1. **Definite Descriptions (Original Proposal)**

As originally proposed, ‘the’ is a functor that takes a common-noun-phrase (type $C$) as input, and delivers a definite-noun-phrase (type $D$) as output, being categorially rendered as follows, where the upside-down iota is the definite-description operator.\(^1\)

\[
\text{[the]} = \lambda P_0 x. P x \quad \text{[type } C \rightarrow D]\]

The following is an example semantic-derivation.

\[
\begin{array}{c|c|c|c|c}
\text{the} & \text{man next to Kay} & (+1) & \text{is tall} \\
\hline
\lambda P_0 x. P x & \lambda x_0 [M x \& N x_k] & \lambda x. x_1 & \lambda x_1 [M x \& N x_k] & \lambda x_1 T x \\
\hline
\lambda x_1 [M x \& N x_k] & & & \lambda x_1 T x \\
\hline
T & \lambda x_1 [M x \& N x_k] & & & \\
\end{array}
\]

What happens when the description\(^2\) is improper? A description $\nu \Phi$ is **proper** if there is exactly-one entity such that $\Phi$ [in the relevant situation]; otherwise, it is **improper**.\(^3\) We propose to treat improper descriptions as **non-starters**, which vitiate every expression in which they appear. In particular, an improper description is null [i.e., denotes nothing], and every expression in which it appears is similarly null. So, in the example above, if no man is next to Kay, or two or more men are next to Kay, then ‘the man next to Kay’ is null, and so is every expression containing it.

Our account follows P.F. Strawson’s theory of descriptions.\(^4\) The underlying idea is that expressions that contain a description **presuppose** the description is proper. When we have presupposition-failure, we also have semantic-failure – not in the sense of being **meaning-less**, but in the sense of being **denotation-less**.

2. **Russell’s Theory of Descriptions**

As is widely known, Bertrand Russell\(^5\) rejected the key premise underlying this approach – that descriptions are singular-terms,\(^6\) and accordingly purport to denote entities. He proposed instead to treat descriptions as quantifier-phrases.

By way of formalizing Russell’s account within our semantic framework, we offer the following categorial rendering of ‘the[$R$]’.

\[
[\text{the}[$R$]] = \lambda P_0 \forall x. P! x \quad \text{[type } C \rightarrow \forall D]\]

Here the exclamation point stands roughly for ‘only’,\(^7\) which is formalized as follows.

\[
P! = \lambda x. \{P x \& \neg \exists y \{P y \& y \neq x\}\}
\]

---

1. Although it is upside-down, we usually refer to it simply as “iota”.
2. As is customary, ‘description’ is often shorthand for ‘definite description’. There are also *indefinite* descriptions like ‘a dog’.
3. See below however, about problems with counting.
6. The term ‘singular-term’ is common in elementary logic, which is usually presented in a way that presupposes all definite-noun-phrases are singular; there are no plural-terms or mass-terms. Our semantic-language also is restricted to singular-terms, **but** we permit them to denote plural-entities and mass-entities.
7. See chapter on exclusive-adverbs for a general semantic treatment of ‘only’.
For example, ‘only Kay is virtuous’ expands as follows.

\[ V! K \overset{=} {=} [\lambda x (V x \land \neg \exists y (V y \land y \neq x))] (K) \]

\[ = V! & \neg \exists y (V y \land y \neq K) \]

In other words, Kay is virtuous, and no one else is.

The following is an example Russellian analysis.

<table>
<thead>
<tr>
<th>the[R]</th>
<th>cat</th>
<th>+1</th>
<th>hates</th>
<th>the[n]</th>
<th>dog</th>
<th>+2</th>
</tr>
</thead>
<tbody>
<tr>
<td>λP₀ ∨ xP!x</td>
<td>C₀</td>
<td></td>
<td></td>
<td>λP₀ ∨ yP!y</td>
<td>D₀</td>
<td></td>
</tr>
<tr>
<td>∀x C!x</td>
<td>λx₁x₁</td>
<td></td>
<td></td>
<td>∀D y</td>
<td>λx₁x₂</td>
<td></td>
</tr>
<tr>
<td>( x₁</td>
<td>C!x )</td>
<td></td>
<td></td>
<td>( y₁</td>
<td>D!y )</td>
<td></td>
</tr>
<tr>
<td>( Hxy</td>
<td>C!x &amp; D!y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∃x ∃y ( C!x &amp; D!y &amp; Hxy )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In other words, there is exactly one cat, and there is exactly one dog, and the former hates the latter. So, if either description is improper, the corresponding uniqueness claim is false, and the sentence evaluates as false. By comparison, according to the Strawsonian analysis, if either description is improper, the sentence is a non-starter, due to presupposition-failure.

3. Free-Logic

In order to compare the Russellian and Strawsonian accounts in more detail, we need to flesh out the latter within our system. All we have said so far is that descriptions have type \( D \), and are null when the description is improper. We have not explained how one reasons with descriptions. We now supply the missing details, following the free logic approach.

Free Logic sounds like a happening in the 60’s, and indeed it came into prominence in the 60’s, but the freedom it trumpets is freedom from presuppositions that haunt classical logic, including the following two principles.

1. The domain is non-empty.
2. Every singular-term denotes something (in the domain).

One cannot do descriptions à la Strawson, as we propose to do, unless one abandons (2), since the central thesis is that improper descriptions are singular-terms, but denote nothing.

