#7A&B Magnetic force between two conductors

This experiment will be done during two lab sessions. In the first we become familiar with the apparatus and make one set of measurements. In the second session, we complete the data-taking and analysis. **The second week lab work cannot be done unless the first week has been completed, and the homework from the first week is completed.**

**Goals**

By doing this experiment, you will

1. perform an experiment based on “null” measurements, and use an “optical lever,”
2. measure how the magnetic force between two parallel conductors varies with current, and with the separation of the conductors,
3. measure the fundamental constant $\mu_0$,
4. learn about the magnetic field of the earth, and measure a component of that field.

**Reading**

The magnetic field and magnetic forces are introduced in Young and Freedman, Sec. 27.2, 27.6, and 27.7 in the 12th ed. Sources of magnetic fields are discussed in Sec. 28.1-2, leading to the force between parallel conductors in Sec. 28.4 in the 12th ed.

**Theory**

As discussed and studied in the previous experiment, a long, straight conductor carrying current $I$ is surrounded by concentric magnetic field lines. The magnitude of the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$, where $r$ is the radius of the circle, *i.e.*, the distance from the conductor, and $\mu_0$ is a constant, equal to $4\pi \times 10^{-7}$ T·m/A. (1T = 1Tesla, the unit of magnetic field strength.) Figure 1 indicates the magnetic field $B$ due to the current $I$ directed to the left. The direction of $B$ is given by the right-hand rule, *i.e.*, when the thumb of the right hand points in the direction of the current, the fingers curl in the direction of $B$.

A conductor carrying a current $I$ in a magnetic field $B$ experiences a force

$$dF_B = I dB \times dL,$$

where $dL$ is the length of the conductor segment, and the vector $dL$ is in the direction of the current in the conductor.

The magnetic field produced by the current in one conductor exerts a force on any nearby conductor carrying a current. For two parallel conductors carrying equal currents $I$ in opposite directions, as in Figure 1, and as in our experiment, $dL$ and $B$ are perpendicular to each other. $B$ at the upper conductor is constant at any given value of $r$, and the force is repulsive. (Not ugly, but the conductors are pushed apart.)

The magnetic force per unit length on the upper conductor is

$$F_B \frac{L}{L} = \frac{\mu_0}{2\pi} \frac{I^2}{r}. \quad \text{(Eq. 1)}$$

Thus the force per unit length between two parallel conductors one meter apart, each carrying precisely 1 A would be exactly $2 \times 10^{-7}$ N/m. This is important because it is the official SI definition of the Ampere. Although Equation 1 is strictly valid only for conductors of infinite length, it is a good approximation if the separation $r$ is small compared to the conductor length $L$. For our experiment, $r/L < 0.1$ is acceptable.
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**Apparatus**

Take some time to appreciate the creative engineering that has gone into the design of the current balance, the principal piece of apparatus that you will use in this experiment. This instrument is as pretty as any you will encounter in a lower division physics lab! As indicated in Fig. 2, the current balance has two parallel conductors, one suspended over the other. Observe how the delicate balance of the top conductor is achieved. This conductor forms one side of a frame that rests on two knife-edge pivots. The equilibrium position is varied by moving the counterweight, but what really clinches this is a magnetic damper consisting of a metal paddle that dips into the field region between two magnets. As its name indicates, the magnetic damper serves to damp out rocking that would otherwise result after the top conductor is displaced.

![Fig. 2. Current balance used for measuring the force between two parallel conductors.](image)

Note, in addition, how currents are brought perpendicularly into and out of the parallel conductor bars. (The simple circuit schematic in Fig. 2 does not represent the actual current path.) The motivation for this is to make the magnetic fields, $\mathbf{B}_{\text{FE}}$, generated from the entry and exit currents parallel or antiparallel to the currents in the conductor bars, so that $dL \times \mathbf{B}_{\text{FE}} = 0$. In this case, the entry and exit currents produce no force on the conductor bars. Furthermore, observe how the entry and exit currents are routed through the knife-edge pivots, thereby minimizing any mechanical drag forces imposed by the required use of some type of conductor to route the currents from one bar to the other.

