1. Indefinite Noun Phrases

In English, and many other languages, a common-noun-phrase may be prefixed by an indefinite article, the resulting phrase being what we shall call an *indefinite noun phrase*. The following are example sentences from English, in which ‘a’ serves as an indefinite article.

<table>
<thead>
<tr>
<th>that is a dog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay owns a dog</td>
</tr>
<tr>
<td>a dog is in the yard</td>
</tr>
<tr>
<td>there is a dog in the yard</td>
</tr>
<tr>
<td>a dog is happy if and only if it is well-fed</td>
</tr>
<tr>
<td>every man who owns a dog feeds it</td>
</tr>
<tr>
<td>if a man owns a dog, then he feeds it</td>
</tr>
<tr>
<td>a dog is a mammal</td>
</tr>
<tr>
<td>a dog can hear sounds a human can’t</td>
</tr>
<tr>
<td>Jay is looking for a dog</td>
</tr>
</tbody>
</table>

Note in particular that, if we delete the word ‘a’, we obtain phrases that standard English rejects as syntactically ill-formed. On the other hand, languages that lack indefinite articles – the biggest of which are Latin, Russian, and Mandarin – have no problem saying sentences like ‘that is dog’. Furthermore, even English eschews indefinite articles when the common-nouns are plural-nouns or mass-nouns, as in the following examples.

<table>
<thead>
<tr>
<th>those are dogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay owns dogs</td>
</tr>
<tr>
<td>dogs are in the yard</td>
</tr>
<tr>
<td>there are dogs in the yard</td>
</tr>
<tr>
<td>dogs are happy if they are well-fed</td>
</tr>
<tr>
<td>if men own dogs, they feed them</td>
</tr>
<tr>
<td>dogs are mammals</td>
</tr>
<tr>
<td>dogs can hear sounds humans can’t</td>
</tr>
<tr>
<td>Jay is looking for dogs</td>
</tr>
<tr>
<td>that is milk</td>
</tr>
<tr>
<td>Jay has milk</td>
</tr>
<tr>
<td>milk is in the refrigerator</td>
</tr>
<tr>
<td>there is milk in the refrigerator</td>
</tr>
<tr>
<td>milk stays fresh if it is refrigerated</td>
</tr>
<tr>
<td>if men have milk, they drink it</td>
</tr>
<tr>
<td>milk is food</td>
</tr>
<tr>
<td>milk can be made into cheese</td>
</tr>
<tr>
<td>Jay is looking for milk</td>
</tr>
</tbody>
</table>

Note carefully, however, that *colloquial spoken* English often employs unstressed ‘some’ [“səm”] as an indefinite article, which can prefix all the nouns above.

Given the strong structural similarities among these examples, and given the existence of languages that lack indefinite articles, we propose to use the term *indefinite noun phrase* in reference to all such phrases, whether prefixed by an overt indefinite article or not.

More generally, we propose to use this term in reference to any common-noun-phrase that plays the role of an NP [subject, object, …], treating the presence of an indefinite article, overt or covert, as secondary.

---

1 Supposing we reject the reading according to which ‘dog’ is a proper-name, and the reading according to which ‘dog’ is a mass-noun [referring presumably to dog-matter].

2 Also note that Spanish has plural indefinite articles – ‘unos’ (masculine), ‘unas’ (feminine), and French has a plural indefinite article ‘des’ and a mass indefinite article ‘de’.
2. Initial Hypothesis – INP's are QP's

First, we consider the following seemingly natural initial hypothesis.

(iH) INPs are QPs; in particular:

‘a’ is a variant of ‘some’;
‘səm’ is a variant of ‘a’, which attaches to plural-nouns and mass-nouns, and which may/must be deleted in the final form [spoken/written].

(iH) accounts for the following examples.\(^3\)

1. Jay owns a dog

<table>
<thead>
<tr>
<th>Jay</th>
<th>owns</th>
<th>a</th>
<th>dog</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J)</td>
<td>(\lambda x \cdot x_1)</td>
<td>(\lambda P_0 \vee {x \mid Px})</td>
<td>(D_0)</td>
</tr>
<tr>
<td>(J_1)</td>
<td>(\lambda y_2 \lambda x_1:Oxy)</td>
<td>(\vee { x \mid Dx })</td>
<td>(\lambda x \cdot x_2)</td>
</tr>
<tr>
<td>(\vee { \lambda y_1 Oyx \mid Dx })</td>
<td>(\vee { ) O(x \mid Dx })</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\exists x { D_0 &amp; O_0 x })</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here, we presume ‘dog’ \([D]\) means dog-individual, and ‘dogs’ \([D]_0\) means dog-plurality.