There are various ways in which one can technically implement Free Logic. In my approach, as presented in my elementary logic works, one posits a class of constants, which are unbound variables (demonstrative pronouns) with the following understanding: although singular-terms generally may denote nothing, constants always denote elements in the domain.

---

9 The “freedom” proposed by Free Logic is more like “reformation”, since it exhorts us to be less indulgent.
10 Russell agrees on this point, but for different reasons; he thinks no description is referential, since descriptions don’t have type \( D \).
11 Note carefully, proper-names are not constants by this account. They of course need not denote.
12 They also denote them rigidly, when we consider modal logic, which is why I call them “constants”. Furthermore, when we apply quantifier-rules, only constants are permitted to be substituted. Complex expressions, including proper-names, get substituted only via Leibniz’s Law.
The inference-rules for descriptions are based on the following underlying principle.

\[ a = \text{the } F \]

iff

\[ a \text{ is } F \]

and

nothing else is F

i.e. nothing other than a is F

Supposing ‘other than’ means ‘≠’, as is plausible to assume, the formalization of this principle goes as follows, which is a bi-directional rule of inference.

\[ c = \nu \Phi \quad \text{if and only if} \quad \Phi[c/\nu] \land \neg \exists \nu \{ \nu \neq c \land \Phi \} \]

In the logical system I propose the latter may be written more succinctly as follows.

\[ c = \nu \Phi \quad \text{if and only if} \quad \forall \nu \{ \Phi \leftrightarrow \nu = c \} \]

Here \( c \) is any constant, \( \nu \) is any variable, and \( \Phi \) is any formula. Remember constants automatically denote entities, so the left side entails that the description is proper.

Notice that using our new operator ‘!’ ["only"] and lambda-abstraction, we can rewrite this rule as follows.

\[ c = \nu \Phi \quad \text{if and only if} \quad [\lambda \nu \Phi]!c \]

For example, \( c = \text{the-dog} \) if and only if c-alone is a-dog.

So far we see how this system recovers Strawson’s account of descriptions. It recovers Russell’s account of descriptions via scoped-singular-terms, formulated as follows.

\[ (\tau/\nu) : \tau \text{ is a } \nu \text{ such that } \ldots \text{ [it is true of } \tau \text{ that } \ldots] \]

For example,

\[ (J/x) : \text{Jay is an } x \text{ such that } \ldots \text{ [it is true of Jay that } \ldots] \]

This is useful for stating things in a de re fashion. More importantly, for present purposes, it is useful for stating description-claims in the manner of Russell, given the way \( (\tau/\nu) \) is contextually defined.

\[ (\tau/\nu) \Phi =_a \exists \nu \{ \nu = \tau \land \Phi \} \]

To use a description in the manner of Russell is, according to this theory, to use it as a scoped-singular-term, given the definition. For example:

\[ (\nu F y/x) G x =_a \exists x \{ x = \nu F y \land G x \} \]

which is equivalent to:

\[ \exists x \{ F!x \land G x \} \]

which is the Russell-formulation of ‘the F is G’.
4. On Deferring – The Middle Way

According to the Strawsonian account of descriptions, a description is a D-term, which purports to denote an entity in the domain. Phrases containing a description presuppose that it is proper; when it is not proper, the phrases are (denotationally-) null. According to Russell, descriptions don’t even purport to denote entities, since they are not D-terms. Rather they are quantifier-phrases, and accordingly denote second-order properties [or junctions].

In this section, we propose an account of descriptions that defers to both Russell and Strawson, producing an account that accords with both!

In order to do this, we propose yet another analysis of ‘the’, as follows.

\[ \lambda P_0 \] \( \Sigma x P!x \)  

As before, the only-operator \([!]\) is formalized as follows.

\[ P! =_{df} \lambda x \{ Px \& \sim \exists y (Py \& y \neq x) \} \]

The following is an example analysis, using a famous example.\(^{13}\)

\[
\begin{array}{cccc}
\text{the} & \text{present king of France} & [+1] & \text{is bald} \\
\lambda P_0 \Sigma x P!x & \lambda x_0 Fx & & \\
\Sigma x F!x & \lambda x, x_1 & & \\
\Sigma \{ x_1 | F!x \} & \lambda x_1 Bx & & \\
\exists x \{ F!x & Bx \} & & & \\
\end{array}
\]

Unlike the earlier rendering of Russellian descriptions, we interpret ‘the F’ not as a quantifier-phrase \( [\forall x F!x] \), but as an indefinite-noun-phrase \( [\Sigma x F!x] \). In the above derivation, the junction is carried to the bitter end, when it is a sum of sentences, which simplifies to an existential, which in particular says that there is exactly one current French king and he is bald. Thus, this derivation reproduces the Russellian analysis, from a slightly different starting point.

So, how does it square with Strawson’s account? Recall that summation has a built-in ambiguity when the items being added are entities.\(^{14}\) The sum of a collection of entities is itself an entity, unless the collection is empty, in which case the sum is null. In the above example, the sum is \( \Sigma x F!x \). So the question is whether this is proper, which amounts to the question whether the collection \( \{ x | F!x \} \) is non-empty, which amounts to the question whether there is exactly one \( F \).

So suppose there is in fact exactly one \( F \), call it \( F \). Then the set \( \{ x | F!x \} \) is non-empty, and indeed \( \{ x | F!x \} = \{ F \} \), so \( \Sigma \{ x | F!x \} = F \). So, if there is in fact exactly one \( F \), then ‘the present king of France’ denotes it! On the other hand, if the description is improper, then \( \{ x | F!x \} = \emptyset \), and so \( \Sigma \{ x | F!x \} \) is null, so \( \Sigma \{ x | F!x \} \) must take on its quantificational guise, resulting in the usual Russellian formula, which in this case is false.

