INTRO LOGIC

DAY 12

Derivations in SL

4

Schedule

✓ Day 09 Introductory Material
✓ Day 10 Direct Derivation (DD)
✓ Day 11 Conditional Derivation (CD)
     Negation Derivation (~D)
Day 12 Indirect Derivation
     show: atomic
     show: disjunction
Day 13 show: conjunction
Day 14 EXAM #2

Exam 2 Format

- 6 argument forms, 15 points each, plus 10 free points
- Symbolic argument forms (no translations)
- For each one, you will be asked to construct a derivation of the conclusion from the premises.
- The rule sheet will be provided.
  1 problem from Set D
  2 problems from Set E
  2 problems from Set F
  1 problem from Set G (91-96)

Inference Rules (so far)

&O

A & B
---
A

B

A & B
---

A

B

&I

A

B

---

A & B

B & A

∨O

A ∨ B
---
A

B

A ∨ B
---
~A

~B

~A

~B

~A

~B

¬A

¬B

¬A

¬B

¬A

¬B

→O

A → B
---
A

~B

B

~A

DN

~A

A

~A

~A
**SHOW Rules (so far)**

### Direct-Derivation Strategy

In Direct Derivation (DD), one **directly** arrives at the very formula one is trying to show.

### Affiliated Rules

**Assumption Rule (CD)**

If one has a line of the form \( \neg d \), then one is entitled to write down the formula \( d \) on the very next line, as an assumption.

**Assumption Rule (~D)**

If one has a line of the form \( \neg \neg d \), then one is entitled to write down the formula \( d \) on the very next line, as an assumption.

**Contradiction-In (\( \lnot \lnot \))**

if you have a formula \( A \) and you have its negation \( \neg A \), then you are entitled to infer a contradiction (absurdity).

### Show-Conditional Strategy

**Show-Conditional Strategy**

In Show-Conditional (CD), one tries to prove a conditional statement by showing that both the antecedent and the consequent hold.
Show-Negation Strategy

Indirect Derivation

Can we show the following?

(1) \( P \rightarrow Q \) Pr
(2) \( \neg P \rightarrow Q \) Pr
(3) SHOW: \( Q \) ??

We are stuck!!

we have \( P \rightarrow Q \)
so to apply \( \rightarrow O \)
we must find \( P \)
or find \( \neg Q \)
we also have \( \neg P \rightarrow Q \)
so to apply \( \rightarrow O \)
we must find \( \neg P \)
or find \( \neg Q \)

Using ID

The difference between ID and \( \neg D \) is that
\( \neg D \) applies only to negations,
whereas ID applies (in principle) to all formulas;
it is a generic rule, like direct-derivation.

Although ID can, in principle, be used on
any formula,
it is best used on two types of formulas.

1. atomic formulas \( P, Q, R, \text{ etc.} \)
2. disjunctions \( A \vee B \)
Show-Atomic Strategy

Example 1

Example 2

Show-Disjunction Strategy
Example 3

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>(1)</td>
<td>( \neg P \rightarrow Q )</td>
</tr>
<tr>
<td>(2)</td>
<td>SHOW: ( P \lor Q )</td>
</tr>
<tr>
<td>(3)</td>
<td>( \neg (P \lor Q) )</td>
</tr>
<tr>
<td>(4)</td>
<td>SHOW: ( \times )</td>
</tr>
<tr>
<td>(5)</td>
<td>( \neg P )</td>
</tr>
<tr>
<td>(6)</td>
<td>( \neg Q )</td>
</tr>
<tr>
<td>(7)</td>
<td>Q</td>
</tr>
<tr>
<td>(8)</td>
<td>( \times )</td>
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</tbody>
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