1. INTRODUCTION

In the present chapter, we discuss how to translate a variety of English statements into the language of sentential logic.

From the viewpoint of sentential logic, there are five standard connectives – ‘and’, ‘or’, ‘if...then’, ‘if and only if’, and ‘not’. In addition to these standard connectives, there are in English numerous non-standard connectives, including ‘unless’, ‘only if’, ‘neither...nor’, among others. There is nothing linguistically special about the five "standard" connectives; rather, they are the connectives that logicians have found most useful in doing symbolic logic.

The translation process is primarily a process of paraphrase – saying the same thing using different words, or expressing the same proposition using different sentences. Paraphrase is translation from English into English, which is presumably easier than translating English into, say, Japanese.

In the present chapter, we are interested chiefly in two aspects of paraphrase. The first aspect is paraphrasing statements involving various non-standard connectives into equivalent statements involving only standard connectives.

The second aspect is paraphrasing simple statements into straightforwardly equivalent compound statements. For example, the statement ‘it is not raining’ is straightforwardly equivalent to the more verbose ‘it is not true that it is raining’. Similarly, ‘Jay and Kay are Sophomores’ is straightforwardly equivalent to the more verbose ‘Jay is a Sophomore, and Kay is a Sophomore’.

An English statement is said to be in standard form, or to be standard, if all its connectives are standard and it contains no simple statement that is straightforwardly equivalent to a compound statement; otherwise, it is said to be non-standard.

Once a statement is paraphrased into standard form, the only remaining task is to symbolize it, which consists of symbolizing the simple (atomic) statements and symbolizing the connectives. Simple statements are symbolized by upper case Roman letters, and the standard connectives are symbolized by the already familiar symbols – ampersand, wedge, tilde, arrow, and double-arrow.

In translating simple statements, the particular letter one chooses is not terribly important, although it is usually helpful to choose a letter that is suggestive of the English statement. For example, ‘R’ can symbolize either ‘it is raining’ or ‘I am running’; however, if both of these statements appear together, then they must be symbolized by different letters. In general, in any particular context, different letters must be used to symbolize non-equivalent statements, and the same letter must be used to symbolize equivalent statements.
2. THE GRAMMAR OF SENTENTIAL LOGIC; A REVIEW

Before proceeding, let us review the grammar of sentential logic. First, recall that statements may be divided into simple statements and compound statements. Whereas the latter are constructed from smaller statements using statement connectives, the former are not so constructed.

The grammar of sentential logic reflects this grammatical aspect of English. In particular, formulas of sentential logic are divided into atomic formulas and molecular formulas. Whereas molecular formulas are constructed from other formulas using connectives, atomic formulas are structureless, they are simply upper case letters (of the Roman alphabet).

Formulas are strings of symbols. In sentential logic, the symbols include all the upper case letters, the five connective symbols, as well as left and right parentheses. Certain strings of symbols count as formulas of sentential logic, and others do not, as determined by the following definition.

Definition of Formula in Sentential Logic:

(1) every upper case letter is a formula;
(2) if $A$ is a formula, then so is $\neg A$;
(3) if $A$ and $B$ are formulas, then so is $(A \& B)$;
(4) if $A$ and $B$ are formulas, then so is $(A \lor B)$;
(5) if $A$ and $B$ are formulas, then so is $(A \rightarrow B)$;
(6) if $A$ and $B$ are formulas, then so is $(A \leftrightarrow B)$;
(7) nothing else is a formula.

In the above definition, the script letters stand for arbitrary strings of symbols. So for example, clause (2) says that if you have a string $A$ of symbols, then provided $A$ is a formula, the result of prefixing a tilde sign in front of $A$ is also a formula. Also, clause (3) says that if you have a pair of strings, $A$ and $B$, then provided both strings are formulas, the result of infixing an ampersand and surrounding the resulting expression by parentheses is also a formula.

As noted earlier, in addition to formulas in the strict sense, which are specified by the above definition, we also have formulas in a less strict sense. These are called unofficial formulas, which are defined as follows.

An unofficial formula is any string of symbols obtained from an official formula by removing its outermost parentheses, if such exist.

The basic idea is that, although the outermost parentheses of a formula are crucial when it is used to form a larger formula, the outermost parentheses are optional when the formula stands alone. For example, the answers to the exercises, at the back of the chapter, are mostly unofficial formulas.
3. CONJUNCTIONS

The standard English expression for conjunction is ‘and’, but there are numerous other conjunction-like expressions, including the following.

(c1) but
(c2) yet
(c3) although
(c4) though
(c5) even though
(c6) moreover
(c7) furthermore
(c8) however
(c9) whereas

Although these expressions have different connotations, they are all truth-functionally equivalent to one another. For example, consider the following statements.

(s1) it is raining, but I am happy
(s2) although it is raining, I am happy
(s3) it is raining, yet I am happy
(s4) it is raining and I am happy

For example, under what conditions is (s1) true? Answer: (s1) is true precisely when ‘it is raining’ and ‘I am happy’ are both true, which is to say precisely when (s4) is true. In other words, (s1) and (s4) are true under precisely the same circumstances, which is to say that they are truth-functionally equivalent.