\(^{13}\) Many years ago, my wife and I attended a Halloween party at the home of Vere Chappell, to which we were instructed to wear philosophically-themed costumes. I went as a bound variable; not that clever. On the other hand, my wife went as follows. She wore a hat with a small wrapped present hanging from it, which was labeled ‘To: Bertrand Russell, from: The Present King of France’. This is a great pun of course, but that was not the point; the philosophical concept was “specious present” [William James, C.D. Broad].

\(^{14}\) Earlier used to account for generic-uses of nouns, in sentences like ‘children like dogs’.
5. The Problem with Both Accounts

There is still a big problem for our account of descriptions, whether following Russell, or following Strawson. Consider the following example.

the dogs are barking

According to both theories, if ‘the dogs’ is proper, then the following is true.

$$\exists x\{ D!x \, \& \, Bx \}$$

Since the terms are plural, quantification is over plural-entities, so this says.\(^{15}\)

there is exactly one dog-plurality, which is/are barking

which implies there is exactly one dog-plurality, which is to say

there is/are a plurality \(x\) such that:

only \(x\) are dogs

But the latter amounts to saying:

\(x\) are dogs

and

no things other than \(x\) are dogs

Now, suppose there are in fact exactly-three dogs in the relevant situation – Penny, Quasi, and Rex, – the plurality of which we refer to as \(PQR\).\(^{16}\) It follows that

\(PQR\) are the dogs,

and so

\(PQR\) are dogs.

But what about the following?

no things other than \(PQR\) are dogs

This is false, since surely we have the following.

<table>
<thead>
<tr>
<th>(PQ) are dogs</th>
<th>(PR) are dogs</th>
<th>(QR) are dogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>and</td>
<td>and</td>
<td>and</td>
</tr>
<tr>
<td>(PQ \neq PQR)</td>
<td>(PR \neq PQR)</td>
<td>(QR \neq PQR)</td>
</tr>
</tbody>
</table>

There are pluralities other than \(PQR\) that are dogs; they include \(PQ\), \(PR\), \(QR\). But then there isn’t exactly one dog-plurality that is barking; there are in fact four – \(PQ\), \(PR\), \(QR\), \(PQR\)!

A parallel counter-example can be formulated using mass-nouns, such as.

the water is boiling

Supposing water is divisible, and supposing that a mass is boiling if and only if all its parts are, we have not one, but many, masses that are boiling.

---

\(^{15}\) Recall our account of non-restrictive relative clauses.

\(^{16}\) This notation is short for \(P+Q+R\), which can be understood as Penny-and-Quasi-and-Roni.
6. Back to the Drawing Board!

In face of these serious difficulties, we must adjust our account of descriptions. Recall the following principle.

\[ \alpha \text{ is the } F \]
\[ \text{iff} \]
\[ \alpha \text{ is } F \]
\[ \text{and} \]
\[ \text{nothing else is } F \]
\[ \text{i.e. nothing other than } \alpha \text{ is } F \]

An alternative, perhaps less obvious, formulation replaces the last clause by:

\[ \text{no two things are } F \]

This states the principle in a singular-logic environment, but there is an associated principle involving plural-logic.

\[ \alpha \text{ are the } F \]
\[ \text{iff} \]
\[ \alpha \text{ are } F \]
\[ \text{and} \]
\[ \text{no other things are } F \]
\[ \text{i.e. no things other than } \alpha \text{ are } F \]

7. Counting is not as easy as you think.

One naturally presumes that ‘other than’ and ‘two’ are pretty straightforward, but they are not! For example, most people believe the following truism, exclaimed by Carmen Miranda in the Marx Brothers movie *Copa Cabana* (1947).

\[ \text{no one can be in two places at the same time!} \]

To this, Groucho coolly replies:

I don't see why not; after all, Boston and Philadelphia
are in two places at the same time!

This is funny, to be sure, but Groucho could equally well have said the following in retort, although it is not so funny.

\[ \text{But you are in two places at this time;} \]
\[ \text{for you are in New York City at this time,} \]
\[ \text{and you are in the U.S. at this time,} \]
\[ \text{and those are surely two places!} \]

The counting-problem arises in other contexts. For example, there is a brain-teaser that asks how many squares there are in the following diagram.\(^{17}\)

---

\(^{17}\) Bear in mind that your monitor or printer may not get the aspect ratio exactly right here.
The quick answer is “four”, but the more-thoughtful answer is “five”, obtained as soon as one realizes that the overall diagram is itself a square.

Another counting-problem arises in measuring mass-entities. One might ask the following question.

how many gallons of milk do we have?

The question is ambiguous. Do we treat gallon-of-milk as a singular noun that presumably refers to gallon-sized cartons of milk, irrespective of how much milk they contain. Or, do we treat gallon-of-milk as a measure-noun, in which case a gallon-of-milk can be scattered all over the kitchen floor.\textsuperscript{18} Let’s pursue the latter interpretation. Perhaps this is a misleading way of asking the question, but let’s see where it goes. Now, to answer the question, it seems we have to know (roughly at least) what a gallon of milk is. Basically, we know what milk is, and we know roughly what a gallon is, so we know roughly what a gallon-of-milk is – it is a gallon-sized amount of milk, perhaps contiguous, but not necessarily.

