MOTION DUE TO GRAVITATIONAL FORCE

In this experiment we will look at the motion of a freely falling object. The experimental apparatus consists of a spark timer producing 60 high voltage pulses per second and connected to two wires, causing a spark to jump between the wires, passing across the rim of the bob and through a waxed paper tape laid against the back wire. The sparks melt small spots on the waxed tape, marking the position of the falling bob at time intervals of 1/60 second. The waxed tape is then removed from the apparatus, and will be your data source. You will use the apparatus to measure the acceleration of the gravitational force on an object.

You will study the motion of a freely falling “bob” in order to determine how its position and velocity change with time. The bob is suspended by a magnetic field created when an electrical current flows through an electromagnet. When the current flow to the electromagnet is stopped, the magnetic field disappears and the bob falls.

A spark timer producing 60 high voltage pulses per second and connected to two wires causes a spark to jump between the wires, passing across the rim of the bob and through a waxed paper tape laid against the back wire. The sparks melt small spots on the waxed tape, marking the position of the falling bob at time intervals of 1/60 second. The waxed tape is then removed from the apparatus, and will be your data source.

1. Tape the ends of the waxed paper tape to the table under moderate tension, similar to that when the sparks were recorded.

2. Take a dot near the top of the waxed paper tape as point #1. We will define this dot as having been made at time \( t = 0 \). The time interval between successive points is 1/60 second.

3. Put the meter stick on the waxed paper tape and align it so you can get the best possible measurement of point #1.

4. For each point (#j), following the first point (which defines \( t = 0 \)), record in the data table provided on the worksheet the value of the time, \( t_j \), and its distance, \( X_j \), from the point defining \( t = 0 \). Be sure to do these measurements without lifting or moving the meter stick.
The instantaneous velocity at the \( j^{th} \) point is approximated from the displacements during the following time interval as:

\[
v_j = \frac{X_{j+1} - X_j}{(t_{j+1} - t_j)} = \frac{(X_{j+1} - X_j) \, m}{1/60 \, \text{sec}}.
\]

\( v_j \) is the average velocity over a time interval of 1/60 second duration. For our purposes, it is a reasonable approximation to the instantaneous velocity at time \( t_j \).

5. Calculate the values of \( v_j \) and enter them into the table.

The instantaneous acceleration at the \( j^{th} \) point is approximated from the velocities during the following time interval as:

\[
a_j = \frac{(v_{j+1} - v_j)}{(t_{j+1} - t_j)} = \frac{(v_{j+1} - v_j) \, m}{1/60 \, \text{sec}}.
\]

\( a_j \) is the average acceleration over a time interval of 1/60 second duration.

6. Calculate the values of \( a_j \) and enter them into the table.

7. Calculate the average value of the acceleration, \( \mu_a \), and enter it into the table at the bottom of the next to last column.

8. Using the calculated value of \( \mu_a \), fill in the last column of the data table with the deviations squared of the accelerations \((a_j - \mu_a)^2\).

9. Calculate the standard deviation of the acceleration, \( \sigma_a \), from the expression

\[
\sigma^2 = \langle (a_j - \mu_a)^2 \rangle = \left( \frac{1}{N-1} \right) \sum_{j=1}^{N} (a_j - \mu_a)^2
\]

10. Graph the velocity, \( v \), as a function of time, \( t \).

The relationship between \( v \) and \( t \) for constant acceleration is \( v = v_0 + \alpha t \). This means that the slope of the \( v \) vs. \( t \) graph is the acceleration, \( \alpha \). In the case of acceleration under the action of the gravitational force, we write the acceleration as \( g \).

11. On the graph of \( v \) vs. \( t \), draw the best straight line through the points. The slope of this line is the value of the gravitational acceleration, \( g \). Compare this value of \( g \) with the average value of the acceleration, \( \mu_a \), determined in 7.
# MOTION DUE TO GRAVITATIONAL FORCE

<table>
<thead>
<tr>
<th>j</th>
<th>( t_j )</th>
<th>( x_j )</th>
<th>( x_{j+1} - x_j )</th>
<th>( v_j = \frac{(x_{j+1} - x_j)}{(t_{j+1} - t_j)} )</th>
<th>( \alpha_j = \frac{(v_{j+1} - v_j)}{(t_{j+1} - t_j)} )</th>
<th>( (\alpha_j - \mu_0)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>9/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>11/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>12/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>13/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>14/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>15/60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
16 16/6 0
17 17/6 0
18 18/6 0

Calculation of avg a, $\mu_a$

Calculation of $\sigma_a = \sqrt{\frac{1}{N-1} \sum_{j=1}^{N} (a_j - \mu_a)^2}$

QUESTIONS

1. State 2 possible sources of **random uncertainty** in your determination of g. What makes these sources random?

2. State 2 possible sources of **systematic uncertainty** in your determination of g. What makes these sources systematic?

3. If the uncertainty in your position measurement were to increase substantially (i.e. it would be less precise), would the measured data points follow a straight line more closely or would there be much more scatter? Explain briefly.
4. Compare the value of $g$ obtained from the slope of your $v$ vs. $t$ graph with the average value of $a_j$ from the table.

At the end of the lab, turn in your worksheet with the data and answered questions as well as your histogram.