INTRO LOGIC
DAY 23
Derivations in PL
2

Overview

- Exam 1  Sentential Logic  Translations (+)
- Exam 2  Sentential Logic  Derivations
- Exam 3  Predicate Logic  Translations
- Exam 4  Predicate Logic  Derivations
- 6 derivations  @ 15 points  + 10 free points
- Exam 5  very similar to Exam 3
- Exam 6  very similar to Exam 4

Sentential Logic Rules

- DD
- ID
- CD
- ~D
- &D
- etc.

Rule Sheet

- provided on exams
- available on course web page (textbook)
- keep this in front of you when doing homework

Sentential Logic Rules

- &I
- &O
- vO
- →O
- ~vO
- etc.
Predicate Logic Rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>Symbol</th>
<th>Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal Derivation</td>
<td>UD</td>
<td>2</td>
</tr>
<tr>
<td>Universal-Out</td>
<td>$\forall \varnothing$</td>
<td>1</td>
</tr>
<tr>
<td>Tilde-Universal-Out</td>
<td>$\sim \forall \varnothing$</td>
<td>3</td>
</tr>
<tr>
<td>Existential-In</td>
<td>$\exists I$</td>
<td>1</td>
</tr>
<tr>
<td>Existential-Out</td>
<td>$\exists O$</td>
<td>2</td>
</tr>
<tr>
<td>Tilde-Existential-Out</td>
<td>$\sim \exists O$</td>
<td>3</td>
</tr>
</tbody>
</table>

Rules to be Introduced Today

<table>
<thead>
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<tr>
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<tr>
<td>Existential-Out</td>
<td>$\exists O$</td>
</tr>
</tbody>
</table>

Example 1

every $F$ is $H$ ; everyone is $F$ / everyone is $H$

<table>
<thead>
<tr>
<th>Rule</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $\forall x(Fx \rightarrow Hx)$</td>
<td>Pr</td>
<td></td>
</tr>
<tr>
<td>(2) $\forall xFx$</td>
<td>Pr</td>
<td></td>
</tr>
<tr>
<td>(3) SHOW: $\forall xHx$</td>
<td>??</td>
<td></td>
</tr>
<tr>
<td>(3) SHOW: $Ha$ &amp; $Hb$ &amp; $Hc$ &amp; …… &amp;. &amp;. &amp; D</td>
<td>??</td>
<td>what is ultimately involved in showing a universal</td>
</tr>
<tr>
<td>(a) SHOW: $Ha$</td>
<td>??</td>
<td></td>
</tr>
<tr>
<td>(?) ??</td>
<td>??</td>
<td></td>
</tr>
<tr>
<td>(b) SHOW: $Hb$</td>
<td>??</td>
<td></td>
</tr>
<tr>
<td>(?) ??</td>
<td>??</td>
<td></td>
</tr>
<tr>
<td>(c) SHOW: $Hc$</td>
<td>??</td>
<td></td>
</tr>
<tr>
<td>(?) ??</td>
<td>??</td>
<td></td>
</tr>
<tr>
<td>……</td>
<td>……</td>
<td></td>
</tr>
</tbody>
</table>

\[ o \] is an OLD name
(more about this later)
Example 1a

1. $\forall x(Fx \to Hx)$ Pr
2. $\forall xFx$ Pr
3. SHOW: $\forall xHx$ ??
4. SHOW: Ha DD
5. Fa $\to$ Ha 1, $\forall O$
6. Fa 2, $\forall O$
7. Ha 5,6, $\to O$

one down, a zillion to go!

Example 1b

1. $\forall x(Fx \to Hx)$ Pr
2. $\forall xFx$ Pr
3. SHOW: $\forall xHx$ ??
4. SHOW: Hb DD
5. Fb $\to$ Hb 1, $\forall O$
6. Fb 2, $\forall O$
7. Hb 5,6, $\to O$

two down, a zillion to go!

Example 1c

1. $\forall x(Fx \to Hx)$ Pr
2. $\forall xFx$ Pr
3. SHOW: $\forall xHx$ ??
4. SHOW: Hc DD
5. Fc $\to$ Hc 1, $\forall O$
6. Fc 2, $\forall O$
7. Hc 5,6, $\to O$

three down, a zillion to go!

But wait – the derivations all look alike!

4. SHOW: Ha DD
5. Fa $\to$ Ha 1, $\forall O$
6. Fa 2, $\forall O$
7. Ha 5,6, $\to O$

4. SHOW: Hb DD
5. Fb $\to$ Hb 1, $\forall O$
6. Fb 2, $\forall O$
7. Hb 5,6, $\to O$

4. SHOW: Hc DD
5. Fc $\to$ Hc 1, $\forall O$
6. Fc 2, $\forall O$
7. Hc 5,6, $\to O$
The Universal-Derivation Strategy

So, all we need to do is do one derivation with one name (say, ‘a’) and then argue that all the other derivations will look the same. To ensure this, we must ensure that the name is general, which we can do by making sure

the name we select is NEW.

a name counts as NEW precisely if it occurs nowhere in the derivation unboxed or uncancelled

Comparison with Universal-Out

a name counts as OLD precisely if it occurs somewhere in the derivation unboxed and uncancelled

