18. NECESSARY CONDITIONS

There are still other words used in English to express conditionals, most importantly the words ‘necessary’ and ‘sufficient’. In the present section, we examine conditional statements that involve ‘necessary’, and in the next section, we do the same thing with ‘sufficient’.

The following expressions are some of the common ways in which ‘necessary’ is used.

\[(n1) \text{ in order that...it is necessary that...} \]
\[(n2) \text{ in order for...it is necessary for...} \]
\[(n3) \text{ in order to...it is necessary to...} \]
\[(n4) \text{...is a necessary condition for...} \]
\[(n5) \text{...is necessary for...} \]

The following are examples of mutually equivalent statements using ‘necessary’.

\[(N1) \text{ in order that I get an A, it is necessary that I take all the exams} \]
\[(N2) \text{ in order for me to get an A, it is necessary for me to take all the exams} \]
\[(N3) \text{ in order to get an A, it is necessary to take all the exams} \]
\[(N4) \text{ taking all the exams is a necessary condition for getting an A} \]
\[(N5) \text{ taking all the exams is necessary for getting an A} \]

Statements involving ‘necessary’ can all be paraphrased using ‘only if’. A more direct approach, however, is first to paraphrase the sentence into the simplest form, which is:

\[(f) \text{ } \mathcal{A} \text{ is necessary for } \mathcal{B} \]

Now, to say that one state of affairs (event) \(\mathcal{A}\) is necessary for another state of affairs (event) \(\mathcal{B}\) is just to say that if the first thing does not obtain (happen), \(\text{then neither}\) does the second. Thus, for example, to say

\[\text{taking all the exams is necessary for getting an A} \]

is just to say that if \(E\) (i.e., taking-the-exams) doesn’t obtain then neither does \(A\) (i.e., getting-an-A). The sentence is accordingly paraphrased and symbolized as follows.

\[\text{if not } E, \text{ then not } A \quad [\neg E \rightarrow \neg A] \]

The general paraphrase principle is as follows.

\[\mathcal{A} \text{ is necessary for } \mathcal{B} \]

\[\text{is paraphrased} \]

\[\text{if not } \mathcal{A}, \text{ then not } \mathcal{B} \]
19. SUFFICIENT CONDITIONS

The natural logical counterpart of ‘necessary’ is ‘sufficient’, which is used in the following ways, completely analogous to ‘necessary’.

(s1) in order that...it is sufficient that...
(s2) in order for....it is sufficient for...
(s3) in order to....it is sufficient to....
(s4) ...is a sufficient condition for...
(s5) ...is sufficient for...

The following are examples of mutually equivalent statements using these different forms.

(S1) in order that I get an A it is sufficient that I get a 100 on every exam

(S2) in order for me to get an A it is sufficient for me to get a 100 on every exam

(S3) in order to get an A it is sufficient to get a 100 on every exam

(S4) getting a 100 on every exam is a sufficient condition for getting an A

(S5) getting a 100 on every exam is sufficient for getting an A

Just as necessity statements can be paraphrased like ‘only if’ statements, sufficiency statements can be paraphrased like ‘if’ statements. The direct approach is first to paraphrase the sufficiency statement in the following form.

(f) A is sufficient for B

Now, to say that one state of affairs (event) A is sufficient for another state of affairs (event) B is just to say that B obtains (happens) provided (if) A obtains (happens). So for example, to say that

getting a 100 on every exam is sufficient for getting an A

is to say that

getting-an-A happens provided (if) getting-a-100 happens

which may be symbolized quite simply as:

H → A

The general principle is as follows.

<table>
<thead>
<tr>
<th>A is sufficient for B</th>
</tr>
</thead>
<tbody>
<tr>
<td>is paraphrased</td>
</tr>
<tr>
<td>if A, then B</td>
</tr>
</tbody>
</table>
20. NEGATIONS OF NECESSITY AND SUFFICIENCY

First, note carefully that necessary conditions are quite different from sufficient conditions. For example,

taking all the exams is necessary for getting an A,
but
taking all the exams is not sufficient for getting an A.

