THE TORSION PENDULUM

In this experiment we will study the torsion constants of three different rods, a brass rod, a thin steel rod and a thick steel rod. We will then see how the moment of inertia and the torsion constant are related to the period of oscillation of a rod.

THE THEORY

In analogy with Hooke’s Law for springs, if we twist a rod about its major axis (i.e. apply a torque to the rod), there will be a restoring torque (proportional to the angle of twist, \( \theta \)) which keeps the rod in static equilibrium.

\[
T_{\text{restoring}} = -C \theta
\]

where \( T_{\text{restoring}} \) is the restoring torque, and \( C \) is called the torsion constant.

If, after applying the torque, we then release the rod, the restoring torque will cause the rod to oscillate in angle. The situation is no longer static, but is dynamic. If we ignore the damping effect of friction, then we have the equation of motion:

\[
T_{\text{restoring}} = l \alpha \rightarrow -C \theta = l \alpha \rightarrow l \alpha + C \theta = 0
\]

Replacing the angular acceleration \( \alpha \) by the \( d^2 \theta / dt^2 \) we have the differential equation describing the oscillation:

\[
\frac{d^2 \theta}{dt^2} + \frac{C}{I} \theta = 0
\]

We can solve equation (2) by trying the solution

\[
\theta = A \sin(\omega t)
\]

where \( A \) is the amplitude of oscillation and the period of oscillation is given by

\[
T = \frac{2\pi}{\omega}
\]

Then we write:

\[
\theta = A \sin(\omega t) \quad \frac{d\theta}{dt} = A \omega \cos(\omega t) \quad \frac{d^2 \theta}{dt^2} = -A \omega^2 \sin(\omega t) = -\omega^2 \theta
\]

Substituting into (2) we have:

\[
-\omega^2 \theta + \left( \frac{C}{I} \right) \theta = 0
\]

We can then solve (5) for \( \omega \) and substitute into (4) to get the dependence of the period of oscillation on \( I \) and \( C \).
Part I: Static situation – measurement of the torsion constant

THE APPARATUS

The apparatus consists of a table clamp with ball-bearing hub to which a wheel is attached. A section of the wheel is graduated in degrees to measure the angle of twist. A rod is fastened to the hub of the wheel at one end and to a fixed socket at the other end. The graduated wheel has a flat peripheral surface around which passes a steel ribbon carrying a weight holder.

![Figure 1](image1.png)

**Figure 1**

**Figure 1** shows the main setup and **Figure 2** is a closeup of the connection of the rod to the wheel. Make sure that the screw holding the rod at each end is aligned and inserted into the **notch** in the respective **rod holder**.

![Figure 2](image2.png)

**Figure 2**

DATA COLLECTION

1. Note which type of rod is inserted into the wheel and the fixed socket.

2. With the weight holder on the end of the steel ribbon, move the vernier arm so that the vernier reads zero.

3. Load the weight holder with the weights provided until the rod is twisted by about 90°. Be careful not to drop the weights. You could get hurt!!

4. Remove the added weights and read the vernier again. If it does not read zero, check the rod to see that it is firmly fastened.
5. When the system is satisfactory, load the weight holder to about one-fifth of the weight that was needed to produce a twist of about 90°. Take the scale and vernier readings.

6. Add successive weights of about one-fifth the maximum, in each case reading the scale and the vernier.

7. Remove the weights in reverse order, so that you end up with two readings of the scale and vernier for each load on the weight holder. Be sure to record the zero-weight scale and vernier reading at the end. Your zero reading will be the average of the two zero-weight readings.

8. Graph the load (in kg) as a function of the twist angle (in radians) using the average of the two values of the angle for each weight value.

9. Measure the radius, r, of the wheel around which the steel ribbon is wrapped.

Draw the best straight line through your data points and determine its slope. The slope is proportional to the torsion constant, C. In order to translate your slope into a measurement of C, convert the load into Newtons and multiply by the wheel radius, r. The torque applied by the hanging weights is then \( \tau = Mgr \). So, the value of C will be the slope times (gr). And its units will be Newton–meters/radian.

Part II: Dynamic situation – measurement of the period of oscillation

THE APPARATUS

In this part of the experiment, we hang the rod vertically, attach a flat disk at one end, apply an initial torque, and measure the period of oscillation of the system. A large ring will also be added to increase the moment of inertia of the system. To compute the total moment of inertia of the system, you will need to:

1. Measure the radius of the flat disk, \( R_D \), and record its mass, \( M_D \).
2. Measure both inner and outer radii of the ring, \( R_1 \) and \( R_2 \), and record the mass of the ring, \( M_R \).
3. Compute the moment of inertia of the disk, \( I_D = \frac{1}{2} M_D R_D^2 \).
4. Compute the moment of inertia of the ring, \( I_R = \frac{1}{2} M_R (R_1^2 + R_2^2) \).
DATA COLLECTION

1. Fasten the upper end of the rod you used to measure C to the wall bracket and attach the flat disk to the lower end of the rod. Leave the ring on the wall bracket. Make sure the disk is securely fastened to prevent any accident!!

2. Twist the disk and release it. Measure the time for the system to complete 10 oscillations.

3. Repeat the measurement in 2. until you obtain between 5 and 10 measurements.

4. Lower the ring onto the disk and repeat steps 2. and 3. Make sure you do not twist the rod too much to prevent the ring from sliding off.

DATA ANALYSIS

For this part, we want to test the functional dependence of the period of oscillation on the ratio $I/C$. Use your data to check three possible functional dependencies:

$$a) \sqrt{\frac{I}{C}}, \quad b) \frac{I}{C}, \quad c) \left(\frac{I}{C}\right)^2$$

Note that if you are using only one rod, then only $I$ is varying and you can not really check the dependence of $I/C$. You are checking to see whether the period depends on $a) \sqrt{I}, \quad b) I, \quad c) I^2$.

If you had used more than one rod, then both $I$ and $C$ would be varying and you could check the dependence on $I/C$. 