PICTORIAL SUMMARY OF RULES OF DERIVATION

SENTENTIAL LOGIC

&I	&O	~&O
$\mathcal{A}$ $\mathcal{B}$ / $\mathcal{A} \,\&\, \mathcal{B}$; $\mathcal{A}$ $\mathcal{B}$ / $\mathcal{B} \,\&\, \mathcal{A}$	$\mathcal{A} \,\&\, \mathcal{B}$ / $\mathcal{A}$; $\mathcal{A} \,\&\, \mathcal{B}$ / $\mathcal{B}$	$\sim(\mathcal{A} \,\&\, \mathcal{B})$ / $\mathcal{A} \rightarrow \sim\mathcal{B}$
∨I	**∨O**	**~∨O**
$\mathcal{A}$ / $\mathcal{A} \vee \mathcal{B}$; $\mathcal{A}$ / $\mathcal{B} \vee \mathcal{A}$	$\mathcal{A} \vee \mathcal{B}$, $\sim\mathcal{A}$ / $\mathcal{B}$; $\mathcal{A} \vee \mathcal{B}$, $\sim\mathcal{B}$ / $\mathcal{A}$	$\sim(\mathcal{A} \vee \mathcal{B})$ / $\sim\mathcal{A}$ $\sim\mathcal{B}$
↔I	**↔O**	**~↔O**
$\mathcal{A} \rightarrow \mathcal{B}$, $\mathcal{B} \rightarrow \mathcal{A}$ / $\mathcal{A} \leftrightarrow \mathcal{B}$; $\mathcal{A} \rightarrow \mathcal{B}$, $\mathcal{B} \rightarrow \mathcal{A}$ / $\mathcal{B} \leftrightarrow \mathcal{A}$	$\mathcal{A} \leftrightarrow \mathcal{B}$ / $\mathcal{A} \rightarrow \mathcal{B}$; $\mathcal{A} \leftrightarrow \mathcal{B}$ / $\mathcal{B} \rightarrow \mathcal{A}$	$\sim(\mathcal{A} \leftrightarrow \mathcal{B})$ / $\sim\mathcal{A} \leftrightarrow \mathcal{B}$
DN	**→O**	**~→O**
$\mathcal{A}$ / $\sim\sim\mathcal{A}$; $\sim\sim\mathcal{A}$ / $\mathcal{A}$	$\mathcal{A} \rightarrow C$, $\mathcal{A}$ / C ; $\mathcal{A} \rightarrow C$, $\sim C$ / $\sim\mathcal{A}$	$\sim(\mathcal{A} \rightarrow C)$ / $\mathcal{A} \,\&\, \sim C$
✕I	**✕O**	**R**
$\mathcal{A}$, $\sim\mathcal{A}$ / ✕	✕ / $\mathcal{A}$	$\mathcal{A}$ / $\mathcal{A}$

DD	CD	ID	ID
SHOW: $\mathcal{A}$ … $\mathcal{A}$	SHOW: $\mathcal{A} \rightarrow C$ … $\mathcal{A}$ SHOW: C …	SHOW: $\sim\mathcal{A}$ … $\mathcal{A}$ SHOW: ✕ …	SHOW: $\mathcal{A}$ … $\sim\mathcal{A}$ SHOW: ✕ …

PREDICATE LOGIC

In the following, v is any variable, a and n are names, and 𝔽 is a formula. Furthermore, 𝔽[a/v] is the formula that results when a is substituted for v at all its *free* occurrences in 𝔽, and similarly, 𝔽[n/v] is the formula that results when n is so substituted.

∀O

$$\frac{\forall v\mathbb{F}}{\mathbb{F}[a/v]}$$

a can be **any** name

∃I

$$\frac{\mathbb{F}[a/v]}{\exists v\mathbb{F}}$$

a can be **any** name

∃O

$$\frac{\exists v\mathbb{F}}{\mathbb{F}[n/v]}$$

n must be a **new** name

UD

~~SHOW~~: ∀v𝔽
| ~~SHOW~~: 𝔽[n/v] n must be a **new** name
| |

~∀O

$$\frac{\sim\forall v\mathbb{F}}{\exists v\sim\mathbb{F}}$$

~∃O

$$\frac{\sim\exists v\mathbb{F}}{\forall v\sim\mathbb{F}}$$