Also not indicated in this circuit schematic is a reversing switch to change the direction of the current. Vertical displacements of the top conductor can be accurately measured using the telescope which views a scale via its reflected image in a mirror mounted on the frame. Cross-hairs within the telescope indicate the precise point on the scale to which the telescope points.

Figure 3 shows how this works. For simplicity, it is assumed that the mirror surface is initially in a vertical plane, and that the telescope is aligned horizontally. Thus the telescope points to the position on the scale at the same height as the telescope. But, as indicated in Fig. 3, should the top conductor move upward, the mirror attached to the conductor tilts accordingly, and the telescope then points to a higher position on the scale.
To be specific, if the conductor moves up distance $a$ and it is distance $b$ from the knife-edge pivots, the mirror tilts through angle $\phi \approx a/b$ in the small angle assumption.

However, as shown in Fig. 3, due to the reflection, the apparent position of the cross-hairs on the scale moves upward through angle $2\phi \approx h/d$.

Thus the height change of the conductor is given by $a \approx h \times (b/2d)$. The amplification of the displacement by the mirror in this way has led to the effect being called an “optical lever”.

**Overview of measurements and analysis**

Considerable care is required in this experiment, because the masses are small, typically 10 to 200 milligrams, and the currents are large.

The very small repulsive magnetic force between the conductors can be measured by knowing the mass on the mass pan that balances the magnetic force. This is a form of a “null” measurement technique. The initial “reference” equilibrium position of the top conductor (with zero current) is determined with the telescope. Then, a small mass $m$ is placed on the pivoted top conductor, causing a deflection. The current is adjusted until the magnetic force $F_B$ balances the added gravitational force $mg$, returning the system to its initial position. Then, we have measured the magnetic force to be $F_B = mg$.

By adjusting the counterweight, the reference (equilibrium) separation distance can be changed from several mm to a few cm. For four different reference separations, measure the current required to balance various masses on the mass pan. (One reference separation is to be completed in the first lab class.)

Change the current direction and record the required current in both directions for each separation and mass point. This provides a correction for the additional small force exerted on the top conductor by the earth’s magnetic field, which has a magnitude of about 0.05 mT at Amherst. Only a component of $B_E$ exerts a vertical force, and you will also be able to extract, from the data, a value for this component of the earth’s magnetic field. In the Appendix we show that an exact correction is obtained by replacing $I_2$ with the product $I_1I_2$. The expression for $B_E$ can be obtained by subtracting the first parts of Equations A3 and A4 from each other.

With the correction for the earth’s magnetic field and Equation 1, from page 1, we expect that $F/L = mg/L = \mu_0 I_1 I_2 / 2\pi r$, so we can write $(I_1 I_2)/mg = (2\pi/\mu_0)(r/L)$.

At each separation, the currents required to balance several different masses have been measured. Calculate the average of $(I_1 I_2)/mg$ at one value of the separation $r$. Repeat this for the other separation values. A graph of $(I_1 I_2)/mg$ as a function of $(r/L)$ is expected to be a straight line with slope $2\pi/\mu_0$. The fundamental constant $\mu_0/2\pi$ can be calculated from the slope of the graph.

Using the average values of $(I_1 - I_2)$ at each separation, you will also be able to calculate a value for a component of the magnetic field of the earth.
**Set up**

- The apparatus has already been wired but **do not turn the current on just yet!** First trace the path of the current through the circuit, including the two switches. One switch merely connects and breaks the current, the other reverses its direction. Designate one position of the reversing switch as the *forward direction*, the other as the *reverse direction*. In our labs the conductors are set approximately East-West. With the reversing switch in the designated forward direction, does the positive current go from East to West through the top conductor, or from West to East?

- The current is to be read on the separate ammeter. In series with the ammeter is a *rheostat*—a low-$R$ variable resistor capable of handling large currents. The rheostat can be used as a fine adjustment to the current; set it initially near its mid-position.

- Measure the lengths of the parallel conducting rods. For the purposes of this experiment, what is the relevant length to measure?

- Also consider the separation parameter $r$ that enters into the equations. What is this distance in the experiment? Measurements of the diameter of the conductors in this lab were all within 0.01 mm of 3.19 mm. Variations this small have no impact on the results. You may use this value, or measure your apparatus with the micrometer.

