BALLISTIC PENDULUM

In this experiment we will study the application of the laws of conservation of momentum and energy in a ballistic pendulum apparatus. The device accelerates a metal ball to a velocity \( v_1 \). The ball then enters and is caught in a metal cylinder suspended from a fixed axle. As a result of the impact, the ball and cylinder swing to some maximum height, \( h \), given by \( h = h_2 - h_1 \), where \( h_2 \) and \( h_1 \) are the heights of the center of mass of the cylinder-ball combination (the cm mark) in the unfired (Fig. 1) and fired (Fig. 2) conditions, shown below.

**Figure 1: Unfired ball**

**Figure 2: Fired ball**

THE THEORY

If the ball has a mass \( m \) and is fired with a velocity \( v_1 \), it has an initial momentum given by \( \mathbf{p}_1 = m\mathbf{v}_1 \). The ball is “caught” by the cylinder (of mass \( M \)) and the cylinder-ball combination will have a momentum given by \( \mathbf{p}_2 = (M + m)\mathbf{v}_2 \) where \( \mathbf{v}_2 \) is their velocity. If, during the impact, the only forces that are important are those between the ball and the cylinder, then the momentum of the ball and cylinder must be the same after the collision as they were before the collision (conservation of momentum):

\[
\mathbf{p}_1 = \mathbf{p}_2 \rightarrow m\mathbf{v}_1 = (M + m)\mathbf{v}_2
\]  

(1)

The height, \( h \), to which the center of mass of the cylinder-ball combination rises can be calculated from the fact that the initial kinetic energy of the cylinder-ball combination is transformed into a change into gravitational potential energy:

\[
K_f = \Delta P.E. \rightarrow \frac{1}{2}(M + m)v_2^2 = (M + m)gh
\]  

(2)
We can combine (1) and (2) to solve directly for the initial velocity, \(v_1\):

\[
m v_1 = (M + m) v_2 \quad \Rightarrow \quad v_1 = \left( \frac{M + m}{m} \right) v_2 \quad \text{and} \quad \frac{1}{2} (M + m) v_2^2 = (M + m) g h
\]

So \( v_2 = \sqrt{2gh} \) and \( v_1 = \left( \frac{M + m}{m} \right) \sqrt{2gh} \) \hspace{1cm} (3)

DATA COLLECTION

I. Pendulum motion

1. Determine the mass of the ball, \(m\). The mass of the cylinder, \(M\), is stamped on the cylinder itself.

2. Cock the ball to either the middle or most compressed position. Make sure the cylinder is vertical and release the ball. Perform the experiment several times until you are comfortable with the technique. Determine \(h\) for the center of mass of the system consisting of the cylinder and the ball (the cm position is marked on the pendulum – see Fig. 1 or 2 above). Do this by averaging the results of at least 5 trials.

3. Calibrate the height by measuring the distance from the table top to the cm mark when the pendulum is in the unfired position (this is \(h_1\)) and the heights above the table top of the cm when the pendulum is resting in a set of notches containing the average notch position you have determined in step 2. above (this will allow you to determine \(h_2\)).

4. Determine \(v_2\) of the cylinder-ball combination using equation (3) above.

5. Using the value of \(v_2\) just determined, find \(v_1\), the initial velocity of the ball, using equation (1) or equation (3) above.

II. Projectile motion

Having determined \(v_1\) in Part 1. of this experiment, it is possible to check this value using our knowledge of projectile motion. If you fire the ball horizontally from an initial elevation \(y\), then you can calculate the horizontal distance, \(x\), that it will travel before it hits the floor.

6. \textbf{Before} actually projecting the ball horizontally, perform the necessary measurements and calculations to predict where the ball will hit the floor.
7. Measure the distance predicted in 6. and place a paper and carbon sandwich at your predicted impact point. Mark your predicted impact location on the paper with a line (indicate the **distance** from the point at which the ball is projected next to the line). Record the predicted **distance** and the **standard deviation** in the predicted distance.

8. Fire the ball at the target at least 4 times.

9. Determine the **average** and **standard deviation** of the locations of your impact (Note: a distance between the prediction and the average measurement of 3 cm or less is excellent!)