24. PARAPHRASING COMPLEX STATEMENTS

As noted earlier, compound statements may be built up from statements which are themselves compound statements. There are no theoretical limits to the complexity of compound statements, although there are practical limits, based on human linguistic capabilities.

We have already dealt with a number of complex statements in connection with the various non-standard connectives. We now systematically consider complex statements that involve various combinations of non-standard connectives. For example, we are interested in what happens when both ‘unless’ and ‘only if’ appear in the same sentence.

In paraphrasing and symbolizing complex statements, it is best to proceed systematically, in small steps. As one gets better, many intermediate steps can be done in one's head. On the easy ones, perhaps all the intermediate steps can be done in one's head. Still, it is a good idea to reason through the easy ones systematically, in order to provide practice in advance of doing the hard ones.

The first step in paraphrasing statements is:

**Step 1:** Identify the simple (atomic) statements, and abbreviate them by upper case letters.

In most of the exercises, certain words are entirely capitalized in order to suggest to the student what the atomic statements are. For example, in the statement ‘JAY and KAY are Sophomores’ the atomic formulas are ‘J’ and ‘K’.

At this stage of analysis, it is important to be clear concerning what each atomic formula stands for; it is especially important to be clear that each letter abbreviates a complete sentence. For example, in the above statement, ‘J’ does not stand for ‘Jay’, since this is not a sentence. Rather, it stands for ‘Jay is a Sophomore’. Similarly, ‘K’ does not stand for ‘Kay’, but rather ‘Kay is a Sophomore’.

Having identified the simple statements, and having established their abbreviations, the next step is:

**Step 2:** Identify all the connectives, noting which ones are standard, and which ones are not standard.

Having identified the atomic statements and the connectives, the next step is:

**Step 3:** Write down the first hybrid formula, making sure to retain internal punctuation.

The first hybrid formula is obtained from the original statement by replacing the simple statements by their abbreviations. A hybrid formula is so called because it
contains both English words and symbols from sentential logic. Punctuation provides important clues about the logical structure of the sentence.

The first three steps may be better understood by illustration. Consider the following example.

**Example 1**

(e1) if neither Jay nor Kay is working, then we will go on vacation.

In this example, the simple statements are:

- J: Jay is working
- K: Kay is working
- V: we go on vacation

and the connectives are:

- if...then (standard)
- neither...nor (non-standard)

Thus, our first hybrid formula is:

(h1) if neither J nor K, then V

Having obtained the first hybrid formula, the next step is to

**Step 4:** Identify the major connective.

Here, the commas are important clues. In (h1), the placement of the comma indicates that the major connective is ‘if...then’, the structure being:

\[
\text{if } \text{neither J nor K,} \quad \text{then } V
\]

Having identified the major connective, we go on to the next step.

**Step 5:** Symbolize the major connective if it is standard; otherwise, paraphrase it into standard form, and go back to step 4, and work on the resulting (hybrid) formula.

In (h1), the major connective is ‘if...then’, which is standard, so we symbolize it, which yields the following hybrid formula.

(h2) \((\text{neither J nor K}) \rightarrow V\)

Notice that, as we symbolize the connectives, we must provide the necessary logical punctuation (i.e., parentheses).

At this point, the next step is:
Step 6: Work on the constituent formulas separately.

In (h2), the constituent formulas are:

(c1) neither J nor K
(c2) V

The latter formula is fully symbolic, so we are through with it. The former is not fully symbolic, so we must work on it further. It has only one connective, ‘neither...nor’, which is therefore the major connective. It is not standard, so we must paraphrase it, which is done as follows.

(c1) neither J nor K
(p1) not J and not K

The latter formula is in standard form, so we symbolize it as follows.

(s1) ~J & ~K

Having dealt with the constituent formulas, the next step is:

Step 7: Substitute symbolizations of constituents back into (original) hybrid formula.

In our first example, this yields:

(s2) (~J & ~K) → V

Once you have a purely symbolic formula, the final step is:

Step 8: Translate the formula back into English and compare with the original statement.

This is to make sure the final formula says the same thing as the original statement. In our example, translating yields the following.

(t1) if Jay is not working and Kay is not working, then we will go on vacation.

Comparing this with the original,

(e1) if neither Jay nor Kay is working, then we will go on vacation

we see they are equivalent, so we are through.

Our first example is simple insofar as the major connective is standard. In many statements, all the connectives are non-standard, and so they have to be paraphrased in accordance with the principles discussed in previous sections. Consider the following example.
Example 2

(e2) you will pass unless you goof off, provided that you are intelligent.

In this statement, the simple statements are:

I: you are intelligent
P: you pass
G: you goof off

and the connectives are:

unless (non-standard)
provided that (non-standard)

Thus, the first stage of the symbolization yields the following hybrid formula.

(h1) P unless G, provided that I

Next, we identity the major connective. Once again, the placement of the comma tells us that ‘provided that’ is the major connective, the overall structure being:

P unless G,
provided that I

We cannot directly symbolize ‘provided that’, since it is non-standard. We must first paraphrase it. At this point, we recall that ‘provided that’ is equivalent to ‘if’, which is a simple variant of ‘if...then’. This yields the following successive paraphrases.

(h2) P unless G, if I
(h3) if I, then P unless G

In (h3), the major connective is ‘if...then’, which is standard, so we symbolize it, which yields:

(h4) I \rightarrow (P unless G)

We next work on the parts. The antecedent is finished, so we more to the consequent.

(c) P unless G

This has one connective, ‘unless’, which is non-standard, so we paraphrase and symbolize it as follows.

(c) P unless G
(p) P if not G,
(p') if not G, then P,
(s) \sim G \rightarrow P

Substituting the parts back into the whole, we obtain the final formula.

