. JAY isn't RESPECTED by anyone.
113. There is someone who doesn't RESPECT anyone.
114. There is no one who isn't RESPECTED by anyone.
115. There is no one who doesn't RESPECT anyone.
116. JAY doesn't RESPECT any POLITICIAN.
117. JAY isn't RESPECTED by any POLITICIAN.
118. There is someone who isn't RESPECTED by any POLITICIAN.
119. There is no one who doesn't RESPECT any POLITICIAN.
120. There is at least one STUDENT who doesn't RESPECT any POLITICIAN.
121. There is no STUDENT who doesn't RESPECT any PROFESSOR.
122. There is no STUDENT who isn't RESPECTED by any PROFESSOR.
123. No STUDENT RESPECTS any POLITICIAN.
124. No STUDENT is RESPECTED by any POLITICIAN.
125. Everyone KNOWS someone who doesn't RESPECT any POLITICIAN.
126. Every STUDENT KNOWS at least one STUDENT who doesn't RESPECT any POLITICIAN.
127. No one who KNOWS JAY RESPECTS anyone who KNOWS KAY.
128. There is someone who doesn't RESPECT anyone who RESPECTS JAY.
129. No STUDENT who KNOWS JAY RESPECTS any PROFESSOR who RESPECTS JAY.
130. There is a PROFESSOR who doesn't RESPECT any STUDENT who doesn't RESPECT him(her).

131. There is a PROFESSOR who doesn't RESPECT any STUDENT who doesn't RESPECT every PROFESSOR.

132. If JAY can CRACK a SAFE, then every PERSON can CRACK it.

133. If KAY can't crack a SAFE, then no PERSON can CRACK it.

134. If a SKUNK ENTERS the room, then every PERSON will NOTICE it.

135. If a CLOWN ENTERS a ROOM, then every PERSON IN the room will NOTICE the clown.

136. If a MAN BITES a DOG, then every WITNESS is SURPRISED at him.

137. If a TRESPASSER is CAUGHT by one of my ALLIGATORS, he/she will be EATEN by that alligator.

138. Any FRIEND of YOURS is a FRIEND of MINE (o=you)

139. Anyone who BEFRIENDS any ENEMY of YOURS is an ENEMY of MINE

140. Any person who LOVES a SLOB is him(her)self a SLOB.

12. ANSWERS TO EXERCISES FOR CHAPTER 7

EXERCISE SET A

1. ∀xRxj
2. ∀xRjx
3. ∃xRxj
4. ∃xRjx
5. ∃x~Rxj
6. ∃x~Rjx
7. ~∃xRxj
8. ~∃xRjx
9. ~∀xRjx
10. ~∀xRxj
11. ∀x∀yRxy
12. ∀x∀yRyx
13. ∀x∃yRxy
14. ∀x∃yRyx
15. ∃x∀yRxy
16. ∃x∀yRyx
17. ∃x∃yRxy
18. ∃x∃yRyx
19. ∀x∃yCyx
20. ∃x∀yCxy

EXERCISE SET B

21. ~∃x∀yRxy
22. ~∃x∀yRyx
23. ∃x~∃yRxy
24. ∃x~∃yRyx
25. ~∀x∀yRxy
26. ~∀x∀yRyx
27. ~∀x∃yRxy
28. ~∀x∃yRyx
29. ~∃x~∃yRxy
30. ~∃x~∃yRyx
31. ~∃x~∀yRxy
32. ~∃x~∀yRyx
33. ~∃x~∃yRyx
34. ~∃x~∃yRxy
35. ~∃x~∃yRyx
36. ~∃x~∃yRxy
37. ∀x∃y~Rxy
38. ∀x∃y~Ryx
39. ∀x∃y~Cyx
40. ~∃x~∃yCyx

