ROTATIONAL DYNAMICS – VARIABLE I, FIXED T

In this experiment we will test Newton’s Second Law for rotational motion and examine how the moment of inertia depends on the properties of a rotating object.

THE THEORY

There is a correspondence between variables in linear motion and those in rotational motion:

<table>
<thead>
<tr>
<th>variable</th>
<th>position</th>
<th>velocity</th>
<th>acceleration</th>
<th>Force/torque</th>
<th>resistance to motion</th>
<th>kinetic energy</th>
<th>Newton’s 2nd Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>x</td>
<td>v</td>
<td>a</td>
<td>F</td>
<td>m</td>
<td>½ mv²</td>
<td>F=ma</td>
</tr>
<tr>
<td>rotational</td>
<td>θ</td>
<td>ω</td>
<td>α</td>
<td>τ</td>
<td>I</td>
<td>½ Iω²</td>
<td>τ=Iα</td>
</tr>
</tbody>
</table>

Torque is defined by \( \tau = r \times \vec{F} = rF \sin \theta \)

The moment of inertia is defined by \( I = \int r^2 \, dm \)

Newton’s Second Law for several bodies rotating about a common axis, can be written:

\[ \sum \tau = I_{tot} \alpha \]

where \( \Sigma \tau \) is the net torque,
\( \alpha \) is the angular acceleration
\( I_{tot} \) is the total moment of inertia about the axis of rotation

APPARATUS

The apparatus consists of a horizontal bar attached to a rotatable drum of radius \( r \). Two brass masses, \( M \), are each placed a distance \( R \) from the axis of rotation, and a mass, \( m \), suspended from a string wound around the drum, causes the apparatus to rotate. A timer is used to measure the time, \( t \), for the mass, \( m \), to fall a distance \( S \), or equivalently, for the apparatus to rotate through an angle \( \theta \). The pulley system increases the available distance, \( S \).
DATA COLLECTION

In this experiment, we keep the torque, $\tau$, constant and vary $I$ by changing the separation, $2R$, between the brass masses, $M$, in order to verify the $R^2$ dependence of the moment of inertia, $I$. Use $m = 0.050\text{kg}$ to apply a constant torque, $\tau = Tr = mgr$.

1. Measure the mass of the drum $M_{\text{drum}}$ and its radius $r$ (to the point of attachment of the string), the length $L$ and mass $M_{\text{bar}}$, and the sum of the brass masses, $2M$.

2. Set the brass masses at their minimum separation and measure $R$ (measure to the center of the brass masses. Note the distance between the holes in the bar).

3. With the drum initially at rest, measure the time for the drum to turn through 5 complete revolutions, $N$. Repeat for a total of 5 time measurements.

4. Use the average elapsed time to calculate the angular acceleration (in units of radian/sec$^2$) as $\alpha = \frac{2\theta}{t^2} = \frac{4\pi N}{t^2}$, where $\theta = 2\pi N$ is the angle, in radians, through which the drum turns in $N$ revolutions.

5. Repeat steps 2 through 4 for 4 larger values of $R$, as well as with the brass masses removed, but with the same value of $m$.

The moment of inertia can be expressed as the sum of two parts,

$$I = I_o + 2MR^2,$$

where $I_o$ is the moment of inertia of the drum and bar without the masses, $2M$, and each $M$ contributes $MR^2$ to the moment of inertia. The sum of drum and bar moments of inertia is

$$I_o = I_{\text{drum}} + I_{\text{bar}} = \frac{1}{2}M_{\text{drum}}r^2 + \frac{1}{12}M_{\text{bar}}L^2$$

where $L$ is the length of the bar. We can rewrite Newton’s Second Law ($\Sigma \tau = I\alpha$) as

$$\sum_{\alpha} \tau = I = I_o + 2MR^2.$$  \hspace{1cm} (3)

If the total torque $\Sigma \tau$ is constant, and $R$ is varied, we expect that the graph of $\Sigma \tau/\alpha$ as a function of $R^2$ should yield a straight line with slope $2M$, and intercept $I_o$.

The net torque is given by $\Sigma \tau = Tr - \tau_i$ where $\tau_i$ is the unknown frictional torque. However, $\tau_i$ is expected to be small and we will assume $\tau_i = 0$, i.e. $\Sigma \tau = Tr$.

Plot $\Sigma \tau/\alpha$ as a function of $R^2$, determine $M$ from the slope, and compare with the actual value of the mass. Determine $I_o$ from the intercept of the graph. Calculate the moment of inertia of the horizontal aluminum bar, and compare with the value of $I_o$ obtained from the
graph.