When we utter (s1)-(s3), we intend to emphasize a contrast that is not emphasized in the standard conjunction (s4), or we intend to convey (a certain degree of) surprise. The difference, however, pertains to appropriate usage rather than semantic content.

Although they connote differently, (s1)-(s4) have the same truth conditions, and are accordingly symbolized the same:

R & H
4. DISGUISED CONJUNCTIONS

As noted earlier, certain simple statements are straightforwardly equivalent to compound statements. For example,

(e1) Jay and Kay are Sophomores

is equivalent to

(p1) Jay is a Sophomore, and Kay is a Sophomore

which is symbolized:

(s1) J & K

Other examples of disguised conjunctions involve relative pronouns (‘who’, ‘which’, ‘that’). For example,

(e2) Jones is a former player who coaches basketball

is equivalent to

(p2) Jones is a former (basketball) player, and Jones coaches basketball,

which may be symbolized:

(s2) F & C

Further examples do not use relative pronouns, but are easily paraphrased using relative pronouns. For example,

(e3) Pele is a Brazilian soccer player

may be paraphrased as

(p3) Pele is a Brazilian who is a soccer player

which is equivalent to

(p3’) Pele is a Brazilian, and Pele is a soccer player,

which may be symbolized:

(s3) B & S

Notice, of course, that

(e4) Jones is a former basketball player

is not a conjunction, such as the following absurdity.

(??) Jones is a former, and Jones is a basketball player

Sentence (e4) is rather symbolized as a simple (atomic) formula.
5. THE RELATIONAL USE OF ‘AND’

As noted in the previous section, the statement,

(c) Jay and Kay are Sophomores,

is equivalent to the conjunction,

Jay is a Sophomore, and Kay is a Sophomore,

and is accordingly symbolized:

J & K

Other statements look very much like (c), but are not equivalent to conjunctions. Consider the following statements.

(r1) Jay and Kay are cousins
(r2) Jay and Kay are siblings
(r3) Jay and Kay are neighbors
(r4) Jay and Kay are roommates
(r5) Jay and Kay are lovers

These are definitely not symbolized as conjunctions. The following is an incorrect translation.

(? J & K WRONG!!)

For example, consider (r1), the standard reading of which is

(r1’) Jay and Kay are cousins of each other.

In proposing J&K as the analysis of (r1’), we must specify which particular atomic statement each letter stands for. The following is the only plausible choice.

J: Jay is a cousin
K: Kay is a cousin

Accordingly, the formula J&K is read

Jay is a cousin, and Kay is a cousin.

But to say that Jay is a cousin is to say that he is a cousin of someone, but not necessarily Kay. Similarly, to say that Kay is a cousin is to say that she a cousin of someone, but not necessarily Jay. In other words, J&K does not say that Jay and Kay are cousins of each other.

The resemblance between statements like (r1)-(r5) and statements like

(c1) Jay and Kay are Sophomores
(c2) Jay and Kay are Republicans
(c3) Jay and Kay are basketball players
is grammatically superficial. Each of (c1)-(c3) states something about Jay independently of Kay, and something about Kay independently of Jay.

By contrast, each of (r1)-(r5) states that a particular relationship holds between Jay and Kay. The relational quality of (r1)-(r5) may be emphasized by restating them in either of the following ways.

(r1') Jay is a cousin of Kay
(r2') Jay is a sibling of Kay
(r3') Jay is a neighbor of Kay
(r4') Jay is a roommate of Kay
(r5') Jay is a lover of Kay

(r1) Jay and Kay are cousins of each other
(r2) Jay and Kay are siblings of each other
(r3) Jay and Kay are neighbors of each other
(r4) Jay and Kay are roommates of each other
(r5) Jay and Kay are lovers of each other

On the other hand, notice that one cannot paraphrase (c1) as

(??) Jay is a Sophomore of Kay
(??) Jay and Kay are Sophomores of each other

Relational statements like (r1)-(r5) are not correctly paraphrased as conjunctions. In fact, they are not correctly paraphrased by any compound statement. From the viewpoint of sentential logic, these statements are simple; they have no internal structure, and are accordingly symbolized by atomic formulas.

[NOTE: Later, in predicate logic, we will see how to uncover the internal structure of relational statements such as (r1)-(r5), internal structure that is inaccessible to sentential logic.]

We have seen so far that ‘and’ is used both conjunctively, as in

Jay and Kay are Sophomores,

and relationally, as in

Jay and Kay are cousins (of each other).

In other cases, it is not obvious whether ‘and’ is used conjunctively or relationally. Consider the following.

(s2) Jay and Kay are married

There are two plausible interpretations of this statement. On the one hand, we can interpret it as

(i1) Jay and Kay are married to each other,

in which case it expresses a relation, and is symbolized as an atomic formula, say: M. On the other hand, we can interpret it as
(i2) Jay is married, and Kay is married, (perhaps, but not necessarily, to each other),
in which case it is symbolized by a conjunction, say: J&K. The latter simply reports the marital status of Jay, independently of Kay, and the marital status of Kay, independently of Jay.