- Place a penny on the mass pan so that the two conductors gently touch. It is unlikely that they will make contact over their entire length. You may need to take the resultant gap into account. Use the feeler gauges (0.1, 0.2, 0.3 mm) to estimate the maximum gap distance between the two conductors. As an approximation, assume that when “in contact”, the average gap distance between the conductors is equal to half the maximum gap. Use this value to calculate $r_c$, the value of $r$ when the two conductors are in contact.

- Adjust the telescope to obtain a sharp image of the cross hairs and scale. Measure the distances $b$ and $d$ indicated in Fig. 3. Record the scale reading with the two conductors touching.

- Remove the penny and, if necessary, adjust the counterweight until the parallel conductors are separated such that $r \approx 6$ mm. This will be the first reference separation. Record the scale reading seen through the telescope.

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**It is easy to make a mistake in setting $r \approx 6$ mm!** What you are looking for is a change $a$ in the height of the top conductor, which, when added to $r_c$, brings you up to $r \approx 6$ mm. Do not use the micrometer to do this — it is not designed for this purpose. Instead use the telescope, remembering that a change in the conductor separation of $a$ corresponds to a change in the scale reading of $h = (2d/b)a$. After you make the adjustment, visually check that the center-to-center separation of the two conductors is indeed about 6 mm.
Measurements

<table>
<thead>
<tr>
<th>Precautions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The set up of the apparatus is delicate. Take great care not to bump the table or apparatus, for once you start a series of measurements, any disturbance may require you to begin again.</td>
</tr>
</tbody>
</table>

1. At a reference separation of ≈ 6 mm, start with 20 mg on the pan. The starting masses for other separations are suggested on the lab report sheet.

2. Check that the voltage knob on the power supply is fully counterclockwise, and then turn on the power supply. Slowly turn the knob to raise the current indicated on the ammeter until the conductor is near the reference position. (The current should be kept below about 14 A, and at no time should the current exceed 15 A.)

3. Tweak the current slightly to restore the conductor to the reference position. Record the value of the current $I_1$ to the nearest 0.1 A or better. Reverse the direction of the current and adjust it as necessary to return to the reference position. Record the current $I_2$.

4. Continue these measurements (items 1 through 3 above) with different masses, increasing the mass in 4 or 5 steps as suggested on the lab report sheet. At each mass, adjust the current to return to the reference position. If more than about 14 A is required, reduce the mass by 20 mg. Do not exceed 15 A under any circumstances. Remember to take measurements with the current in both directions.

5. As you go along, calculate the product $I_1 I_2 / mg$ and difference $|I_1 - I_2|$ at each point.

6. When you have reached the maximum mass and current, remove all masses from the top conductor, and retake the reference and contact scale readings to ensure that nothing has changed. Notify the instructor if the readings differ from your starting values.

7. Repeat the entire sequence of measurements for additional reference separations. Use, $r \approx 6, 8, and 12$ mm. If there is time, do a 4th one at $r \approx 18$ mm.

Analysis

- With the correction for the earth’s magnetic field and Equation 1, from page 1, we can write:

$$F_B = mg = \frac{\mu_0}{2\pi} \frac{I_1 I_2 L}{r} \Rightarrow \frac{I_1 I_2}{mg} = \frac{2\pi r}{\mu_0 L}$$

For each equilibrium separation $r$, the quantity $I_1 I_2 / mg$ should remain constant. (The Appendix shows that analyzing the results in this way eliminates the effect of the earth’s magnetic field.) Calculate the average and standard deviation of $(I_1 I_2)/mg$ for each value of the separation $r$.

- Recall that the sample standard deviation of a quantity $x$ measured $N$ times is defined as:

$$\sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2} \quad \text{where} \quad \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i ,$$

- You now have the averages $(I_1 I_2)/mg$ for each value of the ratio $(r/L)$. Plot a graph of the averages $(I_1 I_2)/mg$ as a function of $(r/L)$. Plot these values with error bars. The uncertainty in $r$ results in an error bar along the horizontal axis for each point, and the standard deviation of $(I_1 I_2)/mg$ is the error bar for each point in the direction of the vertical axis.

- The graph is expected to be a straight line with slope $2\pi/\mu_0$, and the reciprocal of the slope is expected to be $\mu_0/2\pi = 2 \times 10^{-7}$ T·m/A. Determine the value of the fundamental constant $\mu_0/4\pi$.