EXERCISE SET C

41. $\forall x(Fx \rightarrow \exists yRxy)$
42. $\forall x(Fx \rightarrow \exists yRyx)$
43. $\forall x\exists y(Fy \ \& \ Rxy)$
44. $\forall x\exists y(Fy \ \& \ Ryx)$
45. $\exists x(Fx \ \& \ \forall yRxy)$
46. $\exists x(Fx \ \& \ \forall yRyx)$
47. $\exists x\forall y(Fy \rightarrow Rxy)$
48. $\exists x\forall y(Fy \rightarrow Ryx)$
49. $\sim\exists x(Fx \ \& \ \forall yRyx)$
50. $\sim\exists x\forall y(Fy \rightarrow Rxy)$

EXERCISE SET D

51. $\forall x(Px \rightarrow \exists y(Sy \ \& \ Ryx))$
52. $\forall x(Px \rightarrow \exists y(Sy \ \& \ Rxy))$
53. $\forall x(Sx \rightarrow \exists y(Py \ \& \ Ryx))$
54. $\forall x(Sx \rightarrow \exists y(Py \ \& \ Rxy))$
55. $\forall x(Px \rightarrow \exists y(Sy \ \& \ \sim Ryx))$
56. $\forall x(Sx \rightarrow \exists y(Py \ \& \ \sim Ryx))$
57. $\forall x(Px \rightarrow \exists y(Sy \ \& \ \sim Rxy))$
58. $\forall x(Sx \rightarrow \exists y(Py \ \& \ \sim Rxy))$
59. $\exists x(Sx \ \& \ \forall y(Py \rightarrow Rxy))$
60. $\exists x(Px \ \& \ \forall y(Sy \rightarrow Rxy))$
61. $\exists x(Sx \ \& \ \forall y(Py \rightarrow Ryx))$
62. $\exists x(Px \ \& \ \forall y(Sy \rightarrow Ryx))$
63. $\exists x(Sx \ \& \ \sim\exists y(Py \ \& \ Rxy))$
64. $\exists x(Px \ \& \ \sim\exists y(Sy \ \& \ Rxy))$
65. $\exists x(Sx \ \& \ \sim\exists y(Py \ \& \ Ryx))$
66. $\exists x(Px \ \& \ \sim\exists y(Sy \ \& \ Ryx))$
67. $\sim\exists x(Sx \ \& \ \forall y(Py \rightarrow Rxy))$
68. $\sim\exists x(Px \ \& \ \forall y(Sy \rightarrow Rxy))$
69. $\sim\exists x(Sx \ \& \ \forall y(Py \rightarrow Ryx))$
70. $\sim\exists x(Sx \ \& \ \sim\exists y(Py \ \& \ Rxy))$
71. $\sim\exists x(Px \ \& \ \sim\exists y(Sy \ \& \ Rxy))$
72. $\sim\exists x(Sx \ \& \ \sim\exists y(Py \ \& \ Ryx))$
73. $\sim\exists x(Px \ \& \ \sim\exists y(Sy \ \& \ Ryx))$
74. $\exists x(Sx \ \& \ \sim\forall y(Py \rightarrow Rxy))$
75. $\exists x(Px \ \& \ \sim\forall y(Sy \rightarrow Rxy))$
76. $\exists x(Px \ \& \ \sim\forall y(Sy \rightarrow Ryx))$
77. $\exists x(Sx \ \& \ \sim\forall y(Py \rightarrow Ryx))$
78. $\sim\exists x(Sx \ \& \ \forall y(Py \rightarrow Ryx))$
79. $\sim\exists x(Px \ \& \ \forall y(Sy \rightarrow Ryx))$
80. $\sim\exists x(Px \ \& \ \sim\forall y(Sy \rightarrow Ryx))$

