INTRO LOGIC
DAY 15

UNIT 3
Translations in Predicate Logic

Overview

- Exam 1: Sentential Logic Translations (+)
- Exam 2: Sentential Logic Derivations
- Exam 3: Predicate Logic Translations
- Exam 4: Predicate Logic Derivations
- Exam 5: (finals) very similar to Exam 3
- Exam 6: (finals) very similar to Exam 4

Grading Policy

When computing your final grade, I count your four highest scores. (A missed exam counts as a zero.)
Subjects and Predicates

In predicate logic,
every atomic sentence consists of
one predicate
and
one or more “subjects”
including subjects, direct objects,
indirect objects, etc.

in mathematics “subjects” are called “arguments”
(Shakespeare used the term ‘argument’ to mean ‘subject’)

Example 1

<table>
<thead>
<tr>
<th>Subject</th>
<th>Predicate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay</td>
<td>is asleep</td>
</tr>
<tr>
<td>Kay</td>
<td>is awake</td>
</tr>
<tr>
<td>Elle</td>
<td>is a dog</td>
</tr>
</tbody>
</table>

Example 2

<table>
<thead>
<tr>
<th>Subject</th>
<th>Predicate</th>
<th>Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay</td>
<td>respects</td>
<td>Kay</td>
</tr>
<tr>
<td>Kay</td>
<td>is next to</td>
<td>Elle</td>
</tr>
<tr>
<td>Elle</td>
<td>is taller than</td>
<td>Jay</td>
</tr>
</tbody>
</table>

Example 3

<table>
<thead>
<tr>
<th>Subject</th>
<th>Predicate</th>
<th>Direct Object</th>
<th>Indirect Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay</td>
<td>sold</td>
<td>Elle</td>
<td>to Kay</td>
</tr>
<tr>
<td>Kay</td>
<td>bought</td>
<td>Elle</td>
<td>from Jay</td>
</tr>
<tr>
<td>Kay</td>
<td>prefers</td>
<td>Elle</td>
<td>to Jay</td>
</tr>
</tbody>
</table>
What is a Predicate?

A predicate is an "incomplete" expression – i.e., an expression with one or more blanks – such that, whenever the blanks are filled by noun phrases, the resulting expression is a sentence.

Examples

is tall

is taller than

recommends to

Symbolization Convention

1. Predicates are symbolized by upper case letters.
2. Subjects are symbolized by lower case letters.
3. Predicates are placed first.
4. Subjects are placed second.
Examples

<table>
<thead>
<tr>
<th>Statement</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay is tall</td>
<td>Tj</td>
</tr>
<tr>
<td>Kay is tall</td>
<td>Tk</td>
</tr>
<tr>
<td>Jay is taller than Kay</td>
<td>Tjk</td>
</tr>
<tr>
<td>Kay is taller than Elle</td>
<td>Tke</td>
</tr>
<tr>
<td>Jay recommended Kay to Elle</td>
<td>Rjke</td>
</tr>
<tr>
<td>Kay recommended Elle to Jay</td>
<td>Rkiej</td>
</tr>
</tbody>
</table>

Compound Sentences - 1

<table>
<thead>
<tr>
<th>Statement</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay is not tall</td>
<td>~Tj</td>
</tr>
<tr>
<td>Jay is not taller than Kay</td>
<td>~Tjk</td>
</tr>
<tr>
<td>both Jay and Kay are tall</td>
<td>Tj &amp; Tk</td>
</tr>
<tr>
<td>neither Jay nor Kay is tall</td>
<td>~Tj &amp; ~Tk</td>
</tr>
<tr>
<td>Jay is taller than both Kay and Elle</td>
<td>Tjk &amp; Tje</td>
</tr>
</tbody>
</table>

Compound Sentences - 2

<table>
<thead>
<tr>
<th>Statement</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay and Kay are married (individually)</td>
<td>Mj &amp; Mk</td>
</tr>
<tr>
<td>Jay is married, and Kay is married</td>
<td></td>
</tr>
<tr>
<td>Jay and Kay are married (to each other)</td>
<td>Mjk</td>
</tr>
<tr>
<td>and are married</td>
<td></td>
</tr>
</tbody>
</table>

Quantifiers

Quantifiers are linguistic expressions denoting quantity.

Examples
- every, all, any, each, both, either
- some, most, many, several, few
- no, neither
- at least one, at least two, etc.
- at most one, at most two, etc.
- exactly one, exactly two, etc.
Quantifiers – 2

Quantifiers combine common nouns and verb phrases to form sentences.

Examples
- *every* senior is happy
- *no* freshman is happy
- *at least one* junior is happy
- *few* sophomores are happy
- *most* graduates are happy

Predicate logic treats both common nouns and verb phrases as predicates.

The Two Special Quantifiers of Predicate Logic

<table>
<thead>
<tr>
<th>official name</th>
<th>English expressions</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>universal quantifier</td>
<td>every, any</td>
<td>( \forall )</td>
</tr>
<tr>
<td>existential quantifier</td>
<td>some, at least one</td>
<td>( \exists )</td>
</tr>
</tbody>
</table>

Names of Symbols

\( \forall \) upside-down ‘A’  \( \exists \) backwards ‘E’

Actually, they are both upside-down.

How Traditional Logic Does Quantifiers

Quantifier Phrases are Simply Noun Phrases

- every one is happy
- some one is happy
- Jay is happy
- Kay is happy
- subject predicate
How Modern Logic Does Quantifiers

Quantifier Phrases are Sentential Adverbs

Existential Quantifier

- some one is happy
- there is some one who is happy
- there is some one such that he/she is happy
- there is some x such that x is happy

\( \exists x \ H x \) pronunciation

- there is an x (such that) H x

Universal Quantifier

- every one is happy
- every one is such that he/she is happy
- whoever you are you are happy
- no matter who you are you are happy
- no matter who x is x is happy

\( \forall x \ H x \) pronunciation

- for any x H x

Negating Quantifiers

modern logic takes ‘\( \exists \)’ to mean at least one
which means one or more
which means one, or two, or three, or …
if a (counting) number is not one or more it must be zero
thus, the negation of ‘at least one’ is ‘not at least one’
which is ‘none’
Negative-Existential Quantifier

- no one is happy
- there is no one who is happy
- there is no one such that he/she is happy
- there is no x such that x is happy
- there is not some x such that x is happy

\[ \sim \exists x \ H x \]

pronunciation

- there is no x (such that) H x

Quantifying Negations - 1

suppose not everyone is happy
then there is someone who is not happy
i.e., there is some x : x is not happy

\[ \sim \forall x H x \]

the converse argument is also valid

Negative-Universal Quantifier

- not every one is happy
- not every one is such that he/she is H
- it is not true that whoever you are you are H
- it is not true that no matter who you are you are H
- it is not true that no matter who x is x is H

\[ \sim \forall x H x \]

pronunciation

- not for any x H x

Quantifying Negations - 2

suppose no one is happy
then no matter who you are you are not happy
i.e. no matter who x is x is not happy

\[ \sim \exists x H x \]

the converse argument is also valid