- As a byproduct, the data also provide an approximate determination of the component $B_E$ of the earth’s magnetic field that causes the top conductor to deflect in the vertical direction. Study the Appendix and derive an equation that expresses $B_E$ only in terms of measured quantities: $mg$ (the mass on the mass pan), $I_1$, $I_2$, and $L$ (the length of the top conductor).
Appendix

Ignoring the magnetic field of the earth, the magnetic force exerted by the bottom conductor on the top conductor is given by

\[ F = \frac{\mu_0 I_0^2 L}{2\pi r} \]  

(A1)

where \( L \) is the length of the top conductor. In the unavoidable presence of a magnetic field from the earth, the total vertical magnetic force on the top conductor becomes

\[ F = \frac{\mu_0 I_0^2 L}{2\pi r} \pm B_E I \pm L \]  

(A2)

There are a few things to note about this Equation (A2):

- In general, the magnetic force exerted by the earth on the top conductor has both vertical and horizontal components. Our experiment is sensitive only to the component \( B_E \) of the earth’s field that gives a force in the vertical direction.
- The vertical magnetic force exerted by the earth on the top conductor can be upward or downward, depending on the direction of the current \( I \). This accounts for the \( \pm \) signs in Eq. (A2).
- The magnetic force due to the earth is independent of the conductor separation \( r \); it depends only on the current \( I \) in the top conductor. If \( I=0 \), no magnetic forces act on the top conductor.

In our experimental procedure we fix the force on the top conductor by choosing a particular mass \( m \), and then adjusting the current to exactly offset the added weight \( mg \). Different currents will be obtained corresponding to different situations. If \( B_E=0 \) we have Eq. (A1). If the magnetic force exerted by the earth is upward:

\[ F = mg = \frac{\mu_0 I_0^2 L}{2\pi r} + B_E I \pm L = \frac{\mu_0 I_0^2 L}{2\pi r} \]

\[ \Rightarrow \frac{2\pi r B_E}{\mu_0} = \frac{I_0^2 - I_+^2}{I_+} \]

(A3)

When, as obtained by simply reversing the current direction, a magnetic force of equal magnitude is obtained in the downward direction:

\[ F = mg = \frac{\mu_0 I_0^2 L}{2\pi r} - B_E I \pm L = \frac{\mu_0 I_0^2 L}{2\pi r} \]

\[ \Rightarrow \frac{2\pi r B_E}{\mu_0} = \frac{I_0^2 - I_-^2}{I_-} \]

(A4)

Equating Eqs. (A3) and (A4):

\[ \frac{I_0^2 - I_+^2}{I_+} = \frac{I_0^2 - I_-^2}{I_-} \]

\[ \Rightarrow I_0^2 I_- - I_+^2 I_- = I_+^2 I_- - I_0^2 I_+ \]

\[ \Rightarrow I_0^2 (I_+ + I_-) = I_+ I_- (I_+ + I_-) \]

\[ \Rightarrow I_0^2 = I_+ I_- \]

(A5)

Hence, the simple substitution of the product \( I_+ I_- \) for \( I_0^2 \) in Eq. (A1) corrects for the earth’s magnetic field. The correction is exact and therefore applies at all distances, even when \( r \) is so large that the forces the conductors exert on each other are much weaker than the magnetic force imposed by the earth.
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Homework

#7 Magnetic force between two conductors

Name: _______________________________
Lab Section: __________________________      Date: _____________________________

Homework preliminaries:
(Please see the reverse side for sources of information on geomagnetism.)