EXERCISE SET E

81. ∀x(Rxx → ∀yRxy)
82. ∀x(Rxx → ∀yRyx)
83. ∀x(∀yRxy → ∀yRyx)
84. ∀x[∀y(Fy → Rxy) → ∀y(Fy → Ryx)]
85. ∀x[∀y(Jy → Sxy) → Sxj]
86. ∀x[Txk → ∀y(Sy → Txy)]
87. ∀x[∀y(By → Txy) → ∀y(Jy → Txy)]
88. ∀x(Rxk → Rjx)
89. ~∃x(Rxk & Rjx);
90. ∀x(Kxj → ∃y(Kyk & Rxy))
91. ∃x~∃y(Ryj & Rxy)
92. ∃x(Gx & ∀y(Kyx → Fyx))
93. ∃x(Px & ∀y([Sy & Kyx] → Ryx))
94. ∃x(Sx & ∀y([Py & Ryy] → Ryx))
95. ∃x[Px & ∀y({Sy & ∀z([Cz & Oxz] → Eyz)} → Rxy)]
96. ∀x{[Sx & Kxj] → ∀y([Py & Ryj] → Rxy)}
97. ∃x(Px & ~∃y([Sy & ~Ryy] & Rxy))
98. ∃x(Px & ~∃y([Sy & ~∀z(Pz → Ryz)] & Rxy))
99. ~∃x{Px & ~∀y([Sy & ∀z([Cz & Txz) → Eyz)] → Rxy)}
100. ∀x{Sx → ∀y([Py & ∀z(Sz → Ryz)] → Rxy)}
101. ~∃x(~Mx & ∀yHxy)
102. ~∃x(~Sx & ∀yLxy)
103. ~∃x([Mx & ∀yRyx] & ~Sx)
104. ~∃x(~Mx & ∀y(Sy → Ixy))
105. ~∃x(~Mx & ~∃y(~Py & Rxy))

EXERCISE SET F

106. ∀x([Bx & Ljx] → Rjxk)
107. ∀x([Bx & Rkxj] → Ljx)
108. ∀x([Mx & Rjx] → Bjxk)
109. ∀x([Bx & Lkxj] → Skxc)
110. ∀x{Px → ∃y(Sy & ∀z([Bz & Rxzy] → Lyz))}

EXERCISE SET G

111. ∀x~Rjx
112. ∀x~Rxj
113. ∃x∀y~Rxy
114. ~∃x∀y~Ryx
115. ~∃x∀y~Rxy
116. ∀x(Px → ~Rjx)
117. ∀x(Px → ~Rxj)
118. ∃x∀y(Py → ~Ryx)
119. ~∃x∀y(Py → ~Rxy)
120. ∃x(Sx & ∀y(Py → ~Rxy))
121. ~∃x(Sx & ∀y(Py → ~Rxy))
122. ~∃x(Sx & ∀y(Py → ~Ryx))
123. ∀x(Px → ~∃y(Sy & Ryx))
124. ∀x(Px → ~∃y(Sy & Rxy))
125. ∀x∃y(Kxy & ∀z(Pz → ~Ryz))
126. ∀x(Sx → ∃y([Sy & Kxy] & ∀z(Pz → ~Ryz))
127. ∀x(Kxk → ~∃y(Kyj & Ryx))
128. ∃x∀y(Ryj → ~Rxy)
129. ∀x([Px & Rxj] → ~∃y([Sy & Kyj] & Ryx))
130. ∃x(Px & ∀y([Sy & ~Ryx] → ~Rxy))
131. ∃x(Px & ∀y([Sy & ~∀z(Pz → Ryz] → ~Rxy))
132. ∀x([Sx & Cjx] → ∀y(Py → Cyx))
133. ∀x([Sx & ~Ckx] → ~∃y(Py & Cyx))
134. ∀x([Sx & Ex] → ∀y(Py → Nyx))
135. ∀x∀y([(Cx & Ry) & Exy] → ∀z([Pz & Izy]→ Nzx))
136. ∀x∀y([(Mx&Dy) & Bxy] → ∀z(Wz → Szx))
137. ∀x∀y([(Tx & Ay) & Cyx] → Eyx)
138. ∀x(Fxo → Fxm)
139. ∀x∀y([Eyo & Bxy] → Exm)
140. ∀x∀y([Sy & Lxy] → Sx]