We can also say things like the following.

(s3) Jay and Kay are married, but not to each other.

This is equivalent to

(p3) Jay is married, and Kay is married, but Jay and Kay are not married to each other,

which is symbolized:

\[(J \& K) \& \sim M\]

[Note: This latter formula does not uncover all the logical structure of the English sentence; it only uncovers its connective structure, but that is all sentential logic is concerned with.]

6. CONNECTIVE-USES OF ‘AND’ DIFFERENT FROM AMPERSAND

As seen in the previous section, ‘and’ is used both as a connective and as a separator in relation-statements.

In the present section, we consider how ‘and’ is occasionally used as a connective different in meaning from the ampersand connective (\&). There are two cases of this use.

First, sentences that have the form ‘P and Q’ sometimes mean ‘P and then Q’. For example, consider the following statements.

(s1) I went home and went to bed
(s2) I went to bed and went home

As they are colloquially understood at least, these two statements do not express the same proposition, since ‘and’ here means ‘and then’.

Note, in particular, that the above use of ‘and’ to mean ‘and then’ is not truth-functional. Merely knowing that P is true, and merely knowing that Q is true, one does not automatically know the order of the two events, and hence one does not know the truth-value of the compound ‘P and then Q’.

Sometimes ‘and’ does not have exactly the same meaning as the ampersand connective. Other times, ‘and’ has a quite different meaning from ampersand.
(e1) keep trying, and you will succeed
(e2) keep it up buster, and I will clobber you
(e3) give him an inch, and he will take a mile
(e4) give me a place to stand, and I will move the world (Archimedes, in reference to the power of levers)
(e5) give us the tools of war, and we will finish the job (Churchill, in reference to WW2)

Consider (e1) paraphrased as a conjunction, for example:

(?) K & S

In proposing (?) as an analysis of (e1), we must specify what particular statements K and S abbreviate. The only plausible answer is:

K: you will keep trying
S: you will succeed

Accordingly, the conjunction K&S reads:

you will keep trying, and you will succeed

But the original,

keep trying, and you will succeed,

does not say this at all. It does not say the addressee will keep trying, nor does it say that the addressee will succeed. Rather, it merely says (promises, predicts) that the addressee will succeed if he/she keeps trying.

Similarly, in the last example, it should be obvious that Churchill was not predicting that the addressee (i.e., Roosevelt) would in fact give him military aid and Churchill would in fact finish the job (of course, that was what Churchill was hoping!). Rather, Churchill was saying that he would finish the job if Roosevelt were to give him military aid. (As it turned out, of course, Roosevelt eventually gave substantial direct military aid.)

Thus, under very special circumstances, involving requests, promises, threats, warnings, etc., the word ‘and’ can be used to state conditionals. The appropriate paraphrases are given as follows.

(p1) if you keep trying, then you will succeed
(p2) if you keep it up buster, then I will clobber you
(p3) if you give him an inch, then he will take a mile
(p4) if you give me a place to stand, then I will move the world
(p5) if you give us the tools of war, then we will finish the job

The treatment of conditionals is discussed in a later section.
7. NEGATIONS, STANDARD AND IDIOMATIC

The standard form of the negation connective is

it is not true that ______

The following expressions are standard variants.

it is not the case that ______

it is false that ______

Given any statement, we can form its standard negation by placing ‘it is not the case that’ (or a variant) in front of it.

As noted earlier, standard negations seldom appear in colloquial-idiomatic English. Rather, the usual colloquial-idiomatic way to negate a statement is to place the modifier ‘not’ in a strategic place within the statement, usually immediately after the verb. The following is a simple example.

statement: it is raining
idiomatic negation: it is not raining
standard negation: it is not true that it is raining

Idiomatic negations are symbolized in sentential logic exactly like standard negations, according to the following simple principle.

If sentence S is symbolized by the formula $\mathcal{A}$, then the negation of S (standard or idiomatic) is symbolized by the formula $\neg \mathcal{A}$.

Note carefully that this principle applies whether S is simple or compound. As an example of a compound statement, consider the following statement.

(e1) Jay is a Freshman basketball player.

As noted in Section 2, this may be paraphrased as a conjunction:

(p1) Jay is a Freshman, and Jay is a basketball player.

Now, there is no simple idiomatic negation of the latter, although there is a standard negation, namely

(n1) it is not true that (Jay is a Freshman and Jay is a basketball player)

The parentheses indicate the scope of the negation modifier.

However, there is a simple idiomatic negation of the former, namely,

(n1’) Jay is not a Freshman basketball player.

We consider (n1) and (n1’) further in the next section.
Chapter 4: Translations in Sentential Logic

8. NEGATIONS OF CONJUNCTIONS

As noted earlier, the sentence

(s1) Jay is a Freshman basketball player,

may be paraphrased as a conjunction,

(p1) Jay is a Freshman, and Jay is a basketball player,

which is symbolized:

(f1) F & B

Also, as noted earlier, the idiomatic negation of (p1) is

(n1) Jay is not a Freshman basketball player.

