# Translations in Polyadic Predicate Logic

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1. INTRODUCTION

Recall that predicate logic can be conveniently divided into monadic predicate logic, on the one hand, and polyadic predicate logic, on the other. Whereas the former deals exclusively with 1-place (monadic) predicates, the latter deals with all predicates (1-place, 2-place, etc.). In the present chapter, we turn to quantification in the context of polyadic predicate logic.

The reason for being interested in polyadic logic is simple: although monadic predicate logic reveals much more logical structure in English sentences than does sentential logic, monadic logic often does not reveal enough logical structure.

Consider the following argument.

\( (A) \) Every Freshman is a student

/Anyone who respects every student respects every Freshman

If we symbolize this in monadic logic, we obtain the following.

\[ \forall x (Fx \to Sx) \] [every F is S]

/ \[ \forall x (Kx \to Lx) \] [every K is L]

The following is the translation scheme:

\( Fx: \) x is a Freshman

\( Sx: \) x is a student

\( Kx: \) x respects every student

\( Lx: \) x respects every Freshman

The trouble with this analysis, which is the best we can do in monadic predicate logic, is that the resulting argument form is invalid. Yet, the original concrete argument is valid. This means that our analysis of the logical form of \((A)\) is inadequate.

In order to provide an adequate analysis, we need to provide a deeper analysis of the formulas,

\( Kx: \) x respects every student

\( Lx: \) x respects every Freshman

These formulas are logically analyzed into the following items:

\( \text{student: } \) Sy: \ y is a student

\( \text{Freshman: } \) Fy: \ y is a Freshman

\( \text{respects } \) Rxy: \ x respects y

\( \text{every: } \) \( \forall y: \) for any person y

Thus, the formulas are symbolized as follows
Kx: x respects every student
\[ \forall y (Sy \rightarrow Rxy) \]

for any person y:
if y is a student, then x respects y

Lx: x respects every Freshman;
\[ \forall y (Fy \rightarrow Rxy) \]

for any person y:
if y is a Freshman, then x respects y

Thus, the argument form, according to our new analysis is:
\[ \forall x (Fx \rightarrow Sx) / \forall x (\forall y (Sy \rightarrow Rxy) \rightarrow \forall y (Fy \rightarrow Rxy)) \]

This argument form is valid, as we will be able to demonstrate in a later chapter. It is a fairly complex example, so it may not be entirely clear at the moment. Don't worry just yet! The important point right now is to realize that many sentences and arguments have further logical structure whose proper elucidation requires polyadic predicate logic. The example above is fairly complex. In the next section, we start with more basic examples of polyadic quantification.

2. SIMPLE POLYADIC QUANTIFICATION

In the present section, we examine the simplest class of examples of polyadic quantification – those involving an atomic formula constructed from a two-place predicate. First, recall that a two-place predicate is an expression that forms a formula (open or closed) when combined with two singular terms.

For example, consider the two-place predicate ‘...respects...’, abbreviated R. With this predicate, we can form various formulas, including the following.

(1) Jay respects Kay Rjk
(2) he respects Kay Rxk
(3) Jay respects her Rjy
(4) he respects her Rxy
(5) she respects herself Rxx

The particular pronouns used above are completely arbitrary (any third person singular pronoun will do).

Now, the grammar of predicate logic has the following feature: if we have a formula, we can prefix it with a quantifier, and the resulting expression is also a formula. This merely restates the idea that quantifiers are one-place connectives.
Occasionally, however, quantifying a formula is trivial or pointless; for example,

$$\forall x R_{jk}$$ 

everyone is such that Jay respects Kay

says exactly the same thing as

$$R_{jk}$$  
Jay respects Kay

This is an example of trivial (or vacuous) quantification.

In other cases, quantification is significant. For example, beginning with formulas (2)-(5), we can construct the following formulas, which are accompanied by English paraphrases.

