INTRO LOGIC
DAY 05

Schedule for Unit 1

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<th>Day 1</th>
<th>Intro</th>
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<tr>
<td>Day 2</td>
<td>Chapter 1</td>
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<td>Chapter 4</td>
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<td>Day 8</td>
<td>EXAM #1</td>
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- 40% of Exam
- 10 arguments 4 pts each
- 60% of Exam
- 12 translations 5 pts each

CHAPTER 4
TRANSLATIONS IN SENTENTIAL LOGIC

Basic Idea

In order to uncover *logical forms*, we **paraphrase** English sentences so that they contain only **standard connectives**.

We also use special abbreviations and special punctuation.

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40% of Exam
10 arguments 4 pts each
60% of Exam
12 translations 5 pts each
Abbreviation Scheme

1. atomic sentences are abbreviated by upper case letters (of the Roman alphabet).
2. standard connectives are abbreviated by special symbols (logograms).
3. compound sentences are abbreviated by algebraic combinations of 1 and 2.

Standard Connectives

<table>
<thead>
<tr>
<th>connective</th>
<th>symbol</th>
</tr>
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<tbody>
<tr>
<td>and</td>
<td>&amp;</td>
</tr>
<tr>
<td>or</td>
<td>∨</td>
</tr>
<tr>
<td>not</td>
<td>~</td>
</tr>
<tr>
<td>if – then</td>
<td>→</td>
</tr>
</tbody>
</table>

Non-Standard Connectives

... but ...   ... if and only if ...
... although ...    ... unless ...
... or ... but not both ... is necessary for ...
... if ...  ... is sufficient for ...
... only if ...  ... if ... otherwise ...
... unless... in which case

Simple Transformations

Some simple sentences are straightforwardly equivalent to compound sentences.

Jay and Kay are students

Jay is a student and Kay is a student

Kay is a rich student

Kay is rich and Kay is a student
Pitfall #1

Kay is a former student

Kay is former and Kay is a student

F&S

Pitfall #2

Jay and Kay are married ambiguous

[‘ambi’ (also ‘amphi’) means “both” and “around”]

other words containing ‘ambi’ and ‘amphi’

ambidextrous ambivalent
ambience (ambiance) ambitious
amphibian amphitheater

First Reading

Jay and Kay are married

Jay is married (to someone) and Kay is married (to someone)

J & K

Second Reading

Jay and Kay are married

Jay and Kay are married-to-each-other

later, in predicate logic, further parts are revealed

M Mjk
Pitfall #3

Odd use of ‘and’
Sometimes ‘and’ is used as follows.
1) keep trying, **and** you will succeed
2) keep it up buster, **and** I will clobber you
3) give him an inch, **and** he will take a mile
4) give me the money, **and** I’ll give you the product
5) you do that, **and** I’ll kill ya

Other Toughies

Pitfall #3, continued

What happens if we symbolize keep trying **and** you will succeed as:

<table>
<thead>
<tr>
<th>K</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>you will keep trying</td>
<td>you will succeed</td>
</tr>
</tbody>
</table>

appropriately paraphrase

| (1) if you keep trying, **then** you will succeed | \( K \rightarrow S \) |
| (2) if you keep it up buster, **then** I will clobber you | \( K \rightarrow C \) |
| (3) if you give him an inch, **then** he will take a mile | \( G \rightarrow T \) |
| (4) if you give me the money, **then** I’ll give you the product | \( M \rightarrow P \) |
| (5) if you do that, **then** I’ll kill ya | \( D \rightarrow K \) |
**Exclusive ‘Or’**

Ram **xor** Sam will win (the election)
Ram will win, or Sam will win, **but not both**

<table>
<thead>
<tr>
<th>at least one of them win</th>
<th>(R ∨ S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>also</td>
<td>&amp;</td>
</tr>
<tr>
<td>both of them will win...not!</td>
<td>~(R &amp; S)</td>
</tr>
</tbody>
</table>