Although there is no simple idiomatic negation of (p1), its standard negation is:

(n2) it is not true that (Jay is a Freshman, and Jay is a Basketball player),

which is symbolized:

 ~(F & B)

Notice carefully that, when the conjunction stands by itself, the outer parentheses may be dropped, as in (f2), but when the formula is negated, the outer parentheses must be restored before prefixing the negation sign. Otherwise, we obtain:

 ~(F & B),

which is reads:

 Jay is not a Freshman, and Jay is a Basketball player,

which is not equivalent to ~(F&B), as may be shown using truth tables.

How do we read the negation

 ~(F & B)?

Many students suggest the following erroneous paraphrase,

 Jay is not a Freshman, 
 and 
 Jay is not a basketball player, 

which is symbolized:

 ~(J & ~B).

But this is clearly not equivalent to (n1). To say that Jay isn't a Freshman basketball player is to say that one of the following states of affairs obtains.
(1) Jay is a Freshman who does not play Basketball;
(2) Jay is a Basketball player who is not a Freshman;
(3) Jay is neither a Freshman nor a Basketball player.

On the other hand, to say that Jay is not a Freshman and not a Basketball player is to say precisely that the last state of affairs (3) obtains.

We have already seen the following, in a previous chapter (voodoo logic notwithstanding!)

\[ \neg(A \land B) \text{ is NOT logically equivalent to } (\neg A \land \neg B) \]

This is easily demonstrated using truth-tables. Whereas the latter entails the former, the former does not entail the latter.

The correct logical equivalence is rather:

\[ \neg(A \land B) \text{ is logically equivalent to } (\neg A \lor \neg B) \]

The disjunction may be read as follows.

Jay is \textit{not} a Freshman \textit{and/or} Jay is \textit{not} a Basketball player.

One more example might be useful. The colloquial negation of the sentence

Jay and Kay are \textit{both} Republicans \hspace{1cm} J \& K

is

Jay and Kay are \textit{not both} Republicans \hspace{1cm} \neg(J \& K)

This is definitely not the same as

Jay and Kay are both non-Republicans,

which is symbolized:

\[ \neg J \& \neg K. \]

The latter says that \textit{neither} of them is a Republican (see later section concerning ‘neither’), whereas the former says less – that at least one of them isn’t a Republican, \textit{perhaps} neither of them is a Republican.
9. DISJUNCTIONS

The standard English expression for disjunction is ‘or’, a variant of which is ‘either...or’. As noted in a previous chapter, ‘or’ has two senses – an inclusive sense and an exclusive sense.

The legal profession has invented an expression to circumvent this ambiguity – ‘and/or’. Similarly, Latin uses two different words: one, ‘vel’, expresses the inclusive sense of ‘or’; the other, ‘aut’, expresses the exclusive sense.

The standard connective of sentential logic for disjunction is the wedge ‘\( \lor \)’, which is suggestive of the first letter of ‘vel’. In particular, the wedge connective of sentential logic corresponds to the inclusive sense of ‘or’, which is the sense of ‘and/or’ and ‘vel’.

Consider the following statements, where the inclusive sense is distinguished (parenthetically) from the exclusive sense.

(is) Jones will win or Smith will win (possibly both)

(es) Jones will win or Smith will win (but not both)

We can imagine a scenario for each. In the first scenario, Jones and Smith, and a third person, Adams, are the only people running in an election in which two people are elected. So Jones or Smith will win, maybe both. In the second scenario, Jones and Smith are the two finalists in an election in which only one person is elected. In this case, one will win, the other will lose.

These two statements may be symbolized as follows.

\[
(f1) \ J \lor S \\
(f2) \ (J \lor S) \land \neg(J \land S)
\]

We can read (f1) as saying that Jones will win and/or Smith will win, and we can read (f2) as saying that Jones will win or Smith will win but they won't both win (recall previous section on negations of conjunctions).

As with conjunctions, certain simple statements are straightforwardly equivalent to disjunctions, and are accordingly symbolized as such. The following are examples.

\[
(s1) \ \text{it is raining or sleeting} \\
(d1) \ \text{it raining, or it is sleeting} \quad \ R \lor S \\
(s2) \ \text{Jones is a fool or a liar} \\
(d2) \ \text{Jones is a fool, or Jones is a liar} \quad \ F \lor L
\]
10. ‘NEITHER...NOR’

Having considered disjunctions, we next look at negations of disjunctions. For example, consider the following statement.

(e1) Kay isn’t either a Freshman or a Sophomore

This may be paraphrased in the following, non-idiomatic, way.

(p1) it is not true that (Kay is either a Freshman or a Sophomore)

This is a negation of a disjunction, and is accordingly symbolized as follows.

(s1) \( \sim (F \lor S) \)

Now, an alternative, idiomatic, paraphrase of (e1) uses the expression ‘neither...nor’, as follows.

(p1’) Kay is neither a Freshman nor a Sophomore

Comparing (p1’) with the original statement (e1), we can discern the following principle.

