Basic Idea

We start with a few argument forms, which we presume are valid, and we use these to demonstrate that other argument forms are valid.

We demonstrate (show) that a given argument form is valid by deriving (deducing) its conclusion from its premises using a few fundamental modes of reasoning.

Example 1 – Modus Ponens (MP)

\[ \begin{array}{c}
A \rightarrow C \\
A \\
\hline
C \\
\end{array} \quad \begin{array}{c}
\text{if } A \text{ then } C \\
A \\
\hline
C \\
\end{array} \]

Example Argument Form

we can employ \textit{modus ponens} (MP) to derive the conclusion from the premises.

\[ \begin{array}{c}
P \\
Q \\
Q \rightarrow R \\
R \rightarrow S \\
\hline
S \\
\end{array} \]
Example 2 – Modus Tollens (MT)

\[ \mathcal{A} \rightarrow C \]
\[ \sim C \]
\[ \sim \mathcal{A} \]

Example Argument Form
we can employ modus tollens (MT) to derive the conclusion from the premises.

Example 3 – using both MP and MT

\[ \mathcal{A} \rightarrow C \]
\[ \mathcal{A} \]
\[ C \]
\[ \sim \mathcal{A} \]

Example Argument Form
we can employ a combination of MP and MT to derive the conclusion from the premises.
**Example 3 (continued)**

```
~S ; R → S ; ~R → ~T ; P → T ; ~P → ~Q ~Q
```

**Derivations – How to Start**

1. write down premises
2. write down “SHOW:” conclusion

(1) P Pr (premise)
(2) P → Q Pr (premise)
(3) Q → R Pr (premise)
(4) R → S Pr (premise)
(5) SHOW: S what one is trying to derive

**Derivations – How to Continue**

3. Apply rules, as applicable, to available lines until goal is reached.

(1) P Pr (premise)
(2) P → Q Pr (premise)
(3) Q → R Pr (premise)
(4) R → S Pr (premise)
(5) SHOW: S (goal)
(6) Q 1,2,MP follows from lines 1 and 2
(7) R 3,6,MP follows from lines 3 and 6
(8) S 4,7,MP follows from lines 4 and 7

**Derivations – How to Finish**

4. Box and Cancel

(1) P Pr
(2) P → Q Pr
(3) Q → R Pr
(4) R → S Pr
(5) SHOW: S DD Direct Derivation
(6) Q 1,2,MP
(7) R 3,6,MP
(8) S 4,7,MP
Derivation – Example 2

argument : ~S ; R→S ; Q→R ; P→R ; / ~P

(1) ~S           Pr
(2) R → S       Pr
(3) Q → R       Pr
(4) P → Q       Pr
(5) ~P          DD
(6) ~R          1,2,MT
(7) ~Q          3,6,MT
(8) ~P          4,7,MT

Derivation – Example 3

argument : ~S ; R→S ; ~R→~T ; P→T ; ~P→~Q / ~Q

(1) ~S           Pr
(2) R → S       Pr
(3) ~R → ~T     Pr
(4) P → T       Pr
(5) ~P → ~Q     Pr
(6) ~Q          DD
(7) ~R          1,2,MT
(8) ~T          3,7,MP
(9) ~P          4,8,MT
(10) ~Q         5,9,MP

Examples of Modus Ponens

\[ \mathcal{A} \to C \]
\[ \mathcal{A} \]
\[ \mathcal{B} \]

\[ (P \& Q) \to (R \lor S) \]
\[ P \& Q \]
\[ R \lor S \]

\[ \sim P \to \sim Q \]
\[ \sim P \]
\[ \sim Q \]
Examples of Modus Tollens

\[ \begin{array}{c}
A \rightarrow C \\
\sim C \\
\sim A \\
\hline \\
\sim P \rightarrow \sim Q \\
Q \\
P \\
\end{array} \]

valid argument

BUT

NOT an example of MT

Examples of MTP(1)

\[ \begin{array}{c}
A \lor B \\
\sim A \\
B \\
\hline \\
(P \& Q) \lor (R \lor S) \\
\sim (P \& Q) \\
R \lor S \\
\hline \\
\sim P \lor \sim Q \\
\sim Q \\
\end{array} \]

Examples of MTP(2)

\[ \begin{array}{c}
A \lor B \\
\sim B \\
A \\
\hline \\
(P \& Q) \lor (R \lor S) \\
\sim (P \& Q) \\
P \& Q \\
\hline \\
\sim P \lor \sim Q \\
\sim Q \\
\hline \\
\sim P \\
\end{array} \]

Form versus Content/Value

are the dime and ten pennies the same? NO
are they the same in value? YES
actually, in a very important sense, they are not the same in value!

$2,000,000$