Similarly,

getting a 100 is sufficient for getting an A,
but
getting a 100 is not necessary for getting an A.

This suggests that we can combine necessity and sufficiency in a number of ways to obtain various statements about the relation between two events (states of affairs). For example, we can say all the following, with respect to $A$ and $B$.

(c1) $A$ is necessary for $B$
(c2) $A$ is sufficient for $B$
(c3) $A$ is not necessary for $B$
(c4) $A$ is not sufficient for $B$

(c5) $A$ is both necessary and sufficient for $B$
(c6) $A$ is necessary but not sufficient for $B$
(c7) $A$ is sufficient but not necessary for $B$
(c8) $A$ is neither necessary nor sufficient for $B$

We have already discussed how to paraphrase (c1)-(c2). In the present section, we consider how to paraphrase (c3)-(c4), leaving (c5)-(c8) to a later section.

We start with the following example involving ‘not necessary’.

(1) attendance is not necessary for passing logic

This may be regarded as the negation of

(2) attendance is necessary for passing logic

As seen earlier, the latter may be paraphrased and symbolized as follows.

(p2) if I do not attend class, then I will not pass logic

(s2) $\sim A \rightarrow \sim P$

So the negation of (2), which is (1), may be paraphrased and symbolized as follows.

(p1) it is not true that if I do not attend class, then I will not pass logic;

(s1) $\sim (\sim A \rightarrow \sim P)$
Notice, once again, that voodoo does not prevail in logic; there is no obvious simplification of the three negations in the formula. The negations do not simply cancel each other out. In particular, the latter is not equivalent to the following.

\[(\text{voodoo}) \ A \rightarrow P\]

The latter says (roughly) that attendance will ensure passing; this is, of course, not true. Your dog can attend every class, if you like, but it won't pass the course. The former says that attendance is not necessary for passing; this is true, in the sense that attendance is not an official requirement.

Next, consider the following example involving ‘not sufficient’.

\[(3) \ \text{taking all the exams is not sufficient for passing logic}\]

This may be regarded as the negation of

\[(4) \ \text{taking all the exams is sufficient for passing logic.}\]

The latter is paraphrased and symbolized as follows.

\[(p4) \ \text{if I take all the exams, then I will pass logic}\]
\[(s4) \ E \rightarrow P\]

So the negation of (4), which is (3), may be paraphrased and symbolized as follows.

\[(p3) \ \text{it is not true that if I take all the exams, then I will pass logic}\]
\[(s4) \ \sim(E \rightarrow P)\]

As usual, there is no simple-minded (voodoo) transformation of the negation. The negation of an English conditional does not have a straightforward simplification. In particular, it is not equivalent to the following

\[(\text{voodoo}) \ \sim E \rightarrow \sim P\]

The former says (roughly) that taking all the exams does not ensure passing; this is true; after all, you can fail all the exams. On the other hand, the latter says that if you don't take all the exams, then you won't pass. This is not true, a mere 70 on each of the first three exams will guarantee a pass, in which case you don't have to take all the exams in order to pass.
21. YET ANOTHER PROBLEM WITH THE TRUTH-FUNCTIONAL IF-THEN

According to our analysis, to say that one state of affairs (event) \( A \) is not sufficient for another state of affairs (event) \( B \) is to say that it is not true that if the first obtains (happens), then so will the second. In other words,

\[
A \text { is not sufficient for } B
\]

is paraphrased:

it is not true that if \( A \) then \( B \),

which is symbolized:

\(~(A \rightarrow B)\)

As noted in the previous section, there is no obvious simple transformation of the latter formula. On the other hand, the latter formula can be simplified in accordance with the following truth-functional equivalence, which can be verified using truth tables.