(f) I \rightarrow (\sim G \rightarrow P)
Finally, we translate (f) back into English, which yields:

\[(t) \text{ if you are intelligent, then if you do not goof off then you will pass}\]

Although this is not the exact same sentence as the original, it should be clear that they are equivalent in meaning.

Let us consider an example similar to Example 2.

**Example 3**

\[(e3) \text{ unless the exam is very easy, I will make a hundred only if I study}\]

In this example, the simple statements are:

\[
\begin{align*}
E &: \text{ the exam is very easy} \\
H &: \text{ I make a hundred} \\
S &: \text{ I study}
\end{align*}
\]

and the connectives are:

\[
\begin{align*}
\text{unless} & \quad \text{(non-standard)} \\
\text{only if} & \quad \text{(non-standard)}
\end{align*}
\]

Having identified the logical parts, we write down the first hybrid formula.

\[(h1) \text{ unless } E, \text{ H only if } S\]

Next, we observe that ‘unless’ is the principal connective. Since it is non-standard, we cannot symbolize it directly, so we paraphrase it, as follows.

\[(h2) \text{ if not } E, \text{ then H only if } S\]

We now work on the new hybrid formula (h2). We first observe that the major connective is ‘if...then’; since it is standard, we symbolize it, which yields:

\[(h3) \text{ not } E \rightarrow (H \text{ only if } S)\]

Next, we work on the separate parts. The antecedent is simple, and is standard form, being symbolized:

\[(a) \quad \sim E\]

The consequent has just one connective ‘only if’, which is non-standard, so we paraphrase and symbolize it as follows.

\[
\begin{align*}
(c) &: \text{ H only if } S \\
(p) &: \text{ not H if not } S \\
(p') &: \text{ if not } S, \text{ then not } H \\
(s) &: \sim S \rightarrow \sim H
\end{align*}
\]

Next, we substitute the parts back into (h3), which yields:

\[(f) \quad \sim E \rightarrow (\sim S \rightarrow \sim H)\]

Finally, we translate (f) back into English, which yields:
(t) if the exam is not very easy, then if I do not study then I will not get a hundred

Comparing this statement with the original statement, we see that they say the same thing.

The next example is slightly more complicated, being a conditional in which both constituents are conditionals.

**Example 4**

(e4) if Jones will work only if Smith is fired, then we should fire Smith if we want the job finished

In (e4), the simple statements are:

- J: Jones works
- F: we do fire Smith
- S: we should fire Smith
- W: we want the job finished

and the connectives are:

- if...then (standard)
- only if (non-standard)
- if (non-standard)

Next, we write down the first hybrid formula, which is:

(h1) if J only if F, then S if W

The comma placement indicates that the principal connective is ‘if...then’. It is standard, so we symbolize it, which yields:

(h2) (J only if F) → (S if W)

Next, we work on the constituents separately. The antecedent is paraphrased and symbolized as follows.

(a) J only if F
(p) not J if not F
(p') if not F, then not J
(s) ~F → ~J

The consequent is paraphrased and symbolized as follows.

(c) S if W
(p) if W, then S
(s) W → S

Substituting the constituent formulas back into (h2) yields:

(f) (~F → ~J) → (W → S)
The direct translation of (f) into English reads as follows.

\[(t) \quad \text{if we do not fire Smith then Jones does not work, then if we want the job finished then we should fire Smith}\]

The complexity of the conditional structure of this sentence renders a direct translation difficult to understand. The major problem is the "stuttering" at the beginning of the sentence. The best way to avoid this problem is to opt for a more idiomatic translation (just as we do with negations); specifically, we replace some if-then's by simple variant forms. The following is an example of a more natural, idiomatic translation.

\[(t') \quad \text{if Jones will not work if Smith is not fired, then if we want the job finished we should fire Smith}\]

Comparing this paraphrase, in more idiomatic English, with the original statement, we see that they are equivalent in meaning.

Our last example involves the notion of necessary condition.

**Example 5**

\[(e5) \quad \text{in order to put on the show it will be necessary to find a substitute, if neither the leading lady nor her understudy recovers from the flu}\]

In (e5), the simple statements are:

P: we put on the show  
S: we find a substitute  
L: the leading lady recovers from the flu  
U: the understudy recovers from the flu

and the connectives are:

in order to... it is necessary to  
if  
neither...nor

The first hybrid formula is:

\[(h1) \quad \text{in order that P it is necessary that S, if neither L nor U}\]

Next, the principal connective is ‘if’, which is not in standard form; converting it into standard form yields:

\[(h2) \quad \text{if neither L nor U, then in order that P it is necessary that S}\]

Here, the principal connective is ‘if...then’, which is standard, so we symbolize it as follows.

\[(h3) \quad \text{(neither L nor U) → (in order that P it is necessary that S)}\]

We next attack the constituents. The antecedent is paraphrased as follows.
(a) neither L nor U
(p) not L and not U
(s) \( \sim L \& \sim U \)

The consequent is paraphrased as follows.

(c) in order that P it is necessary that S
(p) S is necessary for P
(p') if not S, then not P
(s) \( \sim S \rightarrow \sim P \)

Substituting the parts back into (h3), we obtain:

(f) \( (\sim L \& \sim U) \rightarrow (\sim S \rightarrow \sim P) \)

Translating (f) back into English, we obtain:

(t) if the leading lady does not recover from the flu and her understudy does not recover from the flu, then if we do not find a substitute then we do not put on the show

Comparing (t) with the original statement, we see that they are equivalent in meaning.

By way of concluding this chapter, let us review the basic steps involved in symbolizing complex statements.
### 25. GUIDELINES FOR TRANSLATING COMPLEX STATEMENTS

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