Alternatively,

<table>
<thead>
<tr>
<th>Ram will win, <strong>and</strong> Sam will not win,</th>
<th>(R &amp; ~S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>or</td>
<td>~</td>
</tr>
<tr>
<td>Sam will win, <strong>and</strong> Ram will not win.</td>
<td>(S &amp; ~R)</td>
</tr>
</tbody>
</table>

**Alternative Formulation**

it is **neither** raining nor sleeting
it is **not** raining or sleeting
it is **not** true that (it is raining or it is sleeting)

\[ \sim (R ∨ S) \]

≡

\[ \sim R \& \sim S \]

**‘Neither...Nor’**

it is **neither** raining nor sleeting
it is **not** raining, **and** it is **not** sleeting

\[ \sim R \& \sim S \]

**Neither Jay nor Kay is sleeping**
Jay is **not** sleeping, **and** Kay is **not** sleeping

\[ \sim J \& \sim K \]

**Conditionals**

standard expression:
if A then C
which is symbolized:

\[ A \rightarrow C \]

A is the **antecedent**
C is the **consequent**
Simple Variants of ‘if…then…’
- if A, then C
- if A, C
- C if A

all of these are symbolized the same:

\[ A \rightarrow C \]

‘ if ’ always introduces the antecedent
‘ then ’ always introduces the consequent

‘Only If’

‘ only if ’ is not equivalent to ‘ if ’

I will get an A only if I take all the exams vs.
I will get an A if I take all the exams

I will get an A if I get a hundred vs.
I will get an A only if I get a hundred

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Other Variants of ‘if’

C provided A
C in case A
C supposing A
provided A, C
in case A, C
supposing A, C

all symbolized the same way

A \rightarrow C

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How does ‘only’ work?

1. employees only
2. authorized personnel only
3. cars only
4. right turn only

‘only’ operates as a dual-negative modifier.

for example, (1) means to exclude anyone who is not an employee.

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a more lyrical example of ‘only’

“I only have eyes for you”

The Flamingos
1959
lyrics by Al Dubin
music by Harry Warren
1934

‘only’ is focus-sensitive

I only have eyes for you

alternative focus for ‘only’

I only have eyes for you
alternative focus for ‘only’

I only have eyes for you

How does ‘only’ modify ‘if’?

‘only’ modifies ‘if’ by introducing two negations

A only if B
not A if not B
if not B then not A
~ B → ~ A
Example

I will get an A only if I take all the exams

I will not get an A if I do not take all the exams

if I do not take all the exams then I will not get an A

\[ \sim E \rightarrow \sim A \]

Example of ‘if and only if’

I will pass if and only if I average 50

\[ P \text{ if and only if } A \]

P if A and P only if A

if A, then P

\[ \text{not } P \text{ if not } A \]

if not A then not P

\[ ( A \rightarrow P ) \text{ and } ( \sim A \rightarrow \sim P ) \]

‘If And Only If’

Two approaches to ‘if and only if’

1. treat it as a simple connective (\(\leftrightarrow\))
2. treat it as a complex connective

We will concentrate on (2), according to which if and only if consists of 3 components:

1. if
2. and
3. only if

each of which we already know how to paraphrase.

‘Unless’

The following are all equivalent.

I will pass only if I study

P only if S

I will not pass unless I study

not P unless S

I will not pass if I do not study

not P if not S

\[ \text{only if has two built-in negations} \]

\[ \text{unless has one built-in negation} \]

\[ \text{if has no built-in negation} \]

unless = if not
**Example of ‘unless’**

<table>
<thead>
<tr>
<th>I will not pass unless I study</th>
</tr>
</thead>
<tbody>
<tr>
<td>not P</td>
</tr>
<tr>
<td>not P</td>
</tr>
<tr>
<td>not P</td>
</tr>
<tr>
<td>if not S</td>
</tr>
<tr>
<td>~ S</td>
</tr>
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