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Propagating Uncertainties – a Primer

Suppose you measure the following physical parameters: (they might be lengths, masses, speeds, etc):

$A \pm \Delta A$, $B \pm \Delta B$, and $D \pm \Delta D$,

where ΔA is the (absolute) measurement uncertainty for A, etc. We assume that ΔA is not correlated with ΔB, etc. That is, the values of these random numbers should be independent of one another.

a) Adding a constant: the <u>absolute</u> uncertainty is unchanged.

i.e., if $x = c + A$ *or* $x = c - A$ (where c is a number without uncertainty),

$\rightarrow \Delta x = \Delta A$.

b) Multiplying by a constant: the <u>relative</u> uncertainty is unchanged:

$x = c \times A \rightarrow \Delta x/x = \Delta A/A$. Equivalently, $\Delta x = c \times \Delta A$.

c) Adding or subtracting two measured quantities:

$x = A + B$ *or* $x = A - B$

$\rightarrow \Delta x = \sqrt{(\Delta A)^2 + (\Delta B)^2}$ (adding the absolute uncertainties ‘in quadrature’).

If you add three or more quantities, add more terms inside the √.

(Note that $\Delta x < \Delta A + \Delta B$. This makes sense because it’s unlikely that both A and B will take on extreme values; most likely the two errors will partially cancel so Δx is not so big.)

d) Multiplying or dividing two different, measured quantities:

$x = A \times B$ or $x = A/B$,

$$\rightarrow \frac{\Delta x}{x} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$ (adding the relative uncertainties in quadrature).

(If you multiply three or more quantities, add more terms inside the √.)

e) Raising a measured quantity to a power:

$x = A^n$ or $x = A^{-n}$

$$\rightarrow \frac{\Delta x}{x} = |n| \frac{\Delta A}{A}$$ (the uncertainty is always positive, even if $n < 0$).

f) Multiple operations: break it into individual steps.

e.g., if $x = 5.3 \times \left(\frac{A^2}{B} - D\right)$, then…

Let $y = A^2/B \rightarrow \Delta y = y \times \sqrt{[(2\Delta A/A)^2 + (\Delta B/B)^2]}$.

Let $z = y - D \rightarrow \Delta z = \sqrt{[(\Delta y)^2 + (\Delta D)^2]}$.

And $\Delta x = 5.3 \times \Delta z$.

g) A more general rule: Propagation of error follows the rules of calculus. In general,

if $x = f(A)$, then $\Delta x = \frac{\partial f}{\partial A} \Delta A$.

if $x = f(A,B)$, then $\Delta x = \sqrt{\left(\frac{\partial f}{\partial A}(\Delta A)\right)^2 + \left(\frac{\partial f}{\partial B}(\Delta B)\right)^2}$.