CENTRIPETAL FORCE & UNIFORM CIRCULAR MOTION

In this experiment we will explore the relationship between force and acceleration for the case of uniform circular motion. An object which experiences a constant force perpendicular to the direction of its motion will move in a circular path of radius $r$ and with constant speed $v$. This object is in a state of uniform circular motion, and has a centripetal acceleration,

$$a = \frac{v^2}{r}$$
directed toward the center of the circle.

The constant force causing the centripetal acceleration is also directed toward the center of the circular path, and, according to Newton’s Second Law, is related to the velocity by the equation

$$F = ma = m\frac{v^2}{r}.$$

In this experiment we will not measure the velocity directly, but will measure the period, $T$, of the circular motion. The period is the time it takes for the object to make one trip around the circle. The relationship between $T$ and $v$ is given by

$$T = \frac{2\pi r}{v} \quad \text{so that} \quad v = \frac{2\pi r}{T}.$$

The centripetal acceleration can then be written as

$$a = \frac{F}{m} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}.$$

The apparatus has a vertical shaft which is set spinning by hand, so that a plumb bob of mass $m$ moves in a circular path of radius $r$, where $r$ is defined by a vertical pointer below the bob. If the plumb bob hangs directly below the supporting arm, the centripetal force is supplied only by the stretched horizontal spring.

When the apparatus is at rest, the force exerted by the stretched spring can be measured by suspending a mass $M$ from a string passed over a pulley and attached to the plumb bob, i.e.

$$F = W = Mg,$$

where $g$ is the acceleration of gravity, which we can take to be $g = 9.81 \text{ m/s}^2 = 981 \text{ cm/s}^2$. 
DATA COLLECTION

The radius, \( r \), can be adjusted over a range of approximately 15cm to 22cm. A 0.1kg mass can be added to the top of the plumb bob to give another set of data for a new value of \( m \).

1. Use the balance to determine the mass \( m \) of the plumb bob.

2. Set the pointer at a desired location. Determine the distance \( r \) from the axis of rotation (the center of the vertical shaft). You may use the vernier caliper to measure the diameter of the vertical shaft.

3. Detach the spring and mass \( M \) from the plumb bob and adjust the cross arm so that the bob hangs directly above the pointer. Use the screw eye and locking nut to adjust the height of the bob so that it clears the pointer by 1 to 2mm.

4. **Attach the spring** between the shaft and the plumb bob. **Attach a string** from the bob, over the pulley and to the hanging pan. Add masses to the pan until the bob is again over the pointer. Record the **total** hanging mass, \( M_T = M + 0.05 \text{kg} \) (for the pan).

5. **Remove** the hanging weight and string. **Set the vertical shaft spinning**, increasing the speed until the bob passes directly over the pointer. Practice this several times before starting to take data. Record the time to complete a number of full revolutions, \( N \) (use a value of \( N \) between 5 and 10). Repeat this until you have at least 3 **measurements** with the same \( N \), recording the time for each trial. One lab partner should concentrate on getting the shaft spinning, while the other counts and times the revolutions. Switch tasks periodically, if desired.

6. **Add a 0.1kg** mass to the top of the plumb bob and secure it with the locking nut. Repeat the measurements in step 5 above. Because \( r \) is unchanged, you need not remeasure the spring force.

7. Repeat steps 2 through 6 for a total of 4 different values of \( r \), covering the full range of possible \( r \) values more or less evenly, obtaining two sets of data for each value of \( r \), one without the added mass, and one with the added mass.

QUESTIONS

Answer at least **question 3**, and then as many other questions that you can complete before the end of the lab period.
CALCULATIONS

1. For each trial compute the period, $T$, the time for one revolution. For each radius and plumb bob mass, calculate the average value of $T$, and the centripetal acceleration, $a = \frac{4\pi^2 r}{T^2}$.

2. The centripetal force at each pointer position is determined from the total mass, $M_T$, required to stretch the spring and bring the bob over the pointer by the equation $F = W = M_T g$.

3. **Plot** the centripetal force, $F$, as a function of the centripetal acceleration, $a$, for each of the two plumb bob masses used. **Plot the two sets of data on a single graph** (make sure to differentiate the points from the bare bob from those with the extra mass). Draw two straight lines, passing through the origin, one through each set of data points, and determine the slope of these lines.

CONCLUSIONS

According to Newton’s Second Law, each of the two $F$ vs. $a$ graphs should be a straight line **through the origin** with a slope equal to the corresponding plumb bob mass.

To check this relationship, **make sure you are using a consistent set of units!!**
Additional analysis – error analysis

The acceleration, $a$, is determined by measuring the radius, $r$, of the path and the time, $T$, for one revolution of the apparatus. The uncertainties in $r$ and $T$ must be combined to determine the uncertainty in $a$. It will be necessary to first express the uncertainties as relative uncertainties, then combine them, and finally express the result as an absolute uncertainty in $a$.

1. Estimate the uncertainty in your determination of the radius. Express this as a percent of the radius, and call it $\sigma_r$.

2. Several measurements of $T$ have been made for each radius and plumb bob mass. For each radius and bob mass, calculate the average deviation of the corresponding set of $T$ measurements. Express this average deviation as a percent of $T$ and call it $\sigma_T$.

3. Now that the percent (relative) uncertainties $\sigma_r$ and $\sigma_T$ in $r$ and $T$ have been determined, they can be combined to obtain the uncertainty in the acceleration, $a$.

4. The acceleration, $a$, depends linearly on $r$, and on the square of $T$. A 1% change in $r$ results in a 1% change in $a$, but a 1% change in $T$ results in a 2% change in $a$. As a result, the percent uncertainty in $a$ is given by

$$\sigma_a = \sqrt{\sigma_r^2 + (2\sigma_T)^2}$$

5. Determine the percent uncertainty in each $a$ calculated in step 4. Express these as absolute uncertainties in $a$ (multiply $\sigma_a$ by $a$), and indicate the uncertainties in $a$ as horizontal error bars on your graph of $F$ vs. $a$. Do the error bars alter your conclusions?