1. (a) At the 15 A maximum current used in this experiment, what is the magnetic field at a distance of 10.0 mm from a very long straight conductor? 
\[ B = \ldots \text{mT} \]

(b) What is the ratio of this magnetic field to the magnitude of the earth’s magnetic field at Amherst, approximately \( B_0 = 0.05 \text{ mT} \)?
\[ \frac{B}{B_0} = \ldots \]

2. (a) Two parallel conductors are 25 cm long and \( r = 5.0 \text{ mm} \) apart. If each carries 10 A in the same direction, what is the magnitude of the force between them?
\[ \ldots \text{N} \]

(b) What mass would correspond to a gravitational force equal to the above answer?
\[ \ldots \text{mg} \]

(c) Is the force between the two conductors repulsive or attractive?
\[ \ldots \]

3. (a) From relations given in the Appendix, derive an equation for the magnitude of the component \( B_E \) of the earth’s field that exerts a vertical force on the top conductor. This expression should preferably involve only measured quantities: \( F_B = mg, I_1, I_2, \) and \( L \). Attach a separate sheet with your derivation.

\[ B_E = \ldots \]

(b) Assuming that (positive) current flowed from West to East in the top conductor, what component of the earth’s magnetic field would give an upwards vertical force on it? Circle one.
- East→West
- West→East
- North→South
- South→North
- Upwards
- Downwards

(c) According to the model calculations, what is the vertical component of the earth’s magnetic field at Amherst? Use 3 significant digits. (See reverse side.)
\[ \ldots \text{mT} \]

(d) According to the model calculations, what is the maximum horizontal component of the earth’s magnetic field at Amherst? Use 3 significant digits.
\[ \ldots \text{mT} \]

(e) Which of the following is responsible for the earth’s magnetic field? Circle one.
- The sun
- Earth’s solid inner core
- Coriolis force
- Ionosphere of earth
- Radioactive decay within earth
- Earth’s liquid outer core
- Chemical concentration or differentiation
- Earth’s magnetosphere
- All of these

This homework sheet must be submitted at the start of the second session in order to obtain credit.
The answers to all your questions about the earth’s magnetism – including its origins – are at the National Geomagnetism web site of the USGS: http://geomag.usgs.gov. An introduction to geomagnetism is at http://geomag.usgs.gov/intro.php.

In the Models section http://geomag.usgs.gov/models/ you can run an on-line code to compute the components of the earth’s magnetic field at Amherst (42°23′ N, 72°31′ W, altitude = 40 m). Click “Start GEOMag” and select a model to load. The igrf – 2005 model now seems to be OK. (If not, use IGRF-2000.) Enter the required data before you hit the calculate button.

A description of the coordinate system may be found in the Magnetic Sensors and Infrastructure section of the Operations page http://geomag.usgs.gov/operations.php.
#7A Laboratory Report
Magnetic force between two conductors - A

Name: _______________________________ Partner: _____________________________
Lab Section: __________________________ Date: _____________________________

**Set-up data:** Please record all dimensions in mm

| Effective length of top parallel conductor: | $L =$ |
| Effective diameter of top parallel conductor: |
| Effective diameter of bottom parallel conductor: |
| When the conductors are touching, maximum gap distance between the top of the bottom conductor and the bottom of the top conductor: |
| Approximate average gap distance between conductors when they are touching. |
| Value of contact separation distance $r_c$ when conductors are touching, including 50% of the estimated average gap distance. $r_c =$ |
| Estimated uncertainty in $r_c$. $\sigma(r_c) =$ |
| Distance $b$ (top conductor to pivot): |
| Distance $d$ (mirror to scale): |
| Magnification of the optical lever = $2d/b$ $2d/b = M =$ |

**Magnetic force measurements:**

Reference separation #1: (~ 6 mm)

| $m$ (mg) | $I_1$ (A) | $I_2$ (A) | $I_1 I_2 / mg$ (A²/μN) | $|I_1 - I_2|$ (A) |
|----------|-----------|-----------|--------------------------|-----------------|
| 20       |           |           |                          |                 |
| 50       |           |           |                          |                 |
| 100      |           |           |                          |                 |
| 150      |           |           |                          |                 |
| 180      |           |           |                          |                 |

Average..
Standard Deviation..

$S_{Con} =$ Scale reading at contact = ............mm
$S_{Ref} =$ Initial ref. scale reading = ............mm
$S_{Ref} - S_{Con} =$ Difference = $D_{scale} =$ ............mm
Conductor motion = $D_{scale}/M$ ............mm
Contact separation = $r_c =$ ............mm
$r_c + D_{scale}/M =$ Initial sep. = $r =$ ............mm

Recheck (with $m = 0$):
Final scale reading at contact = ............mm
Final reference scale reading = ............mm
Final separation $r =$ ............mm