But how do we count gallons-of-milk, as suggested by the question? If we only have a quart of milk, then it seems we have no gallon of milk in the following sense – there is nothing that answers to the description ‘gallon of milk that we have’. On the other hand, if we have exactly one gallon of milk, then the answer is “exactly one”. But if there is more than one gallon of milk, then the answer is “infinitely-many”, since there are infinitely-many mass-entities satisfying the description ‘gallon of milk that we have’.\textsuperscript{19}

But this is bizarre – we have infinitely-many gallons of milk, but we surely do not have infinitely-much milk. This seems like Groucho telling you that you can be in infinitely-many places at the same time.

The question then is: how do we count gallons and places? What is it to be two gallons of milk? What is it to be two places?

8. Wholly in Sense

Let us reconsider the underlying principle of ‘the’.

\[
\alpha \text{ is/are the } F
\]

iff

\[
\text{only } \alpha \text{ is/are } F
\]

iff

\[
\alpha \text{ is/are } F
\]

and

\[
\text{no other thing(s) is/are } F
\]

i.e.

\[
\text{no thing(s) other than } \alpha \text{ is/are } F
\]

\textsuperscript{18} Although we are not supposed to cry over it.

\textsuperscript{19} This presupposes milk is infinitely-divisible, which it presumably isn’t. So it is more accurate to substitute ‘arbitrarily-many’.
Perhaps ‘other than’ has a weak sense and a strong sense, and so far we have been using the weak sense, according to which ‘other than’ means ‘logically-distinct from’. But then what is the strong sense? We suggest that it is expressed by the following phrases

- wholly other than
- wholly distinct from

So Carmen Miranda’s response to Groucho should be:

I am in NYC, but I am not in any place that is **wholly-distinct** from NYC;
yes, I am also in the U.S., which is distinct from NYC, but it is not **wholly-distinct** from NYC, since NYC is part of the U.S.

With this in mind, we now go back and tweak our account of ‘the’ as follows.

\[ \alpha \text{ is/are the } F \]
iff \[ \text{only } \alpha \text{ is/are } F \]
iff \[ \alpha \text{ is/are } F \]
and \[ \text{no thing(s) } \textbf{wholly-distinct from } \alpha \text{ is/are } F \]

That leaves us with formally characterizing ‘wholly-distinct’. For this job, we employ **mereological-disjointness** In particular, items \( \alpha \) and \( \beta \) are disjoint if and only if they have no part in common. 20

With disjointness in hand, we now go back and rewrite our definition of ‘only’ [1].

\[ P! =_{\alpha} \lambda x [Px \& \neg \exists y(y \perp x \& Py)] \]

Here we symbolize disjointness using the orthogonality-symbol ‘\( \perp \)’. 21 Notice that, if we concentrate on singular-entities, then disjointness coincides with non-identity, and this principle reduces to our original principle.

With this in hand, let us reconsider our dog-trio PQR. They are the dogs, because they are dogs and there is no dog-plurality wholly-distinct from them that are also dogs. Yes, PQ, PR, and QR are all logically-distinct from PQR, but they are not wholly-distinct from PQR.

Unfortunately, there is a potential counter-example based on the following similar example.

the three dogs

Suppose we understand this via the following semantic analysis.

20 The model-system is set theory, where the part-whole relation is set-inclusion. Set A is included in set B iff every member of A is a member of B. And sets A and B are disjoint if and only if they have no member in common. If A is included in B, A is also said to be a subset of B. The parts of a set are its (non-empty) subsets.

21 The symbol is often read “\( \perp \)”, which is short for ‘perpendicular’, which is borrowed from geometry – or more to the point, quantum geometry. Quantum-mechanical states live in an infinite-dimensional vector space, which has a built-in orthogonality relation. If two states are orthogonal, they are wholly-distinct; if they are not orthogonal, they are not wholly-distinct. What the latter means (very roughly!) is that a system in one state can "instantly appear" in the other state [a quantum jump]. Classical mechanics does not permit such shenanigans – state-transitions of this sort are prohibited; orthogonality is just non-identity.
So, the question is whether the description is proper, which amounts to whether some plurality \( A \) is/are such that:

\[
A = \lambda x [\exists x \& D_x] \quad \text{\( A \) is/are the three dogs}
\]

By the iota-principle this amounts to

\[
A \text{ is/are three dogs}
\]

and

\[
\text{there is no } X \text{ such that}
\]

\[
X \perp A \text{ and } X \text{ is/are three dogs}
\]

Now suppose as before that \( PQR \) are a dog-trio, but further suppose there is an extra dog, Sonny. So we have a dog-quartet \( SPQR \), so the description ‘the three dogs’ is improper. Nevertheless

\[
PQR \text{ is/are three dogs}
\]

and

\[
\text{there is no } X \text{ such that}
\]

\[
X \perp PQR \text{ and } X \text{ is/are three dogs}
\]

To see the latter, suppose otherwise. Then there are three dogs that are wholly-distinct from \( PQR \). But that means there are in fact six dogs, which contradicts our premise that there are only four dogs total. It follows that \( PQR \) are the-three-dogs, even though they are not the-dogs.