(2a) $$\forall x R_{xk}$$  
everyone respects Kay

(2b) $$\exists x R_{xk}$$  
someone respects Kay

(3a) $$\forall y R_{yj}$$  
Jay respects everyone

(3b) $$\exists y R_{yj}$$  
Jay respects someone

(4a) $$\forall x R_{xy}$$  
everyone respects her

(4b) $$\exists x R_{xy}$$  
someone respects her

(4c) $$\forall y R_{xy}$$  
he respects everyone

(4d) $$\exists y R_{xy}$$  
he respects someone

(5a) $$\forall x R_{xx}$$  
everyone respects him(her)self

(5b) $$\exists x R_{xx}$$  
someone respects him(her)self

Now, (4a)-(4d) have variables that can be further quantified in a significant way. So prefixing (4a)-(5b) yields the following formulas.

(4a1) $$\forall y \forall x R_{xy}$$

(4a2) $$\exists y \forall x R_{xy}$$

(4b1) $$\forall y \exists x R_{xy}$$

(4b2) $$\exists y \exists x R_{xy}$$

(4c1) $$\forall x \forall y R_{xy}$$

(4c2) $$\exists x \forall y R_{xy}$$

(4d1) $$\forall x \exists y R_{xy}$$

(4d2) $$\exists x \exists y R_{xy}$$

How do we translate such formulas into English. As it turns out, there is a handy step-by-step procedure for translating formulas (4a1)-(4d2) into colloquial English – supposing that we are discussing people exclusively, and supposing that the predicate is ‘...respects...’ This procedure is given as follows.

**Step 1:** Look at the first quantifier, and read it as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>universal ($\forall$)</td>
<td>everyone</td>
</tr>
<tr>
<td>(b)</td>
<td>existential ($\exists$)</td>
<td>there is someone who</td>
</tr>
</tbody>
</table>

**Step 2:** Look to see which variable is quantified (is it ‘x’ or ‘y’?), then check where that variable appears in the quantified formula; does it appear in
the first (active) position, or does it appear in the second (passive) position? If it appears in the first (active) position, then read the verb in the active voice as ‘respects’. If it appears in the second (passive) position, then read the verb in the passive voice as ‘is respected by’ (passive voice).

(a) active respects
(b) passive is respected by

Step 3: Look at the second quantifier, and read it as follows:

(a) universal (∀) everyone
(b) existential (∃) someone or other

Step 4: String together the components obtained in steps (1)-(3) to produce the colloquial English sentence.

With this procedure in mind, let us do a few examples.

Example 1: \( \forall x \exists y Rxy \)

(1) the first quantifier is universal, so we read it as: everyone
(2) the variable x appears in the active position, so we read the verb in the active voice: respects
(3) the second quantifier is existential, so we read it as: someone (or other)
(4) altogether: everyone respects someone (or other)

Example 2: \( \exists x \forall y Ryx \)

(1) the first quantifier is existential, so we read it as: there is someone who
(2) the variable x appears in the passive position, so we read the verb in the passive voice: is respected by
(3) the second quantifier is universal, so we read it as: everyone
(4) altogether: there is someone who is respected by everyone

By following the above procedure, we can translate all the above formulas in colloquial English as follows.
(4a1) \( \forall y \forall x Rxy: \) everyone is respected by everyone

(4a2) \( \exists y \forall x Rxy: \) there is someone who is respected by everyone

(4b1) \( \forall y \exists x Rxy: \) everyone is respected by someone or other

(4b2) \( \exists y \exists x Rxy: \) there is someone who is respected by someone or other

(4c1) \( \forall x \forall y Rxy: \) everyone respects everyone

(4c2) \( \exists x \forall y Rxy: \) there is someone who respects everyone

(4d1) \( \forall x \exists y Rxy: \) everyone respects someone or other

(4d2) \( \exists x \exists y Rxy: \) there is someone who respects someone or other

Before continuing, it is important to understand the significance of the expression ‘or other’. In Example 1, the final translation is

\( \forall x \exists y Rxy: \) everyone respects someone or other

Dropping ‘or other’ yields

\( \forall x \exists y Rxy: \) everyone respects someone.

This is fine so long as we are completely clear what is meant by the last sentence – namely, that everyone respects someone, not necessarily the same person in each case.

A familiar grammatical transformation converts active sentences into passive ones; for example,

Jay respects Kay

can be transformed into

Kay is respected by Jay.