As a bonus, (iH) also accounts for predicative uses of ‘a’ as in:

3. Rex is a dog

<table>
<thead>
<tr>
<th>Rex</th>
<th>is</th>
<th>a</th>
<th>dog</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>(\lambda x_1)</td>
<td>(\lambda P_0 \vee {x \mid Px})</td>
<td>(D_0)</td>
</tr>
<tr>
<td>(R_1)</td>
<td>(\lambda y_2 \lambda x_1[x=y])</td>
<td>(\vee { x \mid Dx })</td>
<td>(\lambda x \cdot x_2)</td>
</tr>
<tr>
<td>(\vee { \lambda y_1 [y=x] \mid Dx })</td>
<td>(\vee { ) O(x \mid Dx })</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\exists x { D_0 &amp; R = x })</td>
<td>(D_0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notice in particular that being a dog is equivalent to being identical to some dog.

3. Problems with the Initial Hypothesis

Although (iH) accounts for some data, it has trouble accounting for other data, including the following examples.

- a dog is a mammal
- Jay is looking for a dog
- a dog is happy if it is well-fed

Let’s see what happens when we apply (iH) to these examples.

---

\(^3\) A parallel example can be provided with a mass-noun; for example: Jay owns \(səm\) land.
1. a dog is a mammal

<table>
<thead>
<tr>
<th>a dog [+1]</th>
<th>is</th>
<th>a mammal [+2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>∃x D x</td>
<td>λy λx(x=y)</td>
<td>∀ y M y</td>
</tr>
<tr>
<td></td>
<td>∀ λx(x=y) M y</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∃ x (x=y</td>
<td>D x )</td>
</tr>
<tr>
<td></td>
<td>∃ y (M y &amp; x=y)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x (x=y</td>
<td>D x )</td>
</tr>
</tbody>
</table>

Other sentences with similar form do seem to work this way – for example:

a dog is in the yard

∃x (D x & Y x)

But, unlike the latter, the former seems to be nomic (law-like), and to be about kinds? The following is similar in the latter respect.

2. Jay is looking for a dog

<table>
<thead>
<tr>
<th>Jay [+1] is-looking-for</th>
<th>a dog [+2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>J λy λx(x=y)</td>
<td>∀ y D y</td>
</tr>
<tr>
<td></td>
<td>λx(x=y) D y</td>
</tr>
<tr>
<td></td>
<td>∃ y (D y &amp; L y)</td>
</tr>
</tbody>
</table>

According to this reading, there is a (particular) dog that Jay is looking for. Although this is an admissible reading, there is another reading according to which Jay is not looking for a particular dog. Rather, ‘a dog’ is better understood as indicating the kind of thing Jay is looking for. The following seems similarly generic.

3. a dog is happy if it is well-fed

<table>
<thead>
<tr>
<th>a dog [+]</th>
<th>is happy if</th>
<th>it [+] is well-fed</th>
</tr>
</thead>
<tbody>
<tr>
<td>∃x D x</td>
<td>λx H x</td>
<td>λx λQ (P→Q) λx W x</td>
</tr>
<tr>
<td>β x x1</td>
<td>λx λQ (W x→Q)</td>
<td></td>
</tr>
<tr>
<td>∃ x (D x &amp; (W x→ H x))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This reading says there is at least one dog who is happy if well-fed, which is probably better said using the word ‘some’ rather than ‘a’. It seems that ‘a dog’ seems more generic in this example.

In conclusion, treating ‘a’ as synonymous with ‘some’ fails to capture what may be described as the generic use of ‘a’.

4 Indeed, there is a usage of ‘some’ according to which it means in effect “special”. For example: that is some dog you have there
This usage is often conveyed phonetically by emphasizing the word ‘some’. Another example, from a TV ad. this sale is not for some people; it is for all people.
4. The New Proposal

In order to account for indefinite noun phrases, we pursue a three-part approach.