Numerical-adjectives are odd, this being a further illustration! First, the-three-dogs are the-dogs, so long as these are both proper. By contrast, the-brown-dogs are not the-dogs, unless all the dogs are brown. This observation affords us a way out. We need merely reject the grammatical analysis given above for ‘the three dogs’. What is wrong is that it treats the adjective ‘three’ as restrictive, when it should be treated as non-restrictive, as in the following.\(^{23}\)

\[\begin{array}{|c|c|c|c|c|}
\hline
\text{the} & \text{three} & \text{dogs} & \text{[+0]} & \text{[+1]} & \text{are barking} \\
\hline
\lambda P_{0 \alpha} \alpha P x & \lambda x_0 3x & \lambda x_0 D_x & \lambda x_0 \lambda x_1, x_1 & \lambda x_1 B x \\
\hline
\end{array}\]

\(^{22}\) Written in this order because they are patriotic Romans!

\(^{23}\) Curiously, our current framework does not allow us to draw a tree for ‘the three dogs’ in isolation. In a later section, we correct this by offering an even better account of descriptions and non-restrictive adjectives.
9. We Demand a Recount

We now go back and reconsider number-words. In our original account, we analyzed them using primitive adjectives in the semantic language, whose exact nature we did not elucidate. This seems like an opportune time to remedy this shortcoming, hoping it can help us adjudicate the dispute between Carmen Miranda and Groucho Marx.

Originally, we treated number-words as bare-adjectives, being categorially rendered as follows.

\[
\begin{align*}
\text{one} & = \lambda x_0 \lambda x.1x \\
\text{two} & = \lambda x_0 \lambda x.2x \\
\text{etc.}
\end{align*}
\]

Furthermore, to be three-dogs is just to be three (individuals) and to be dogs. So long as we concentrate on count-nouns this is fine. But number-words also apply to measure-nouns like ‘gallon’, ‘acre’, and ‘mile’. It is pretty clear that to be three-gallons is not to be three (what?) and to be gallons. Rather, ‘three’ is a non-conjunctive modifier-adjective.

We accordingly replace the previous account of number-words by the following more general, and more accurate, account.

\[
\begin{align*}
\text{one} & = \lambda M_0 \lambda x_0 \lambda x.1(M)[x] \\
\text{two} & = \lambda M_0 \lambda x_0 \lambda x.2(M)[x] \\
\text{etc.}
\end{align*}
\]

Here, ‘M’ ranges over measure-nouns, which include the usual ones like ‘gallon’, ‘acre’, ‘mile’, but also sortal-nouns, including the generic one ‘individual’.

What counts as one \( M \) – for example, one gallon, one acre, or one individual – is taken to be primitive. On the other hand, what counts as two \( M \), three \( M \), etc., is logically constrained by the following.

\[\begin{align*}
\text{Fundamental Additive Measurement Principle} \\
(m+n)(M)[\alpha] & \leftrightarrow \exists x \exists y \{ x \perp y \land \alpha = x \oplus y \land m(M)[x] \land n(M)[y] \}
\end{align*}\]

Here, \( \alpha \), \( x \), and \( y \) are entities, \( \perp \) is mereological-disjointness, and \( \oplus \) is mereological-addition. Also, \( + \) is numerical-addition, and \( m \) and \( n \) are positive rational numbers. We can also apply this principle to count-entities, which amounts to replacing ‘M’ [the “units”] by a sortal expression, the most general one being ‘I’ (individuals), the latter being optionally pronounced. In that case \( m \) and \( n \) are natural numbers.

---

24 Obviously, one cannot measure length, or area, or volume, without specifying the units. But it has been proposed that one also cannot measure count-entities without units, which is to say without reference to a sortal.

25 These words have traditionally been defined ostensively. For example, ‘1 kilogram’ officially refers to a particular platinum-iridium bar – the standard kilogram – residing the International Bureau of Weights and Measures, in Paris. However, most measure-nouns are currently defined by reference to reproducible experiments very exactly described.

26 This principle applies exclusively to additive scales. Not all measurement scales are additive; for example, temperature is not additive; a 100-degree mass does not consist of two 50-degree sub-masses. Also, note that individuals are occasionally measured using mass-like measures. For example, one can buy a bushel of apples. This kind of measurement only approximately fits the fundamental principle, since the count objects do not divide arbitrarily. Apples are “atomic” for these purposes, and also don’t have equal volume.

27 Rational numbers suffice for actual measurements, although real numbers (rational numbers plus irrational numbers) are much easier to work with theoretically.
Note carefully that the mereological notions (⊥ and ⊕) can be tricky, since there are many ways in which part-whole ideas may be applied.

(1) natural-mereology one individual is naturally part of another; e.g., an elbow is part of an arm; a proton is part of a nucleus, which is part of an atom.

(2) plural-mereology one plurality is/are part-of/among -another; e.g., the senators are among the politicians.

(3) mass-mereology one portion of matter is part of another; e.g. the gold in a statue is part of (all) the gold.

(4) hybrid-mereology a gold statue consists of its gold,\textsuperscript{28} which consists of gold-atoms, which consist of elementary particles, which consist of quarks, which consist of fundamental matter [energy].