Both are symbolized the same way.

\( Rjk \)

If we perform the same grammatical transformation on

\( \forall x \exists y Rxy: \) everyone respects someone,

we obtain:

\( \exists x \exists y Rxy: \) someone is respected by everyone,

which might be thought to be equivalent to

\( \exists x \forall y Rxy: \) there is someone who is respected by everyone.

The following lists the various sentences.
Chapter 7: Translations in Polyadic Predicate Logic

(1) everyone respects someone or other
(2) everyone respects someone
(3) someone is respected by everyone
(4) there is someone who is respected by everyone

The problem we face is simple: (1) and (4) are not equivalent; although (4) implies (1), (1) does not imply (4).

In order to see this, consider a very small world with only three persons in it: Adam (a), Eve (e), and Cain (c). For the sake of argument, suppose that Cain respects Adam (but not vice versa), Adam respects Eve (but not vice versa), and Eve respects Cain (but not vice versa). Also, suppose that no one respects him(her)self (although the argument does not depend upon this). Thus, we have the following state of affairs.

| Symbol | Predicate
|--------|----------------|
| Rae    | Adam respects Eve
| Rec    | Eve respects Cain
| Rca    | Cain respects Adam
| ~Rea   | Eve doesn't respect Adam
| ~Rce   | Cain doesn't respect Eve
| ~Rac   | Adam doesn't respect Cain
| ~Rcc   | Cain doesn't respect himself
| ~Raa   | Adam doesn't respect himself
| ~Ree   | Eve doesn't respect herself

Now, to say that everyone respects someone or other is to say everyone respects someone, but not necessarily the same person in each case. In particular, it is to say all of the following:

Adam respects someone  $\exists x Rax$
Eve respects someone  $\exists x Rex$
Cain respects someone  $\exists x Rcx$

The first is true, since Adam respects Eve; the second is true, since Eve respects Cain; finally, the third is true, since Cain respects Adam. Thus, in the very small world we are imagining, everyone respects someone or other, but not necessarily the same person in each case.

They all respect someone, but there is no single person they all respect. To say that there is someone who is respected by everyone is to say that at least one of the following is true.

Adam is respected by everyone  $\forall x Rxa$
Eve is respected by everyone  $\forall x Rxe$
Cain is respected by everyone  $\forall x Rxc$

But the first is false, since Eve doesn't respect Adam; the second is false, since Cain doesn't respect Eve, and the third is false, since Adam doesn't respect Cain. Also, in this world, no one respects him(her)self, but that doesn't make any
difference. Thus, in this world, it is not true that there is someone who is respected by everyone, although it is true that everyone respects someone or other.

Thus, sentences (1) and (4) are not equivalent. It follows that the following can’t all be true:

1. (1) is equivalent to (2)
2. (2) is equivalent to (3)
3. (3) is equivalent to (4)

For then we would have that (1) and (4) are equivalent, which we have just shown is not the case.

The problem is that (2) and (3) are ambiguous. Usually, (2) means the same thing as (1), so that the ‘or other’ is not necessary. But, sometimes, (2) means the same thing as (4), so that the ‘or other’ is definitely necessary to distinguish (1) and (2). It is best to avoid (2) in favor of (1), if that is what is meant. On the other hand, (3) usually means the same thing as (4), but occasionally it is equivalent to (1).

In other words, it is best to avoid (2) and (3) altogether, and say either (1) or (4), depending on what is meant.

3. NEGATIONS OF SIMPLE POLYADIC QUANTIFIERS

What happens when we take the formulas considered in Section 2 and introduce a negation (~) at any of the three possible positions? That is what we consider in the present section.