‘neither...nor’

is the negation of

‘either...or’

This suggests introducing a non-standard connective, neither-nor with the following defining property.

neither \( A \) nor \( B \)

is logically equivalent to

\( \sim (A \lor B) \)

Note carefully that neither-nor in its connective guise is highly non-idiomatic. In particular, in order to obtain a grammatically general reading of it, we must read it as follows.

neither \( A \) nor \( B \)

is officially read:

neither is it true that \( A \)

nor is it true that \( B \)

This is completely analogous to the standard (grammatically general) reading of ‘not P’ as ‘it is not the case that P’.

For example, if R stands for ‘it is raining’ and S stands for ‘it is sleeting’, then ‘neither R nor S’ is read

neither is it true that it is raining

nor is it true that it is sleeting
This awkward reading of neither-nor is required in order to insure that ‘neither P nor Q’ is grammatical irrespective of the actual sentences P and Q. Of course, as with simple negation, one can usually transform the sentence into a more colloquial form. For example, the above sentence is more naturally read

neither is it raining nor is it sleeting,
or more naturally still,

it is neither raining nor sleeting.

We have suggested that neither-nor is the negation of either-or. Other uses of the word ‘neither’ suggest another, equally natural, paraphrase of neither-nor. Consider the following sentences.

neither Jay nor Kay is a Sophomore

Jay is not a Sophomore, and neither is Kay

A bit of linguistic reflection reveals that these two sentences are equivalent to one another. Further reflection reveals that the latter sentence is simply a stylistic variant of the more monotonous sentence

Jay is not a Sophomore, and Kay is not a Sophomore

The latter is a conjunction of two negations, and is accordingly symbolized:

\[ \sim J \& \sim K \]

Thus, we see that a neither-nor sentence can be symbolized as a conjunction of two negations. This is entirely consistent with the truth-functional behavior of ‘and’, ‘or’, and ‘not’, since the following pair are logically equivalent, as is easily demonstrated using truth-tables.

\[ \sim(A \lor B) \text{ is logically equivalent to } (\sim A \& \sim B) \]

We accordingly have two equally natural paraphrases of sentences involving neither-nor, given by the following principle.

neither A nor B
may be paraphrased
\[ \sim(A \lor B) \]
or equivalently
\[ \sim A \& \sim B \]
11. CONDITIONALS

The standard English expression for the conditional connective is ‘if...then’. A standard conditional (statement) is a statement of the form

if \( A \), then \( C \),

where \( A \) and \( B \) are any statements (simple or compound), and is symbolized:

\[ A \rightarrow C \]

Whereas \( A \) is called the antecedent of the conditional, \( C \) is called the consequent of the conditional. Note that, unlike conjunction and disjunction, the constituents of a conditional do not play symmetric roles.

There are a number of idiomatic variants of ‘if...then’. In particular, all of the following statement forms are equivalent (\( A \) and \( C \) being any statements whatsoever).

(c1) if \( A \), then \( C \)
(c2) if \( A \), \( C \)
(c2') \( C \) if \( A \)
(c3) provided (that) \( A \), \( C \)
(c3') \( C \) provided (that) \( A \)
(c4) in case \( A \), \( C \)
(c4') \( C \) in case \( A \)
(c5) on the condition that \( A \), \( C \)
(c5') \( C \) on the condition that \( A \)

In particular, all of the above statement forms are symbolized in the same manner:

\[ A \rightarrow C \]

As the reader will observe, the order of antecedent and consequent is not fixed: in idiomatic English usage, sometimes the antecedent goes first, sometimes the consequent goes first. The following principles, however, should enable one systematically to identify the antecedent and consequent.

| ‘if’ always introduces the antecedent |
| ‘then’ always introduces the consequent |
| ‘provided (that)’, ‘in case’, and ‘on the condition that’ are variants of ‘if’ |
12. ‘EVEN IF’

The word ‘if’ frequently appears in combination with other words, the most common being ‘even’ and ‘only’, which give rise to the expressions ‘even if’, ‘only if’.

In the present section, we deal very briefly with ‘even if’, leaving ‘only if’ to the next section.

The expression ‘even if’ is actually quite tricky. Consider the following examples.

(e1) the Allies would have won even if the U.S. had not entered the war (in reference to WW2)

(i1) the Allies would have won if the U.S. had not entered the war

These two statements suggest quite different things. Whereas (e1) suggests that the Allies did win, (i1) suggests that the Allies didn't win. A more apt use of ‘if’ would be:

(i2) the Axis powers would have won if the U.S. had not entered the war.