So one's left-elbow and one's left-arm count are plurally-disjoint, but not naturally-disjoint, since the left-elbow is a natural part of the left-arm.\textsuperscript{29}

So, what do we say about Miranda vs. Marx? Can one be in two places at the same time? It depends upon how we construe places mereologically. Are places regions of space? In that case, we should measure/count them as masses. Or, are they like rooms and houses? In that case we should measure/count them as individuals. But even then, do we use plural-mereology or natural-mereology?

\begin{itemize}
  \item \textbf{mass} \quad \text{places } P_1 \text{ and } P_2 \text{ are two} \iff P_1 \text{ and } P_2 \text{ do not overlap } [i.e., P_1\bot P_2].
  \item \textbf{count} \quad \text{places } P_1 \text{ and } P_2 \text{ are two} \iff P_1 \text{ and } P_2 \text{ are logically distinct } [i.e., P_1\neq P_2].
  \item \textbf{natural} \quad \text{places } P_1 \text{ and } P_2 \text{ are two} \iff P_1 \text{ and } P_2 \text{ do not overlap } [i.e., P_1\bot P_2].
\end{itemize}

A similar counting problem arises for the question

\begin{center}
how many gallons of milk do we have?
\end{center}

Are we asking how \textit{many} gallon-sized-cartons of milk we have, irrespective of how much milk it is; or, are we asking how-\textit{much} milk [as measured in gallons] we have, irrespective of its spatial containment. The answer to the first question can be any non-negative integer, but it can't be a fractional number like 1.25. On the other hand, the answer to the second question can be any non-negative rational number.\textsuperscript{30}

\textsuperscript{28} Note that a (pure) gold statue is logically distinct from the gold that makes up the statue, since they don't have the same history. The statue came into existence a few years ago, whereas the gold came into existence in one or more supernova explosions billions of years ago.

\textsuperscript{29} This distinction may be described by saying that although my left-elbow is part of my left-arm, my left-elbow is not among my left-arm.

\textsuperscript{30} If milk were infinitely-divisible, then it would be logically possible for the correct answer to be $\pi$ gallons. But the context of utterance generally involves a level of precision – say to the nearest fluid ounce. There is a road-race at Georgia Tech called “The PI Mile Road Race”. Since they are mostly engineering/science types, I am sure they know that the distance is really a rational approximation of $\pi$!
10. Counting Calories and Kelvins

When light (lite?) beer was introduced in the 70's, one heard two kinds of claims.

our light beer has 30% less calories than our regular beer.
our light beer has 30% fewer calories than our regular beer.

Grammar mavens object to ‘less’ when applied to plural-nouns, insisting that ‘less’ is reserved for singular-nouns and mass-nouns. For example, the following sounds illiterate.

I have less dogs than you

But the following sound fine.

I have one less dog than you
I have less money than you

The issue, though, is not just syntax. It is also metaphysics. What are calories? Are they physical objects like tables, chairs, protons, and galaxies? Are they social-complexes like states, armies, and bands? Are they abstract particulars like numbers? In other words, are they what we propose to call entities?

We hereby propose a grammatical test for thinginess for a common-noun – does it admit demonstration? For example, in the case of ‘calorie’, do the following make sense?\(^{31}\)

this calorie, these calories, that calorie, those calories

They don’t. So, it seems that ‘calories’ is plural, but ‘calorie’ is not a sortal. How do we count calories? I mean literally.\(^{32}\) Can we (even in principle) have them parade before us, one at a time, while we count them, one at a time?\(^{33}\)

Thus, measure-nouns involve plural-morphology, but they don’t denote classes of entities, so they don’t allow counting. What they measure – distance, area, volume, heat, etc. – cannot be counted either, although they can of course be measured. So the proper way to speak is to say

our light beer has 30% less available-energy than our regular beer

although this sounds strange, since people mostly don’t understand what calories measure.\(^{34}\)

Thus, the beer ads that use ‘fewer’ rather than ‘less’ get the grammar right, perhaps, but they get the metaphysics wrong! Calories can’t be counted (at least literally).

Finally, we note that the pluralization of measure-nouns has expanded in recent years; in particular, temperature is now measured in science using the expression ‘Kelvins’ [formerly ‘degrees Kelvin’]. On the other hand, no sane person would suggest ‘Fahrenheits’. Why? Count-objects can’t be negative; one cannot have minus-three fish. But one cannot have minus-three Kelvins either, since the Kelvin-scale is absolute. Still one might wonder what these odd little critters are that completely disappear when a tank of Helium, say, gets as cold as possible!

---

\(^{31}\) Bear in mind that demonstrative pronouns are also used anaphorically. Try to hear these as including an external-pointing act.

\(^{32}\) Similarly, how does one pinch pennies?

\(^{33}\) The beginning of Mozart’s The Marriage of Figaro (Le nozze di Figaro) presents a scene in which Figaro and Susanna are presumably counting – cinque, dieci, venti, trenta, trentasei, quarantatre. They are measuring the space for their wedding bed. A spoof (on NPR no less!) overlays the following commentary: “the curtain rises, and we find Figaro and Susanna busy counting their blessings.”

\(^{34}\) It’s further confused by the fact that the usage of ‘calorie’ in pure-science is different from the usage of ‘calorie’ in nutrition-science. They are both units of heat, but the latter equals 1000 of the former; 1 Calorie = 1000 calories; note capitalization. Here, it sounds like I am talking count-objects, but I am not! It would be more obvious if, for some bizarre reason, 1 C were equal to 1005.67 c.
11. Unitary Account of Nouns [including yet another account of ‘the’]

Our current semantic theory is based on the orthodox account of nouns, according to which definite-noun-phrases have type $D$, which denote entities, whereas common-noun-phrases have type $C$, which denote classes of entities. This demarcation is based mainly on the syntax of Western European languages, and the syntax of modern symbolic logic.

In what follows, we propose an account of nouns that:

(1) is not so WE-centric, and
(2) offers a leaner and cleaner semantic theory.

In particular, we propose an account based on a unitary account of nouns, according to which common-nouns and proper-nouns, including descriptions, have type $C$.