The quantified formulas obtainable from the atomic formulas ‘Rxy’ and ‘Ryx’ are the following:

1. \(\forall x \forall y Rxy\)  \(\forall y \forall x Ryx\) everyone respects everyone
2. \(\forall y \forall x Rxy\)  \(\forall x \forall y Ryx\) everyone is respected by everyone
3. \(\exists x \exists y Rxy\)  \(\exists y \exists x Ryx\) someone respects someone
4. \(\exists y \exists x Rxy\)  \(\exists x \exists y Ryx\) someone is respected by someone
5. \(\forall x \exists y Rxy\)  \(\forall y \exists x Ryx\) everyone respects someone
6. \(\exists y \forall x Rxy\)  \(\exists x \forall y Ryx\) someone is respected by everyone
7. \(\forall y \exists x Rxy\)  \(\forall x \exists y Ryx\) everyone is respected by someone or other
8. \(\exists x \forall y Rxy\)  \(\exists y \forall x Ryx\) someone respects everyone

Now, at any stage in the construction of these formulas, we could interpolate a negation connective. That gives us not just 8 formulas but 64 distinct formulas (plus alphabetic variants). The basic form is the following.
SIGN..QUANTIFIER..SIGN..QUANTIFIER..SIGN..FORMULA

Each sign is either negative or positive (i.e., negated or not negated); each quantifier is either universal or existential; finally, the formula has the first quantified variable in active or passive position. All told, there are 64 \((2 \times 2 \times 2 \times 2 \times 2)\) combinations!

Let us consider two examples.

(e1) \(~\forall x \sim \exists y \sim Rxy\)

In this formula, all the signs are negative, the first quantifier is universal, the second quantifier is existential, the first quantified variable (‘x’) is in active position.

(e2) \(~\exists x \forall y \sim Rxy\)

In this formula the first and third signs are negative, the second sign is positive, the first quantifier is existential, the second quantifier is universal, the first quantified variable (‘x’) is in passive position.

There are 54 more combinations! We have seen the latter two combinations, not to mention the original eight, which are the combinations in which every sign is positive.

But how does one translate formulas with negations into colloquial English? This is considerably trickier than before. The problem concerns where to place the negation operator in the colloquial sentence. Consider the following sentences.

(1) j dislikes k;
(2) j doesn't like k;
(3) it is not true that j likes k.

The problem is that sentence (2) is actually ambiguous in meaning between the sentence (1) and sentence (3). Furthermore, this is not a harmless ambiguity, since (1) and (3) are not equivalent. In particular, the following is not valid in ordinary English.

\[\text{it is not true that Jay likes Kay;}\]
\[\text{therefore, Jay dislikes Kay.}\]

The premise may be true simply because Jay doesn't even know Kay, so he can't like her. But he doesn't dislike her either, for the same reason – he doesn't know her.

Now, the problem is that, when someone utters the following,

I don't like spinach,

he or she usually means,

I dislike spinach,
although he/she might go on to say,
but I don't dislike spinach, either (since I've never tried it),

Given that ordinary English seldom provides us with simple negations, we need some scheme for expressing them. Toward this end, let us employ the somewhat awkward expression ‘fails to...’ to construct simple negations. In particular, let us adopt the following translation.

\[ x \text{ fails to Respect } y \equiv \text{not}(x \text{ Respects } y) \]

With this in mind, let us proceed. Recall that a simple double-quantified formula has the following form.

\[ \text{SIGN..QUANTIFIER..SIGN..QUANTIFIER..SIGN..FORMULA} \]

Let us further parse this construction as follows.

\[ \text{[SIGN-QUANTIFIER]..[SIGN-QUANTIFIER]..[SIGN-FORMULA]} \]

In particular, let us use the word quantifier to refer to the combination sign-quantifier. In this case, there are four quantifiers (plus alphabetic variants):

\[ \forall x, \sim \forall x, \exists x, \sim \exists x \]

We are now, finally, in a position to offer a systematic translation scheme, given as follows.