\[
~(A \rightarrow B)
\]

is truth-functionally equivalent to

\( A \& ~B \)

Consider our earlier example,

(1) taking all the exams is not sufficient for passing logic

Our proposed paraphrase and symbolization is:

(p1) \( \text{it is not true that if I take all the exams then I will pass logic} \)

(s1) \( ~ (E \rightarrow P) \)

But this is truth-functionally equivalent to:

(s2) \( E \& ~P \)

(p2) I will take all the exams, and I will not pass

However, to say that taking the exams is not sufficient for passing logic is not to say you will take all the exams yet you won’t pass; rather, it says that it is possible (in some sense) for you to take the exams and yet not pass.

However, possibility is not a truth-functional concept; some falsehoods are possible; some falsehoods are impossible. Thus, possibility cannot be analyzed in truth-functional logic.

We have dealt with negations of conditionals, which lead to difficulties with the truth-functional analysis of necessity and sufficiency. Nevertheless, our paraphrase technique involving ‘if...then’ is not impugned, only the truth-functional analysis of ‘if...then’.
Recall that the possible combinations of statements about necessity and sufficiency are as follows.

(c1) \( A \) is necessary for \( B \)
(c2) \( A \) is sufficient for \( B \)
(c3) \( A \) is not necessary for \( B \)
(c4) \( A \) is not sufficient for \( B \)
(c5) \( A \) is both necessary and sufficient for \( B \)
(c6) \( A \) is necessary, but not sufficient, for \( B \)
(c7) \( A \) is sufficient, but not necessary, for \( B \)
(c8) \( A \) is neither necessary nor sufficient for \( B \)

We have already dealt with (c1)-(c4). We now turn to (c5)-(c8).

First, notice carefully that (c1)-(c4) are less informative than (c5)-(c8). For example, if I say \( A \) is necessary for \( B \), and leave it at that, I am not saying whether \( A \) is sufficient for \( B \), one way or the other. Similarly, if I say that Jay is a Sophomore, and leave it at that, I have said nothing concerning whether Kay is a Sophomore, one way or the other.

Consider the following example of combination (c5).

(e5) averaging at least 50 is both necessary and sufficient for passing

This is quite clearly the conjunction of a necessity statement and a sufficiency statement, as follows.

average at least fifty is necessary for passing,
and
average at least fifty is sufficient for passing

The latter is symbolized:

\[ (\sim F \rightarrow \sim P) & (F \rightarrow P) \]

Reading this back into English, we obtain

if I do not average at least fifty, then I will not pass,
and
if I do average at least fifty, then I will pass

Next, consider the following example of combination (c6).

(e6) taking all the exams is necessary, but not sufficient, for getting an A

This is a somewhat more complex conjunction:

taking all the exams is necessary for getting an A,

but

taking all the exams is not sufficient for getting an A
which is symbolized:

\((\neg T \rightarrow \neg A) \& \neg(T \rightarrow A)\)

Reading this back into English, we obtain

*if I do not take all the exams, then I will not get an A, but it is not true that if I do take all the exams then I will get an A*

Next, consider the following example of combination (c7).

(e7) getting 100 on every exam *is sufficient, but not necessary, for getting an A*

This too is a conjunction:

getting 100 on every exam *is sufficient for getting an A,*

but

giving 100 on every exam *is not necessary for getting an A*

which is symbolized:

\((H \rightarrow A) \& \neg(\neg H \rightarrow \neg A)\)

Reading this back into English, we obtain

*if I get a 100 on every exam, then I will get an A,
but it is not true that
if I do not get a 100 on every exam then I will not get an A*

Finally, consider the following example of combination (c8).