1. Product and Sum

We expand the type-formation rules so that, if $\mathcal{S}$ is a type, then so are $\Pi\mathcal{S}$ and $\Sigma\mathcal{S}$, and we expand the syntax of type-theory to include all expressions of the following forms.

$$
\begin{align*}
\Pi\{\varepsilon | \Phi\} & \quad \text{the product of all } \varepsilon \text{ such that } \Phi \\
\Sigma\{\varepsilon | \Phi\} & \quad \text{the sum of all } \varepsilon \text{ such that } \Phi
\end{align*}
$$

Here, $\varepsilon$ is any expression [of type $\mathcal{S}$], $\Phi$ is any formula, and the resulting expression has type $\Pi\mathcal{S}$ [respectively, $\Sigma\mathcal{S}$]. The following are the associated type-identities.

$$
\Sigma\mathcal{D} = \mathcal{D} \quad \Sigma\mathcal{S} = \mathcal{S} \quad \Pi\mathcal{S} = \mathcal{S}
$$

And the following are the associated composition-rules.

$$
\begin{align*}
\alpha & \quad \text{if } \alpha \text{ is } \Sigma\text{-promoting} \\
\{\alpha, \beta\} & \vdash \gamma \quad \beta, \gamma \text{ any expressions} \\
\Pi\{\beta | \Phi\} & \quad \Phi \text{ any formula} \\
\Pi\{\gamma | \Phi\} & \quad \Sigma\{\gamma | \Phi\}
\end{align*}
$$

5. This was proposed earlier in our treatment of number-words, where we proposed that number-words are fundamentally adjectives, and ‘a’ is synonymous with ‘one’.

6. As usual, we also have abbreviated forms: $\Pi\nu\Phi$, $\Sigma\nu\Phi$.

7. In other words, we propose that a sum of entities is itself an entity. We are not committed either way, but $\Sigma x F x$ might be the set of all $F$’s, or it might be the mereological-sum of all $F$’s.

8. There are no admissibility restrictions. Even relative pronouns are admitted, unlike all other junctions.

9. Concerning promotion, we postulate that every phrase that promotes $\Pi$ also promotes $\Sigma$, but not conversely.

10. In which case, the node in question is assertional. The transformation is based on the plausible intuition that to assert a product of sentences is to assert all those sentences.
2. Common-Noun-Phrases Transform into Entity-Sums

As before, we propose that common-noun-phrases are fundamentally 0-marked predicates, which is to say they have type $D_0 \rightarrow S$. We further propose that every such phrase may be transformed into an associated entity-sum, in accordance with the following rule.\(^{11}\)

$$\lambda \nu \Phi \ // \ \Sigma \nu \Phi$$

3. The Indefinite Article ‘a’

We propose that ‘a’ is fundamentally a number-word, which is an adjective, semantically rendered as follows.

$$[a] = \lambda x_0 : 1x$$

where ‘1’ is understood as follows.

$$1x =_a x \text{ is a "unit"}$$

which is regarded as a primitive notion.\(^{12}\) If we do not admit compound-nouns, plural-nouns, or mass-nouns, as is common in elementary logic, then the domain consists only of "units", and ‘a’ is redundant.

5. Examples

In the following, we concentrate on singular-nouns, and accordingly treat ‘a’ as redundant.

1. Jay owns a dog

$$\exists y (Dy \& Oy)$$

In this example, ‘a dog’ ultimately gets treated just like ‘some dog’, since ‘a dog’ begins as a common-noun-phrase, which transforms into an entity-sum, which eventually gets simplified to an existential.

The following follows a similar path, treating ‘a dog’ pretty much just like ‘some dog’.

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\(^{11}\) This is not an identity, since the objects don’t have the same type. Rather, it is a bi-directional rule that authorizes transforming a nullative-predicate into an entity-sum, and conversely.

