SIMPLE HARMONIC MOTION – I

(A short report is required for this lab. Just fill in the worksheet, make the graphs, and provide answers to the questions. Be sure to include the units associated with your measurements, and estimates of uncertainties.)

Simple Harmonic Motion (SHM) is a type of periodic motion that is described by a single sine/cosine function with a unique frequency. It occurs whenever there is a linear restoring force, i.e., when the magnitude of the restoring force is directly proportional to the displacement of the system from equilibrium. In this case, the magnitude of the acceleration $a$ will also be proportional to the displacement $x$ from equilibrium, but always opposite in direction.

Let $C$ be the proportionality constant between acceleration and displacement, i.e., $a = -Cx$. Then it is easy to see that the solution for $x$ is simply $A \cos(\omega t + \phi)$ where $A$ is the amplitude of the oscillation and $\omega$ the angular frequency, with $C = \omega^2$. The period of the motion, $T$, is related to $C$ by the equation

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{C}}$$

(1)

For many systems this is strictly true only for small amplitude oscillations.

For a mass $m$ attached to a spring moving in one dimension without friction, we have $a = F/m$, where $F$ is the spring force. A Hookean spring obeys $F = -kx$, where $k$ is called the spring constant. So $a = -(k/m)x$, i.e., $C = k/m$. Hence

$$k/m = C = \frac{4\pi^2}{T^2}$$

(2)

In this experiment we will use masses and springs to study simple harmonic motion. Please refer to your text or lecture notes for the full derivation of the above equations.

A. Spring Constant from Hooke’s Law  (5 points)

First we want to find out if springs obey Hooke’s law: $F = -kx$. This can be done by measuring the extension of a spring $x$ as a function of the force applied to the spring. However, we shall use two stretched springs on opposite sides of the mass instead of just one spring on one side. The two springs may be considered identical.

Attach a spring between each end of the air track and the cart. Turn on the air blower and observe the equilibrium position of the mass. Since the air track is nearly frictionless, even the
A tiniest disturbance will set the cart into oscillation. This should not prevent you from obtaining the equilibrium position. (Why?)

Turn off the air so that the mass stays still. Attach a fine string to the one end of the cart, and pass it over the pulley. Suspend a mass at the other end of the string. Its weight is the applied force $F$. Turn on the air again and find the new equilibrium position of the cart due to this force. Repeat this for several masses, recording the mass and the position of the cart, as determined from the pin attached to the cart and the ruler attached to the air track. Also record the uncertainty in your measurement of the position each time. Note that if the air tracks have air pulleys, a light weight ribbon (a recording tape) is used in place of the string.

Make a plot of your mass vs. position data. In equilibrium the magnitude of the force exerted by the stretched springs is equal to the suspended weight $mg$, so a Hookean spring would obey.

$$mg = F = -kx,$$

i.e., the graph of $m$ vs. $x$ should be linear and the slope is given by $-k/g$.

Draw the “best” straight line through the data points, and measure the slope of the line. From the slope calculate the effective spring constant $k$ for the two springs combined. From the uncertainty in the slope determine the uncertainty in the spring constant.

**Part A Questions**

1. Is your data in the graph of mass as a function of position consistent with a straight line?

2. What is your value and uncertainty for the spring constant $k$ for two springs?

**B. Spring Constant from Period (5 points)**

_Gently_ remove one spring from the apparatus in part (A) and _gently_ hang it on the horizontal small “finger” of the vertical stand. Position the stand such that the spring hang over the edge of the lab bench. (The key word here is GENTLE because the springs are very delicate and they are IRREPLACEABLE.) Suspend different masses $M$ on the spring and measure the period $T$ for small vertical oscillations with the stop clock. In order to not over-stretch the spring, keep the mass $M$ under 25 g, make sure the amplitude of oscillation is small and vertical. Do not let the spring swing like a pendulum.

The period of oscillation is given by Eq. (2), which can be rewritten as

$$M = \left(\frac{k}{4\pi^2}\right)T^2.$$

Thus, the graph of $M$ versus $T^2$ is expected to have a slope equal to $k/4\pi^2$.

Measure the period of oscillation for several masses $M$, and plot $M$ as a function of $T^2$. Draw the “best” straight line through the data. Determine the slope of the line ($k/4\pi^2$). Find $k$ and its uncertainty.

**Part B Questions:**

1. To measure $T$, you might let the mass go up and down 10 cycles. Although it is natural to use the top or bottom positions to mark the beginning and end, it is actually more accurate to use the midpoint. Why?

2. What is the value and uncertainty for the spring constant, as determined from the period?
3. Compare the values of the spring constant found in parts A and B. Note that $k_A$ in Part (A) is due to two springs, but $k_B$ in Part (B) is due to one spring. Assuming that the two springs are identical, what is the theoretical relationship between $k_A$ and $k_B$? Is this borne out by your data?

C. EXTRA CREDIT:

Repeat the experiment in Part A with one spring, using the one left on the air track, i.e., the one not used in Part (B). Find the spring constant $k_C$ for this spring. How does it compare with $k_B$? Are the springs identical? If not, how are they related to $k_A$? (3 points)
A. Spring Constant from Hooke’s Law

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<th>Mass (m)</th>
<th>Position (trial 1)</th>
<th>Position (trial 2)</th>
<th>Position (trial 3)</th>
<th>Position (trial 4)</th>
<th>Position (trial 5)</th>
<th>Average X</th>
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Attach the graph of suspended mass, m, as a function of position of the cart.

1. Is your data in the graph of mass as a function of position consistent with a straight line?

What is the slope of the straight line that best represents the data?

Slope = ___________ ± ___________

2. What is your value and uncertainty for the spring constant $k$ for two springs?

$k_A = ___________ ± ___________

Show the calculation of the spring constant below.
B. Spring Constant from Period

Number of full cycles used = ___________

| Mass M ( ) | Time ( ) | | | | | |
|------------|----------|---|---|---|---|
|            |          |---|---|---|---|
|            |          |---|---|---|---|
|            |          |---|---|---|---|
|            |          |---|---|---|---|

1. Why is it suggested that you use the position of the cart when it is near the middle of the oscillations as the reference, rather than using the end point of the motion as the reference?

2. Attach the graph of M as a function of $T^2$.
   What is the slope of the straight line that best represents the data?

   Slope = ___________ ± ___________

   What is the value and uncertainty for the spring constant, as determined from the period?

   $k_B = ___________ ± ___________

3. Compare the values of the spring constant found in parts A and B, and comment on the agreement or lack thereof.