Notwithstanding the pragmatic matters of appropriate, sincere usage, it seems that the pure semantic content of ‘even if’ is the same as the pure semantic content of ‘if’. The difference is not one of meaning but of presupposition, on the part of the speaker. In such examples, we tend to use ‘even if’ when we presuppose that the consequent is true, and we tend to use ‘if’ when we presuppose that the consequent is false. This is summarized as follows.

\[
\text{it would have been the case that } B \\
\quad \text{if} \\
\quad \text{it had been the case that } A \\
\quad \text{pragmatically presupposes} \\
\quad \sim B
\]

\[
\text{it would have been the case that } B \\
\quad \text{even if} \\
\quad \text{it had been the case that } A \\
\quad \text{pragmatically presupposes} \\
\quad B
\]

To say that one statement \( A \) \textit{pragmatically presupposes} another statement \( B \) is to say that when one (sincerely) asserts \( A \), one takes for granted the truth of \( B \).
Given the subtleties of content versus presupposition, we will not consider ‘even if’ any further in this text.

13. ‘ONLY IF’

The word ‘if’ frequently appears in combination with other words, the most common being ‘even’ and ‘only’, which give rise to the expressions ‘even if’, ‘only if’.

The expression ‘even if’ is very complex, and somewhat beyond the scope of intro logic, so we do not consider it any further. So, let us turn to the other expression, ‘only if’, which involves its own subtleties, but subtleties that can be dealt with in intro logic.

First, we note that ‘only if’ is definitely not equivalent to ‘if’. Consider the following statements involving ‘only if’.

(o1) I will get an A in logic only if I take all the exams
(o2) I will get into law school only if I take the LSAT

Now consider the corresponding statements obtained by replacing ‘only if’ by ‘if’.

(i1) I will get an A in logic if I take all the exams
(i2) I will get into law school if I take the LSAT

Whereas the ‘only if’ statements are true, the corresponding ‘if’ statements are false. It follows that ‘only if’ is not equivalent to ‘if’.

The above considerations show that an ‘only if’ statement does not imply the corresponding ‘if’ statement. One can also produce examples of ‘if’ statements that do not imply the corresponding ‘only if’ statements. Consider the following examples.

(i3) I will pass logic if I score 100 on every exam
(i4) I am guilty of a felony if I murder someone

(o3) I will pass logic only if I score 100 on every exam
(o4) I am guilty of a felony only if I murder someone

Whereas both ‘if’ statements are true, both ‘only if’ statements are false. Thus, ‘A if B’ does not imply ‘A only if B’, and ‘A only if B’ does not imply ‘A if B’.

So how do we paraphrase ‘only if’ statements using the standard connectives? The answer is fairly straightforward, being related to the general way in which the word ‘only’ operates in English – as a special dual-negative modifier.

As an example of ‘only’ in ordinary discourse, a sign that reads ‘employees only’ means to exclude anyone who is not an employee. Also, if I say ‘Jay loves only Kay’, I mean that he does not love anyone except Kay.
In the case of the connective ‘only if’, ‘only’ modifies ‘if’ by introducing two negations; in particular, the statement

\[ \mathcal{A} \text{ only if } \mathcal{B} \]

is paraphrased

\[ \text{not } \mathcal{A} \text{ if not } \mathcal{B} \]

In other words, the ‘if’ stays put, and in particular continues to introduce the antecedent, but the ‘only’ becomes two negations, one in front of the antecedent (introduced by ‘if’), the other in front of the consequent.

With this in mind, let us go back to original examples, and paraphrase them in accordance with this principle. In each case, we use a colloquial form of negation.

\( (p1) \) I will not get an A in logic if I do not take all the exams
\( (p2) \) I will not get into law school if I do not take the LSAT

Now, \( (p1) \) and \( (p2) \) are not in standard form, the problem being the relative position of antecedent and consequent. Recalling that ‘\( \mathcal{A} \text{ if } \mathcal{B} \)’ is an idiomatic variant of ‘\( \mathcal{B}, \text{ then } \mathcal{A} \)’, we further paraphrase \( (p1) \) and \( (p2) \) as follows.

\( (p1') \) if I do not take all the exams, then I will not get an A in logic
\( (p2') \) if I do not take the LSAT, then I will not get into law school

These are symbolized, respectively, as follows.

\( (s1) \) \( \sim T \rightarrow \sim A \)
\( (s2) \) \( \sim T \rightarrow \sim L \)

Combining the paraphrases of ‘only if’ and ‘if’, we obtain the following principle.

\[ \mathcal{A} \text{ only if } \mathcal{B} \]

is paraphrased

\[ \text{not } \mathcal{A} \text{ if not } \mathcal{B} \]

which is further paraphrased

\[ \text{if not } \mathcal{B}, \text{ then not } \mathcal{A} \]

which is symbolized

\[ \sim \mathcal{B} \rightarrow \sim \mathcal{A} \]
14. A PROBLEM WITH THE TRUTH-FUNCTIONAL IF-THEN

The reader will recall that the truth-functional version of ‘if...then’ is characterized by the truth-function that makes ‘\(A \rightarrow B\)’ false precisely when \(A\) is true and \(B\) is false. As noted already, this is not a wholly satisfactory analysis of English ‘if...then’; rather, it is simply the best we can do by way of a truth-functional version of ‘if...then’. Whereas the truth-functional analysis of ‘if...then’ is well suited to the timeless, causeless, eventless realm of mathematics, it is not so well suited to the realm of ordinary objects and events.