\(^{12}\) ‘unit’ is scare-quoted because its application is heavily context-dependent. First, in the case of mass-nouns, the units are ratio-measures such as gallons, acres, miles; in which case ‘a’ and ‘one’ are modifier-adjectives, not bare adjectives. Second, sometimes compound entities count as "units" as in:

- a man and woman
- a family
2. every man who owns a dog is happy

\begin{align*}
\lambda P_0 \land \{ x_1 \mid Px \} & \quad \land \{ x_1 \mid Mx \land \exists y \{ Dy \land Oxy \} \} \\
& \quad \land \{ Hx \mid Mx \land \exists y \{ Dy \land Oxy \} \} \\
& \quad \forall x \{ \{ Mx \land \exists y \{ Dy \land Oxy \} \rightarrow Hx \} \}
\end{align*}

The following is an alternative account of the above sentence, which does not treat ‘a dog’ as ‘some dog’, although it produces an equivalent reading.

\begin{align*}
\lambda P_0 \land \{ x_1 \mid Px \} & \quad \lambda x_0, x_1 \\
& \quad \lambda x_0 \{ Mx \land \exists y \{ Dy \land Oxy \} \} \\
& \quad \forall x \{ \{ Mx \land \exists y \{ Dy \land Oxy \} \rightarrow Hx \} \}
\end{align*}

Note that \( \Sigma \) admits [who], and note that [every] is \( \Sigma \)-promoting and accordingly converts \( \Sigma \) to \( \Pi \), which grants wide scope to ‘a dog’.

In the previous example, ‘a dog’ can be narrow-existential or wide-universal, the resulting formulas being logically equivalent. In the following example, ‘a dog’ can only be a wide-universal.

3. every man who owns a dog feeds it

\begin{align*}
\lambda P_0 \land \{ x_1 \mid Px \} & \quad \lambda x_0, x_1 \\
& \quad \lambda x_0 \{ Mx \land \exists y \{ Dy \land Oxy \} \} \\
& \quad \forall y \{ Dy \rightarrow \forall x \{ \{ Mx \land Oxy \} \rightarrow Fxy \} \}
\end{align*}

13 Here, in order to save space, we "pre-inflect" the quantifier. Officially, this is syntactically inadmissible, since case-markers attach only to NPs. We propose to ease this restriction, and allow anticipatory case-marking.
Notice, in particular, that a narrow-scope reading of ‘a dog’ is impossible because \( \Sigma \)-simplification is not available at any node, because the sum is not a sum of sentences. In order to accomplish the latter, we must remove the anaphoric marker \([-1]\), but then ‘a dog’ does not bind ‘it’.

The following is another example in which ‘a dog’ gains wide-scope in order to bind ‘it’.

4. a dog is happy if it is well-fed

\[
\begin{array}{|c|c|c|c|}
\hline
\text{a dog} & [+1, -1] & \text{is happy} & \text{if} \\
\hline
\Sigma[x | \text{D}x] & \lambda x_1 x_2 x_1 & \lambda x_1, Hx & \lambda P, Q [P \rightarrow Q] \\
\hline
\hline
\Sigma[Hx x_1 x_2 | \text{D}x] & \lambda x_1, Hx & \lambda P, Q [Wx \rightarrow Q] & \lambda x_1 Wx \\
\hline
\end{array}
\]

Notice that \([\text{if}…]\) is \( \Sigma \)-promoting, and accordingly converts \( \Sigma \) to \( \Pi \).

The following is an example of semantic-binding that is not syntactically-admissible.

5. it is happy if a dog is well-fed

\[
\begin{array}{|c|c|c|c|}
\hline
\text{it} & [+1] & \text{is happy} & \text{if} \\
\hline
\lambda x_1 x_2 & \lambda x_1, Hx & \lambda P, Q [P \rightarrow Q] & \lambda x_1 Wx \\
\hline
\Sigma[Wx x_1 x_2 | \text{D}x] & \lambda x_1 Wx & \lambda Q [Wx \rightarrow Q] & \lambda x_1, Hx \\
\hline
\Pi[Wx \rightarrow Hx | \text{D}x] & \lambda x_1, Hx & \lambda P, Q [Wx \rightarrow Q] & \lambda x_1 Wx \\
\hline
\forall x [\text{D}x \rightarrow \{Wx \rightarrow Hx\}] \\
\end{array}
\]

By contrast, both the following are OK syntactically.