**Step 1:** Look at the first quantifier, and read it as follows:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>universal (( \forall ))</td>
</tr>
<tr>
<td>(b)</td>
<td>existential (( \exists ))</td>
</tr>
<tr>
<td>(c)</td>
<td>negation universal (( \sim \forall ))</td>
</tr>
<tr>
<td>(d)</td>
<td>negation existential (( \sim \exists ))</td>
</tr>
</tbody>
</table>

**Step 2:** Check the quantified formula, and check whether the first quantified variable occurs in the active or passive position, and read the verb as follows:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>positive active</td>
</tr>
<tr>
<td>(b)</td>
<td>positive passive</td>
</tr>
<tr>
<td>(c)</td>
<td>negative active</td>
</tr>
<tr>
<td>(d)</td>
<td>negative passive</td>
</tr>
</tbody>
</table>

**Step 3:** Look at the second quantifier, and read it as follows:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>universal (( \forall ))</td>
</tr>
<tr>
<td>(b)</td>
<td>existential (( \exists ))</td>
</tr>
</tbody>
</table>
(c) negation universal (~∀) not...everyone*
(d) negation existential (~∃) no one

*Here, it is understood that ‘not’ goes in front of the verb phrase.

Step 4: String together the components obtained in steps (1)-(3) to produce the colloquial English sentence.

With this procedure in mind, let us do a few examples.

Example 1:  
\[ \exists x \sim \exists y Rxy \]

(1) the first quantifier is existential, so we read it as: there is someone who
(2) the quantified formula is positive, and the first quantified variable ‘x’ is in the passive position, so we read the verb as: is respected by
(3) the second quantifier is negation-existential, so we read it as: no one
(4) altogether: there is someone who is respected by no one

Example 2:  
\[ \sim \forall x \exists y \sim Rxy \]

(1) the first quantifier is negation-universal, so we read it as: not everyone
(2) the quantified formula is negative, and the first quantified variable ‘x’ is in the active position, so we read the verb as: fails to respect
(3) the second quantifier is existential, so we read it as: someone (or other)
(4) altogether: not everyone fails to respect someone (or other)

Example 3:  
\[ \exists x \sim \forall y Rxy \]

(1) the first quantifier is existential, so we read it as: there is someone who
(2) the quantified formula is positive, and the first quantified variable ‘x’ is in active position, so we read the verb as: respects
(4) the second quantifier is negation-universal, so we read it as: not...everyone

(5) altogether: there is someone who respects not...everyone

(5*) or, more properly: there is someone who does not respect everyone

4. THE UNIVERSE OF DISCOURSE

The reader has probably noticed a small discrepancy in the manner in which the quantifiers are read. On the one hand, the usual readings are the following.

\[ \forall x: \text{everything } x \text{ is such that...} \]
\[ \text{for any } x... \]

\[ \exists x: \text{something } x \text{ is such that...} \]
\[ \text{there is at least one } \text{thing } x \text{ such that...} \]

On the other hand, in the previous sections in particular, the following readings are used.

\[ \forall x: \text{every person } x \text{ is such that...} \]
\[ \text{for any person } x... \]

\[ \exists x: \text{some person } x \text{ is such that...} \]
\[ \text{there is at least one person } x \text{ such that...} \]

In other words, depending on the specific example, the various quantifiers are read differently. If we are talking exclusively about persons, then it is convenient to read ‘\( \forall x \)’ as ‘everyone’ and ‘\( \exists x \)’ as ‘someone’, rather than the more general ‘everything’ and ‘something’. If, on the other hand, we are talking exclusively about numbers (as in arithmetic), then it is equally convenient to read ‘\( \forall x \)’ as ‘every number’ and ‘\( \exists x \)’ as ‘some number’.

The reason that this is allowed is that, for any symbolic context (formula or argument), we can agree to specify the associated universe of discourse. The universe of discourse is, in any given context, the set of all the possible things that the constants and variables refer to.

Thus, depending upon the particular universe of discourse, U, we read the various quantifiers differently.

In symbolizing English sentences, one must first establish exactly what U is. For sake of simplifying our choices, in the exercises, we allow only two possible choices for U, namely:

\[ U = \text{things (in general)} \]
\[ U = \text{persons} \]
In particular, if the sentence uses ‘everyone’ or ‘someone’, then the student is allowed to set \( U=\text{persons} \), but if the sentence uses ‘every \text{person}’ or ‘some \text{person}’, then the student must set \( U=\text{things} \).

In some cases (but never in the exercises) both ‘every(some)one’ and ‘every(some)thing’ appear in the same sentence. In such cases, one must explicitly supply the predicate ‘...is a person’ in order to symbolize the sentence.