In the present section, we examine one of the problems resulting from the truth-functional analysis of ‘if...then’, a problem specifically having to do with the expression ‘only if’.

We have paraphrased ‘\(A\) only if \(B\)’ as ‘not \(A\) if not \(B\)’, which is paraphrased ‘if not \(B\), then not \(A\)’, which is symbolized ‘\(~B \rightarrow ~A\)’. The reader may recall that, using truth tables, one can show the following.

\[
\begin{array}{c}
\sim B \rightarrow \sim A \\
is \text{equivalent to} \\
A \rightarrow B
\end{array}
\]

Now, \(~B \rightarrow ~A\) is the translation of ‘\(A\) only if \(B\)’, whereas \(A \rightarrow B\) is the translation of ‘if \(A\), then \(B\)’. Therefore, since \(~B \rightarrow ~A\) is truth-functionally equivalent to \(A \rightarrow B\), we are led to conclude that ‘\(A\) only if \(B\)’ is truth-functionally equivalent to ‘if \(A\), then \(B\)’.

This means, in particular that our original examples,

(o1) I will get an A in logic only if I take the exams
(o2) I will get into law school only if I take the LSAT

are truth-functionally equivalent to the following, respectively:

(e1) if I get an A in logic, then I will take the exams
(e2) if I get into law school, then I will take the LSAT

Compared with the original statements, these sound odd indeed. Consider the last one. My response is that, if you get into law school, why bother taking the LSAT!

The oddity we have just discovered further underscores the shortcomings of the truth-functional if-then connective. The particular difficulty is summarized as follows.
To paraphrase ‘\( \mathcal{A} \) only if \( \mathcal{B} \)’ as ‘if \( \mathcal{A} \) then \( \mathcal{B} \)’ is at the very least misleading in cases involving temporal or causal factors. Consider the following example.

(o3) my tree will grow only if it receives adequate light

is best paraphrased

(p3) my tree will not grow if it does not receive adequate light

which is quite different from

(e3) if my tree grows, then it will receive adequate light.

The latter statement may indeed be true, but it suggests that the growing leads to, and precedes, getting adequate light (as often happens with trees competing with one another for available light). By contrast, the former suggests that getting adequate light is required, and hence precedes, growing (as happens with all photosynthetic organisms).

A major problem with (e1)-(e3) is with the tense in the consequents. The word ‘then’ makes it natural to use future tense, probably because ‘then’ is used both in a logical sense and in a temporal sense (for example, recall ‘and then’).

If we insist on translating ‘only if’ statements into ‘if... then’ statements, following the method above, then we must adjust the tenses appropriately. So, for example, getting adequate light precedes growing, so the appropriate tense is not simple future but future perfect. Adjusting the tenses in this manner, we obtain the following re-paraphrases of (e1)-(e3).

(p1') if I get an A in logic, then I will have taken the exams

(p2') if I get into law school, then I will have taken the LSAT

(p3') if my tree grows, then it will have received adequate light

Unlike the corresponding statements using simple future, these statements, which use future perfect tense, are more plausible paraphrases of the original ‘only if’ statements.
Nonetheless, ‘not \( A \) if not \( B \)’ remains the generally most accurate paraphrase of ‘\( A \) only if \( B \)’.

15. ‘IF AND ONLY IF’

Having examined ‘if’, and having examined ‘only if’, we next consider their natural conjunction, which is ‘if and only if’. Consider the following sentence.

\[(e) \text{ you will pass if and only if you average at least fifty} \]

This is naturally thought of as dividing into two halves, a promise-half and a threat-half. The promise is

\[(p) \text{ you will pass if you average at least fifty,} \]

and the threat is

\[(t) \text{ you will pass only if you average at least fifty,} \]

which we saw in the previous section may be paraphrased:

\[(t') \text{ you will not pass if you do not average at least fifty.} \]

So (e) may be paraphrased as a conjunction:

\[(t'') \text{ you will pass if you average at least fifty,} \]
\[\text{and} \]
\[\text{you will not pass if you do not average at least fifty.} \]

The first conjunct is symbolized:

\[A \rightarrow P \]

and the second conjunct is symbolized:

\[\sim A \rightarrow \sim P \]

so the conjunction is symbolized:

\[(A \rightarrow P) \& (\sim A \rightarrow \sim P) \]

The reader may recall that our analysis of the biconditional connective \( \leftrightarrow \) is such that the above formula is truth-functionally equivalent to

\[P \leftrightarrow A \]

So \( P \leftrightarrow A \) also counts as an acceptable symbolization of ‘\( P \) if and only if \( A \)’, although it does not do full justice to the internal logical structure of ‘if and only if’ statements, which are more naturally thought of as conjunctions of ‘if’ statements and ‘only if’ statements.
16. ‘UNLESS’

There are numerous ways to express conditionals in English. We have already seen several conditional-forming expressions, including ‘if’, ‘provided’, ‘only if’. In the present section, we consider a further conditional-forming expression – ‘unless’.