(e8) attending class is neither necessary nor sufficient for passing

which may be paraphrased as a complex conjunction:

attending class *is not necessary for passing,*

and

attending class *is not sufficient for passing*

which is symbolized:

\(\neg(\neg A \rightarrow \neg P) \& \neg(A \rightarrow P)\)

Reading this back into English, we obtain

*it is not true that
if I do not attend class
then I will not pass,*

nor is it true that
*if I do attend class
then I will pass*
23. ‘OTHERWISE’

In the present section, we consider two three-place connective expressions that are used to express conditionals in English. The key words are ‘otherwise’ and ‘in which case’.

First, the general forms for ‘otherwise’ statements are the following:

(o1) if $A$, then $B$; otherwise $C$
(o2) if $A$, $B$; otherwise $C$
(o3) $B$ if $A$; otherwise $C$

The following is a typical example.

(e1) if it is sunny, I'll play tennis
    otherwise, I'll play racquetball

This statement asserts what the speaker will do if it is sunny, and it further asserts what the speaker will do otherwise, i.e., if it is not sunny. In other words, (e1) can be paraphrased as a conjunction, as follows.

(p1) if it is sunny, then I'll play tennis,
    and
    if it is not sunny, then I'll play racquetball

The latter statement is symbolized:

(s1) $(S \rightarrow T) \& (\sim S \rightarrow R)$

The general principle governing the paraphrase of ‘otherwise’ statements is as follows.

<table>
<thead>
<tr>
<th>if $A$, then $B$; otherwise $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>is paraphrased</td>
</tr>
<tr>
<td>if $A$, then $B$, and if not $A$, then $C$,</td>
</tr>
<tr>
<td>which is symbolized</td>
</tr>
<tr>
<td>$(A \rightarrow B) &amp; (\sim A \rightarrow C)$</td>
</tr>
</tbody>
</table>

A simple variant of ‘otherwise’ is ‘else’, which is largely interchangeable with ‘otherwise’. In a number of high level programming languages, including BASIC and PASCAL, ‘else’ is used in conjunction with ‘if...then’ to issue commands. For example, the following is a typical BASIC command.

(c) if $X<=100$ then goto 300 else goto 400

This is equivalent to two commands in succession:

if $X<=100$ then goto 300
if not($X<=100$) then goto 400
In a computer language, such as BASIC, there is always a "default" ‘else’ command, namely to go to the next line and follow that command. So, for example, the command line

if \( X \leq 100 \) then goto 400

standing alone means

if \( X \leq 100 \) then goto 400 else goto next line

Unlike ‘if...then’ statements in computer languages, English ‘if...then’ statements do not incorporate default ‘else’ clauses. For example, the statement

(e2) I'll go to the doctor if I break my arm

says nothing about what the speaker will or won't do if he/she does not break an arm. Similarly, if I say I won't play tennis if it is raining, and leave it at that, I am not committing myself to anything in case it is not raining; I leave that case open, or undetermined.

That brings us to an expression that is very similar to ‘otherwise’ – namely, ‘in which case’. Consider the following example.

(e2) I'll play tennis unless it is raining, in which case I'll play squash

Recall that ‘unless’ is equivalent to ‘if not’. So, as with ‘otherwise’ statements, there are two cases considered – it rains; it doesn't rain. Statement (e2) asserts what the speaker will do in each case – in case it is not raining, and in case it is raining. Recall ‘in case’ is a variant of ‘if’.

The paraphrase of (e2) is similar to that of (e1).

(p) if it is not raining, then I'll play tennis, and if it is raining, then I'll play squash

The latter is symbolized:

(s) \( \sim R \rightarrow T \) & \( R \rightarrow S \)

The overall paraphrase pattern is given by the following principle.

\[
\begin{align*}
\mathcal{A} \text{ unless } B, \text{ in which case } C \\
\text{is paraphrased} \\
\text{if not } B, \text{ then } \mathcal{A}, \text{ and if } B \text{ then } C \\
\text{which is symbolized} \\
(\sim B \rightarrow \mathcal{A}) & (B \rightarrow C)
\end{align*}
\]