Consider the following example.

there is some one who hates every thing,

which means

there is some \text{person} who hates every \text{thing}.

The following is not a correct translation.

\[ \exists x \forall y \text{Hxy} \quad \text{WRONG}!!! \]

In translating this back into English, we first must specify the reading of the quantifiers, which is to say we must specify the universe of discourse. In the present context at least, there are only two choices; either \( U=\text{persons} \) or \( U=\text{things} \). So the two possible readings are:

there is some \text{person} who hates every \text{person}

there is some \text{thing} that hates every \text{thing}

Neither of these corresponds to the original sentence. In particular, the following is not an admissible reading of the above formula.

there is some \text{person} who hates every \text{thing} WRONG!!!

The principle at work here may be stated as follows.

One cannot change the universe of discourse in the middle of a sentence.

All the quantifiers in a sentence must have a uniform reading

5. QUANTIFIER SPECIFICATION

So, how do we symbolize

there is some one (some \text{person}) who hates every thing.

First, we must choose a universe of discourse that is large enough to encompass everything that we are talking about. In the context of intro logic, if we are talking about anything whatsoever that is not a person, then we must set \( U=\text{things} \). In
that case, we have to specify which things in the sentence are persons by employing the predicate ‘...is a person’. The following paraphrase makes significant headway.

there is something such that
it is a person who hates everything

Now we have a sentence with uniform quantifiers. Continuing the translation yields the following sequence.

there is something such that
it is a person and
it hates everything

Let's do another example much like the previous one.

everyone hates something (or other)

This means

every person hates something (or other)

which can be paraphrased pretty much like every other sentence of the form ‘every A is B’:

everything is such that
if it is a person,
then it hates something (or other)

At this point, let us compare the sentences.

The general forms of the above may be formulated as follows.

We have already seen this particular transition – from completely general claims to more specialized claims. This maneuver, which might be called quantifier specification, still works.
everything is B: \( \forall x \ldots Bx \)

every A is B: \( \forall x (Ax \to Bx) \)

something is B: \( \exists x \ldots Bx \)

some A is B: \( \exists x (Ax \& Bx) \)

Quantifier specification is the process of modifying quantifiers by further specifying (or delimiting) the domain of discussion. The following are simple examples of quantifier specification.

converting ‘everything’ into ‘every physical object’

converting ‘everyone’ into ‘every student’

converting ‘something’ into ‘some physical object’

converting ‘someone’ into ‘some student’

The general process (in the special case of a simple predicate P) is described as follows.

**SIMPLE QUANTIFIER SPECIFICATION:**

Where \( v \) is any variable, \( P \) is any one-place predicate, and \( F \) is any formula, quantifier specification involves the following substitutions.

substitute \( \forall v (Pv \to F) \) for \( \forall v F \)

substitute \( \exists v (Pv \& F) \) for \( \exists v F \)

Note carefully the use of ‘→’ in one and ‘&’ in the other.

**Examples**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Symbolic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>something is evil</td>
<td>( \exists x E(x) )</td>
</tr>
<tr>
<td>some physical thing is evil</td>
<td>( \exists x (P(x) &amp; E(x)) )</td>
</tr>
<tr>
<td>everything is evil</td>
<td>( \forall x E(x) )</td>
</tr>
<tr>
<td>every physical thing is evil</td>
<td>( \forall x (P(x) \to E(x)) )</td>
</tr>
<tr>
<td>someone respects everyone</td>
<td>( \exists x \forall y R(x,y) )</td>
</tr>
<tr>
<td>some student respects everyone</td>
<td>( \exists x (S(x) &amp; \forall y R(x,y)) )</td>
</tr>
<tr>
<td>everyone respects someone</td>
<td>( \forall x \exists y R(x,y) )</td>
</tr>
<tr>
<td>every student respects someone</td>
<td>( \forall x (S(x) \to \exists y R(x,y)) )</td>
</tr>
</tbody>
</table>

So far we have dealt exclusively with the outermost quantifier. However, we can apply quantifier specification to any quantifier in a formula. Consider the following example:

everyone respects someone (or other) \( \forall x \exists y R(x,y) \)
versus everyone respects some student (or other) ℮?