6. if a dog is well-fed, then it is happy

7. if it is well-fed, then a dog is happy

The following is a variation with two occurrences of ‘a’ with corresponding pronouns.

8. if a man owns a dog, then he feeds it

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{if} & \text{a man} [+1, -1] & \text{owns} & \text{a dog} [+2, -2] & \text{then} & \text{feeds} \\
\hline
\Sigma[x_1, x_2 | \text{M}x & \text{D}y] & \lambda y_1, x_1, Oxy & \Sigma[y_2 x_1, y_2 | \text{D}y] & \emptyset & \lambda y_1, x_1 & \lambda y_2, x_1, Fxy \\
\hline
\Sigma[Oxy x_1, x_2 | \text{M}x & \text{D}y] & \Sigma[y_1, x_2, y_2 | \text{D}y] & \lambda x_1, x_2 & \lambda y_1, x_1, Fxy & \lambda y_2, x_1, Fxy \\
\hline
\Pi[\lambda Q(Oxy \rightarrow Q) x_1, x_2 | \text{M}x & \text{D}y] & \lambda y_1, x_1, Fxy & \lambda y_1, x_1, Fxy & \lambda y_2, x_1, Fxy \\
\hline
\Pi[\lambda Q(Oxy \rightarrow Q) x_1, x_2 | \text{M}x & \text{D}y] & \lambda y_1, x_1, Fxy & \lambda y_1, x_1, Fxy & \lambda y_2, x_1, Fxy \\
\hline
\forall x \forall y \{\text{M}x & \text{D}y \rightarrow \{Oxy \rightarrow Fxy\}\} \\
\end{array}
\]

Notice that the two \( \Sigma \)-junctions combine into a big one, and \([\text{if}…]\) is \( \Sigma \)-promoting, and accordingly converts the latter to a big \( \Pi \).
6. Sometimes ‘some’ is Indefinite

When applied to special domains, quantifier phrases occasionally take on special forms, including ‘always’, ‘never’, ‘everywhere’, and ‘somewhere’. When the special domain is persons, we have the following replacements.\(^\text{14}\)

| every person | ↩ | everyone |
| any person   | ↩ | anyone   |
| some person  | ↩ | someone  |
| no person    | ↩ | no one    |

The morphological rule is clear. However, when we apply it to ‘a person’, we obtain ‘a one’, which is inadmissible.\(^\text{15}\) What we have instead is the following.

a person ↩ someone

which means that ‘someone’ is ambiguous between the QP ‘some person’ and the INP ‘a person’, which can be a source of confusion for semantic-theorists.

A potential exception to this rule is the following.

a person is a moral agent

This philosophical claim is presumably nomic (law-like), and accordingly licenses counterfactual reasoning. But if we replace ‘a person’ by ‘someone’, we obtain

someone is a moral agent

which does not seem to be nomic.

The following illustrates using ‘someone’ as an indefinite noun phrase.

1. if someone owns a dog, then he/she is happy

| if | someone [+1][-1] owns a dog then (-1) he/she [+1] is happy |
|---|---|---|
| \(\lambda P \land Q[P \to Q]\) | \(\sum [ x_1 \times x_{-1} | P x ] \) | \(\lambda x_1 \exists y(Dy & Oxy)\) | \(\emptyset\) | \(\lambda x_{-1}, x_1\) | \(\lambda x_1 Hx\) |
| \(\sum [ \exists y(Dy & Oxy) \times x_{-1} | P x ] \) | \(\Pi [ \lambda Q[\exists y(Dy & Oxy) \to Q] \times x_{-1} | P x ] \) | \(\lambda x_{-1}, Hx\) |

The following is a similar example, but with two bound pronouns

2. if someone owns a dog, then he/she feeds it

which can be done similarly, and is left as an exercise.

---

\(^\text{14}\) Note that these phrases should be distinguished from similar forms that have a slight pause before ‘one’. For example, ‘every…one’ is analogous to ‘this one’, as in the following example.

I have many dogs; every one is smart; this one is very smart

\(^\text{15}\) Also, the rule does not apply to plural quantifiers; for example, ‘several persons’ does not abbreviate as ‘several ones’; also, the rule does not apply to numerical quantifiers; for example, ‘exactly one person’ does not abbreviate as ‘exactly one one’.