Since ‘the’ provides the principal connection between $C$ and $D$, we must first re-categorize ‘the’, which we do as follows.

$$[\text{the}] = \lambda P_0 \lambda x_0 P!x \quad [\text{type} = C \to C]$$

In other words, ‘the’ is a modifier-adjective, which is moreover non-conjunctive. Since ‘the $F$’ is now a CNP, the ‘is’ in ‘is the $F$’ may now be understood as predicative, as seen in the following example.\(^{35}\)

Jay is the man next to Kay

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Jay } [+1] & \text{is} & \text{the} & \text{man-next-to-Kay} \\
\hline
\lambda x_1 \lambda x_0 P!x & \lambda x_0 M!x & \lambda x_0 M!x \\
\lambda x_1 M!x & \lambda x_0 M!x \\
J_1 & \lambda x_0 M!x \\
\hline
\end{array}
\]

This says that Jay is a man-next-to-Kay, and no one else is. Even though this is not an identity claim, it can nevertheless be flipped to obtain the following equivalent claim.

the man-next-to-Kay is Jay

Here, since ‘Jay’ and ‘the man next to Kay’ are NPs, ‘is’ is a transitive-verb, the sentence being analyzed as follows.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{the} & \text{man-next-to-Kay} & [+1] & \text{is} & \text{Jay } [+2] \\
\hline
\lambda P_0 \lambda x_0 P!x & \lambda x_0 M!x & \lambda y_1 \lambda x_1 [x = y_1] & J_2 \\
\lambda x_0 M!x & \Sigma M!x & \lambda x_0 M!x \\
\Sigma \{ x_1 | M!x \} & \lambda x_0 M!x & \lambda x_0 M!x \\
\Sigma \{ x = J | M!x \} & \exists x [ M!x \& x = J] & M! \\
\hline
\end{array}
\]

\(^{35}\) Note carefully, in this example, we start with descriptions. We do proper-names later. For the moment, we treat proper-names as type $D$.  

The semantic analysis above parallels our analysis of INPs, which start as CNPs, but which transform into junctions when called upon to serve as NPs. This means that this account of descriptions is very similar to Russell's account. The difference is that, when the description is not called upon to serve as an NP, it remains a CNP.

The latter may seem like a trifle. But treating ‘the F’ as a CNP permits a completely straightforward and simple analysis of the following.

Obama wants to be the greatest U.S. president

If the description is a singular-term, then ‘be’ is identity, so if Lincoln is in fact the greatest U.S. president, then this says Obama wants to be Lincoln!\(^{36}\) According to the new analysis, ‘be’ is predicative, and we do not face this difficulty.\(^{37}\)

Along completely analogous lines, consider the expression:

if I were you, …

It seems daft to suppose that the following are true.\(^{38}\)

if I were you, then you would be me

if I were you, and you were him, then I would be him

The daftness of these claims suggests that although the subjects are genuine NPs, the predicate-nominatives are not. Otherwise, ‘were’ [the subjunctive form of ‘be’] would be identity, and the daft sentences would be true. But if the predicate-nominatives are common-nouns, then ‘were’ is predicative, and we are not faced with issues of symmetry and transitivity.

Indeed, treating predicate-nominatives as common-nouns makes sense, since it seems that

doesn’t mean

if I were you

if I were identical to you

Rather, it means more like:

if I were in your shoes

or more literally:

if I were in your circumstances

Still another advantage of the new approach, although largely theoretical, is that it leads to a much more straightforward and intuitive analysis of relative clauses. For example, according to the existing account, the phrase

the woman – who is tall

where the relative clause is non-restrictive, has no semantic-value by itself. But according to the new account, it can be analyzed as follows.

<table>
<thead>
<tr>
<th>the</th>
<th>woman who is tall</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda x_0 P!x)</td>
<td>(\lambda x_0 W!x)</td>
</tr>
<tr>
<td>(\lambda x_0 W!x)</td>
<td>(\lambda x_0 T!x)</td>
</tr>
<tr>
<td>(\lambda x_0 {W!x &amp; T!x})</td>
<td></td>
</tr>
</tbody>
</table>

\(^{36}\) There are many ways to avoid this conclusion, even while treating ‘the greatest U.S. president’ as an NP. The point is, the issue does not even arise if ‘the greatest U.S. president’ is a CNP.

\(^{37}\) The new account still permits the goofy reading, but it is no longer seen as natural.

\(^{38}\) A favorite joke of mine – If I were you, I would be giving myself advice!
On the other hand, the restrictive reading of ‘who’ produces the following analysis.

\[
\begin{array}{c|c|c}
\text{the woman-who-is-tall} & \text{the} & \text{woman whose is tall} \\
\hline
\lambda x_0 W & \lambda x_0 T & \lambda x_0 (W x & T x) \\
\lambda P_0 \lambda x_0 P ! x & \lambda x_0 [W y & T y] ! x \\
\hline
\end{array}
\]

Notice that ‘who’ is given the same reading in both sentences. This suggests that whether ‘who’ is restrictive or non-restrictive is not a matter of lexicon, as we originally proposed, but rather a matter of (structural) scope. The scopal account is furthermore consistent with the phonology of these two readings, since the intended reading is conveyed by placing a pause after ‘woman’ [non-restrictive] or after ‘the’ [restrictive].