‘Unless’ is very similar to ‘only if’, in the sense that it has a built-in negation. The difference is that, whereas ‘only if’ incorporates two negations, ‘unless’ incorporates only one. This means, in particular, that in order to paraphrase ‘only if’ statements using ‘unless’, one must add one explicit negation to the sentence. The following are examples of ‘only if’ statements, followed by their respective paraphrases using ‘unless’.

(o1) I will graduate only if I pass logic
(u1) I will not graduate unless I pass logic
(u1’) unless I pass logic, I will not graduate

(o2) I will pass logic only if I study
(u2) I will not pass logic unless I study
(u2’) unless I study, I will not pass logic

Let us concentrate on the first one. We already know how to paraphrase and symbolize (o1), as follows.

(p1) I will not graduate if I do not pass logic
(p1’) if I do not pass logic, then I will not graduate
(s1) \( \sim P \rightarrow \sim G \)

Now, comparing (u1) and (u1’) with the last three items, we discern the following principle concerning ‘unless’.

\[
\text{‘unless’} \\
\text{is equivalent to} \\
\text{‘if not’}
\]

Here, ‘if not’ is short for ‘if it is not true that’. Notice that this principle applies when ‘unless’ appears at the beginning of the statement, as well as when it appears in the middle of the statement.

The above principle may be restated as follows.

\[
\begin{array}{|c|c|}
\hline
\text{A unless B} & \text{unless A, B} \\
\text{is equivalent to} & \text{is equivalent to} \\
\text{A if not B} & \text{if not A, then B} \\
\text{which is symbolized} & \text{which is symbolized} \\
\sim B \rightarrow A & \sim A \rightarrow B \\
\hline
\end{array}
\]
17. THE STRONG SENSE OF ‘UNLESS’

As with many words in English, the word ‘unless’ is occasionally used in a way different from its "official" meaning. As with the word ‘or’, which has both a weak (inclusive) sense and a strong (exclusive) sense, the word ‘unless’ also has both a weak and strong sense.

Just as we opt for the weak (inclusive) sense of ‘or’ in logic, we also opt for the weak sense of ‘unless’, which is summarized in the following principle.

```
the weak sense of
‘unless’
is equivalent to
‘if not’
```

Unfortunately, ‘unless’ is not always intended in the weak sense. In addition to the meaning ‘if not’, various Webster Dictionaries give ‘except when’ and ‘except on the condition that’ as further meanings.

First, let us consider the meaning of ‘except’; for example, consider the following fairly ordinary ‘except’ statement, which is taken from a grocery store sign.

(e1) open 24 hours a day except Sundays

It is plausible to suppose that (e1) means that the store is open 24 hours Monday-Saturday, and is not open 24 hours on Sunday (on Sunday, it may not be open at all, or it may only be open 8 hours). Thus, there are two implicit conditionals, as follows, where we let ‘open’ abbreviate ‘open 24 hours’.

(c1) if it is not Sunday, then the store is open
(c2) if it is Sunday, then the store is not open

These two can be combined into the following biconditional.

(b) the store is open if and only if it is not Sunday

which is symbolized:

(s) \( O \leftrightarrow \neg S \)

Now, similar statements can be made using ‘unless’. Consider the following statement from a sign on a swimming pool.

(u1) the pool may not be used unless a lifeguard is on duty

Following the dictionary definition, this is equivalent to:

(u1’) the pool may not be used except when a lifeguard is on duty
which amounts to the conjunction,

(c) the pool may not be used if a lifeguard is not on duty, and the pool may be used if a lifeguard is on duty.

which, as noted earlier, is equivalent to the following biconditional,

(b) the pool may be used if and only if a lifeguard is on duty

By comparing (b) with the original statement (u1), we can discern the following principle about the strong sense of ‘unless’.

<table>
<thead>
<tr>
<th>the strong sense of</th>
<th>‘unless’</th>
<th>is equivalent to</th>
<th>‘if and only if not’</th>
</tr>
</thead>
</table>

Or stating it using our symbols, we may state the principle as follows.

\[ \mathcal{A} \text{ unless } \mathcal{B} \]

(in the strong sense of unless)

is equivalent to

\[ \mathcal{A} \leftrightarrow \neg \mathcal{B} \]

It is not always clear whether ‘unless’ is intended in the strong or in the weak sense. Most often, the overall context is important for determining this. The following rules of thumb may be of some use.

- Usually, if it is intended in the strong sense, ‘unless’ is placed in the middle of a sentence; (the converse, however, is not true).
- Usually, if ‘unless’ is at the beginning of a statement, then it is intended in the weak sense.
- If it is not obvious that ‘unless’ is intended in the strong sense, you should assume that it is intended in the weak sense.

**Note carefully:** Although ‘unless’ is occasionally used in the strong sense, you may assume that every exercise uses ‘unless’ in the weak sense.

**Exercise** (an interesting coincidence): show that, whereas the weak sense of ‘unless’ is truth-functionally equivalent to the weak (inclusive) sense of ‘or’, the strong sense of ‘unless’ is truth-functionally equivalent to the strong (exclusive) sense of ‘or’.