Example 2

every F is H / if everyone is F, then everyone is H

(1) \( \forall x(Fx \rightarrow Hx) \)  Pr
(2) \( \forall xFx \rightarrow \forall xHx \)  CD
(3) \( \forall xFx \)  As
(4) \( \forall xHx \)  UD  a new
(5) \( \forall xHx \)  UD
(6) Fa \rightarrow Ha  1, \( \forall O \)  a old
(7) Fa  3, \( \forall O \)  a old
(8) Ha  6,7, \rightarrow O
Example 3

every $F$ is $G$ ; every $G$ is $H$ / every $F$ is $H$

(1) $\forall x(Fx \rightarrow Gx)$  Pr
(2) $\forall x(Gx \rightarrow Hx)$  Pr
(3) SHOW: $\forall x(Fx \rightarrow Hx)$  UD  a new
(4) SHOW: $Fa \rightarrow Ha$  CD
(5) $Fa$  As
(6) SHOW: $Ha$  DD
(7) $Fa \rightarrow Ga$  1, $\forall O$  a old
(8) $Ga \rightarrow Ha$  2, $\forall O$  a old
(9) $Ga$  5, 7, $\rightarrow O$
(10) $Ha$  8, 9, $\rightarrow O$

Example 4

everyone $R$'s everyone / everyone is $R$'ed by everyone

(1) $\forall x\forall yRxy$  Pr
(2) SHOW: $\forall x\forall yRxy$  UD  a new
(3) SHOW: $\forall yRya$  UD  b new
(4) SHOW: $Rba$  DD
(5) $\forall yRby$  1, $\forall O$  b old
(6) $Rba$  5, $\forall O$  a old

Existential-Out ($\exists O$)

any variable (z, y, x, w ...)

$\exists uF[u]$  $\rightarrow n$  $n$ replaces $u$

any NEW name (a, b, c, d, ...)

Comparison with Universal-Out

$\forall O$

$\forall uF[u]$  $\rightarrow o$  $O$ name

$\exists uF[u]$  $\rightarrow n$  $N$ name

a name counts as OLD
precisely if it occurs

somewhere
unboxed and uncancelled

a name counts as NEW
precisely if it occurs

nowhere
unboxed or uncancelled
Example 5

**every \( F \) is un-H / no \( F \) is H**

1. \( \forall x(Fx \rightarrow \sim Hx) \)  Pr
2. Show: \( \sim \exists x(Fx \& Hx) \) \( \sim D \)
3. \( \exists x(Fx \& Hx) \) As
4. Show: \( \times \) DD
5. \( Fa \& Ha \) 3, \( \exists O \) new
6. \( Fa \rightarrow \sim Ha \) 1, \( \forall O \) old
7. \( Fa \)
8. \( Ha \)
9. \( \sim Ha \) 6,7, \( \rightarrow O \)
10. \( \times \) 8,9, \( \times \)

Example 7

**every \( F \) is G ; some \( F \) is H / some \( G \) is H**

1. \( \forall x(Fx \rightarrow Gx) \)  Pr
2. \( \exists x(Fx \& Hx) \) Pr
3. Show: \( \exists x(Gx \& Hx) \) DD
4. \( Fa \& Ha \) 2, \( \exists O \) new
5. \( Fa \rightarrow Ga \) 1, \( \forall O \) old
6. \( Fa \)
7. \( Ha \)
8. \( Ga \) 5,6, \( \rightarrow O \)
9. \( Ga \& Ha \) 7,8, \( &I \)
10. \( \exists x(Gx \& Hx) \) 9, \( \exists I \)

Example 6

**some \( F \) is not H / not every \( F \) is H**

1. \( \exists x(Fx \& \sim Hx) \) Pr
2. Show: \( \sim \forall x(Fx \rightarrow Hx) \) ID
3. \( \forall x(Fx \rightarrow Hx) \) As
4. Show: \( \times \) DD
5. \( Fa \& \sim Ha \) 1, \( \exists O \) new
6. \( Fa \rightarrow Ha \) 3, \( \forall O \) old
7. \( Fa \)
8. \( \sim Ha \)
9. \( Ha \) 6,7, \( \rightarrow O \)
10. \( \times \) 8,9, \( \times \)

Example 8

**if anyone is \( F \) then everyone is H / if someone is \( F \), then everyone is H**

1. \( \forall x(Fx \rightarrow \forall xHx) \) Pr
2. Show: \( \exists Fx \rightarrow \forall xHx \) CD
3. \( \exists Fx \) As
4. Show: \( \forall xHx \) UD new
5. Show: \( Ha \) DD
6. \( Fb \) 3, \( \exists O \) new
7. \( Fb \rightarrow \forall xHx \) 1, \( \forall O \) old
8. \( \forall xHx \) 6,7, \( \rightarrow O \)
9. \( Ha \) 8, \( \forall O \) old
Example 9

if someone is F, then everyone is H
/ if anyone is F then everyone is H

(1) \( \exists x Fx \rightarrow \forall x Hx \)
(2) \( \text{SHOW: } \forall x(Fx \rightarrow \forall y Hy) \)
(3) \( \text{SHOW: } Fa \rightarrow \forall y Hy \)
(4) \( Fa \)
(5) \( \text{SHOW: } \forall y Hy \)
(6) \( \text{SHOW: } Hb \)
(7) \( \exists x Fx \)
(8) \( \forall x Hx \)
(9) \( Hb \)

Example 10 (a fragment)

someone R's someone
??missing premises??
/ everyone R's everyone

(1) \( \exists x \exists y Rxy \)
(2) \( ?? \)
(3) \( \text{SHOW: } \forall x \forall y Rxy \)
(4) \( \text{SHOW: } \forall y Ray \)
(5) \( \text{SHOW: } Rab \)
(6) \( \exists y Rcy \)
(7) \( Rd \)
(8) \( ?? \)