When we apply these ideas to proper-names, we reap further theoretical benefits. The underlying idea is that all nouns are common-nouns, so names like ‘Jay’ and ‘Kay’ are common-nouns. This is born out in ordinary language, where one is apt to combine proper-names with determiners like ‘my’ or ‘the’. The latter is usually, but not always, combined with plural forms, like ‘the Smiths’. The admission of determiners and plural-forms makes syntactic sense (only) if proper-names can serve as common-nouns. The easiest way to accomplish this is to maintain that fundamentally they are common-nouns.

Of course, usually, proper-names refer to just one entity, or at least they purport to. We propose to understand such usage as containing an implicit definiteness operator, as employed earlier in connection with genitive-nouns.\(^{39}\) What is new here is that definiteness, which is an unpronounced form of ‘the’, has a new meaning; in particular, ‘Jay’ is a common-noun, and so is ‘the Jay’, and so is ‘the my Jay’.\(^{40}\)

The following shows how these ideas are fleshed out in a simple example.

\[
\begin{array}{c|c|c}
\text{Jay respects Kay} & \text{Jay} & \text{[def]} & \text{[+1]} & \text{respects} & \text{Kay} & \text{[def]} & \text{[+2]} \\
\hline
\lambda x_0 J x & \lambda P_0 \lambda x_0 P ! x & \lambda x_0 J x & \lambda P_0 \lambda x_0 P ! x & \lambda x_0 K x & \lambda x_0 K ! x & \lambda x_0 & \lambda x_0 \\
\hline
\Sigma \{ x_1 \mid J ! x \} & \Sigma \{ x_1 \mid J ! x \} & \Sigma \{ y_2 \mid K ! y \} & \Sigma \{ y_2 \mid K ! y \} & \Sigma \{ \lambda x_0 R x y \mid K ! y \} & \Sigma \{ \lambda x_0 R x y \mid K ! y \} \\
\hline
\exists x \exists y \{ R x y \mid J ! x & K ! y \} & \exists x \exists y \{ J ! x & K ! y & R x y \} & \exists x \exists y \{ J ! x & K ! y & R x y \} & \exists x \exists y \{ J ! x & K ! y & R x y \} & \exists x \exists y \{ J ! x & K ! y & R x y \} \\
\hline
\end{array}
\]

The last transformation needs explaining. The basic idea goes as follows, which pursues a mathematical-logical custom to its logical extreme.\(^{41}\) The idea is that every n-place function-sign is deep-down a special kind of n+1-place predicate. So a 0-place (!) function-sign is a 1-

\(^{39}\) For example, ‘my mother’ can be definite in intent, which means it contains a tacit [def].

\(^{40}\) The phrase ‘my Jay’ may also be non-restrictive adjectival-modification, as in ‘Jay, who is mine’. The use we have in mind occurs when distinguishing my Jay from your Jay in situations in which ‘Jay’ is multiply-instantiated.

\(^{41}\) And is utilized in Kalish and Montague’s classic introduction to symbolic logic. In particular, they use zero-place function-signs to symbolize proper-names.
Categorial analysis can be quite complicated using our original analysis. It is considerably easier using the new account of nouns. In particular, notice in the penultimate line above that it says there is an $x$ and a $y$ such that $x$ alone is $J$, and $y$ alone is $K$, and $x$ respects $y$. This means $J$ has a solitary instance, which we call $I$, and $K$ has a solitary instance, which we call $K$. This yields the final transformation above.

This seems like a very complicated way to reach such a simple formula! But this will always be the problem we face when we use nouns that assert, rather than presuppose, existence. In particular, if $J$ or $K$ has more than one instance, the above sentence is false, unless [def] is dropped.

Finally, we reconsider how adjectives can variously apply to definite-nouns, including descriptions and proper-names. The following are examples.

- the treacherous Vogons
- the poet Homer
- Zeno of Elea
- Black Bart

Categorial analysis can be quite complicated using our original analysis. It is considerably easier using the new account of nouns. In particular, notice in the following examples that restrictive versus non-restrictive readings come down to structural ambiguity. We also get more readings, in general, although some of them don't seem very plausible.

\[
\begin{array}{|c|c|c|}
\hline
\text{the} & \text{treacherous} & \text{Vogons} \\
\hline
\lambda x_0 Tx & \lambda x_0 Vx & \lambda P_0,\lambda x_0 P!x \\
\lambda P_0,\lambda x_0 P!x & \lambda x_0 \{Tx \& Vx\} \\
\lambda x_0 [\lambda y \{Ty \& Vy\}] !x & \lambda x_0 \{Ty \& Vy\} !x \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{the} & \text{poet} & \text{Homer} \\
\hline
\lambda P_0,\lambda x_0 P!x & \lambda x_0 Px & \lambda x_0 Hx \\
\lambda P_0,\lambda x_0 P!x & \lambda x_0 \{Px \& Hx\} \\
\lambda x_0 [\lambda y \{Py \& Hy\}] !x & \lambda x_0 \{Py \& Hy\} !x \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Zeno} & \text{of Elea} & [\text{def}] \\
\hline
\lambda x_0 Zx & \lambda x_0 Ex & \lambda P_0,\lambda x_0 P!x \\
\lambda x_0 \{Zx \& Ex\} & \lambda P_0,\lambda x_0 P!x \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Black} & \text{Bart} & [\text{def}] \\
\hline
\lambda x_0 Kx & \lambda x_0 Bx \\
\lambda x_0 \{ Kx \& Bx \} & \lambda P_0,\lambda x_0 P!x \\
\hline
\end{array}
\]