In applying quantifier specification, we note the following.

overall formula: \( \exists y Rxy \)
specified quantifier: \( \exists y \)
specifying predicate: \( Sy \)
modified formula: \( Rxy \)

So applying the procedure, we obtain:

resulting formula: \( \exists y(Sy & Rxy) \)

So plugging this back into our original formula, we obtain

everyone respects some student (or other) \( \forall x \exists y(Sy & Rxy) \).

The more or less literal reading of the latter formula is:

for any person x,
there is a person y such that,
y is a student
and x respects y.

More colloquially,

for any person, there is a person such that
the latter is a student and the former respects the latter.

Still more colloquially,

for any person, there is a person such that
the latter is a student whom the former respects.

We can deal with the following in the same way.

there is someone who respects every student

This results from

there is someone who respects everyone
\( \exists x \forall y Rxy \),

by specifying the second quantifier, as follows:

overall formula: \( \forall y Rxy \)
specified quantifier: \( \forall y \)
specifying predicate: \( Sy \)
modified formula: \( Rxy \)

So applying the procedure, we obtain:
resulting formula: \( \forall y (Sy \rightarrow Rxy) \)

So plugging this back into our original formula, we obtain

there is someone who respects every student
\[ \exists x \forall y (Sy \rightarrow Rxy) \]

The more or less literal reading of the latter formula is:

there is a person \( x \) such that,
for any person \( y \),
if \( y \) is a student,
then \( x \) respects \( y \).

More colloquially,

there is a person such that,
for any person,
if the latter is a student
then the former respects the latter.

Still more colloquially,

there is a person such that,
for any student,
the former respects the latter.

So far, we have only done examples in which a single quantifier is specified by a predicate. We can also do examples in which both quantifiers are specified, and by different predicates. The principles remain the same; they are simply applied more generally. Consider the following examples.

(1) there is someone who respects everyone
(1a) there is a student who respects every professor
(1b) there is a professor who respects every student

(2) there is someone who is respected by everyone
(2a) there is a student who is respected by every professor
(2b) there is a professor who is respected by every student

(3) everyone respects someone or other
(3a) every student respects some professor or other
(3b) every professor respects some student or other

(4) everyone is respected by someone or other
(4a) every student is respected by some professor or other
(4b) every professor is respected by some student or other

The following are the corresponding formulas; in each case, the latter two are obtained from the first one by specifying the quantifiers appropriately.
6. COMPLEX PREDICATES

In order to further understand the translations that appear in the previous sections, and in order to be prepared for more complex translations still, we now examine the notion of complex predicate.

Roughly, complex predicates stand to simple (ordinary) predicates as complex (molecular) formulas stand to simple (atomic) formulas. Like ordinary predicates, complex predicates have places; there are one-place, two-place, etc., complex predicates. However, we are going to concentrate exclusively on one-place complex predicates.

The notion of a complex one-place predicate depends on the notion of a free occurrence of a variable. This is discussed in detail in an appendix. Briefly, an occurrence of a variable in a formula is bound if it falls inside the scope of a quantifier governing that variable; otherwise, the occurrence is free.

Examples

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>$Fx$</td>
<td>the one and only occurrence of ‘$x$’ is free.</td>
</tr>
<tr>
<td>2</td>
<td>$\forall x(Fx \to Gx)$</td>
<td>all three occurrences of ‘$x$’ are bound by ‘$\forall x$’.</td>
</tr>
<tr>
<td>3</td>
<td>$\forall x Rxy$</td>
<td>every occurrence of ‘$x$’ is bound; the one and only occurrence of ‘$y$’ is free.</td>
</tr>
</tbody>
</table>

Next, to say that a variable (say, ‘$x$’) is free in a formula $F$ is to say that at least one occurrence of ‘$x$’ is free in $F$; on the other hand, to say that ‘$x$’ is bound in $F$ is to say that no occurrence of ‘$x$’ is free in $F$. For example, in the following formulas, ‘$x$’ is free, but ‘$y$’ is bound.