Name: ________________________ Student Number:

**Exam 2A**

**Introductory Statistics**
**Resource Economics 211**

**Please read the following rules and sign below to indicate that you do understand our rules. Thanks!**

- ❖ Please have nothing but a pencil/pen and your calculator on your desk. All book bags, backpacks need to be stored at the front of the class.
- ❖ You cannot share calculators.
- ❖ You cannot talk with those around you. Raise your hand, if you have any questions or problems.
- ❖ No cell phones. If you are seen with a cell phone, we will take your exam.
- ❖ Please check your Exam; there should be **9 pages** and three parts:
  - o A – information for us.
  - o B – 15 MC questions.
  - o C – Problems, **questions 1-7**. You must **provide exact formulas where required to earn full credit**.
- ❖ We will provide you with a formula sheet and a bubble sheet. Only the bubble sheet answers count for the MC questions.
- ❖ You will need a #2 pencil for the bubble sheet.
- ❖ Neatness counts! **If we cannot read your answers, then we cannot give you credit**.
- ❖ Please keep the parts stapled together! To be safe, please put your name on all pages!
- ❖ Relax, you'll think better. Stress is bad. Good Luck.

**Okay, Dan. I've read and agree to these rules.** ______________________________________________

**Part A. No points here, just necessary information. Declare your weights**. You can revise these weights at Exam 2 and again at the Final. Answer the following questions on the ScanTron Bubble Answer sheet with a number 2 pencil.

These weights must sum to 90%.

1. What is the color of your exam?
   a. Yellow
   b. Blue

2. What weight do you wish to place on Exam 1?
   a. 10%
   b. 20%
   c. 30%

3. What weight do you wish to place on Exam 2?
   a. 10%
   b. 20%
   c. 30%

4. What weight do you wish to place on the Final Exam?
   a. 20%
   b. 30%
   c. 40%

5. What weight do you wish to place on OWL quizzes?
   a. 10%
   b. 20%

Name: ________________________ Student Number:

**Part B: 2 points each. Answer the following MC questions on the ScanTron Bubble Answer sheet.**

1. A list of events for the experiment "a draw from a deck of cards" is shown below. Which events are mutually exclusive?

a. K and H.
b. H and F.
c. C and A.
d. A and N.
e. None of the above.

N = Number card
K = King
H = Heart
C = Club
A = Ace
R = Red card
F = Face card

2. Customers entering a store may do one of three things: buy nothing, buy a small amount, or buy a large amount. If a customer buys a large amount, he or she cannot also buy a small amount or buy nothing. Given this information, we can conclude that these events constitute:

a. outcomes.
b. the sample space.
c. mutually exclusive events.
d. all of the above.
e. none of the above.

3. You are dealt two cards successively (without replacement) from a shuffled deck of 52 playing cards. Find the probability that the first card is a king and the second card is a queen.

a. 13/102
b. 2/13
c. 1
d. 1/663
e. 4/663

4. Please define in words the following notation: P(A | B)

a. The probability that A or B occurred
b. The probability that A and B occurred.
c. The probability of A occuring when you already know B occured first.
d. The probability of B given A occured.
e. None of the above.

5. Choose the answer below that represents ${}_5C_3$ (or equivalently, $C_3^5$ )

a. 20
b. 3!/5!
c. 10
d. 5!/3!
e. None of the above.

6. The general shape of a binomial random variable that has a probability of success of 0.7 and a very large n will be: (choose the best answer)

a. symmetric
b. uniform or rectangular
c. right skewed
d. left skewed
e. normal or bell shaped

Name: _______________________ Student Number:

When a quarter is tossed four times, 16 outcomes are possible:
HHHH; HHHT; HHTH; HHTT; HTHH; HTHT; HTTH; HTTT
THHH; THHT; THTH; THTT; TTHH; TTHT; TTTH; TTTT

Here, for example, HTTH represents the outcome that the first toss is heads, the next two tosses are tails, and the fourth toss is heads. Answer the following two questions using these outcomes.

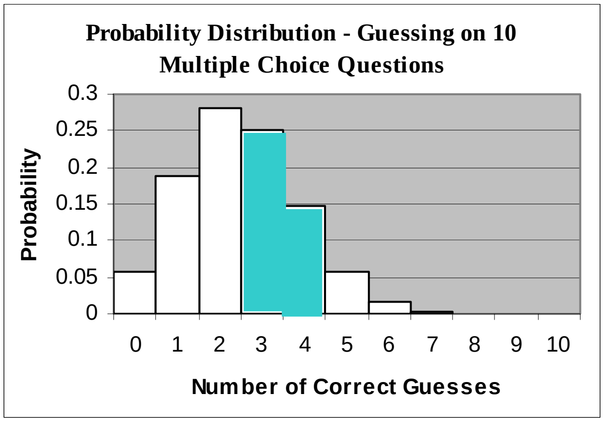
7. The event **A** is defined as follows: **A** = event the first two tosses are heads
List all the outcomes that comprise the event (**not A**)
a. TTHH, TTHT, TTTH, TTTT **(This answer *is* event A)**
b. THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT
c. HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT
d. TTHT, TTHH
e. HHHH, HHHT, HHTH, HHTT

8. The events **B** and **C** are defined as follows: **B** = event exactly two tails are tossed; and
**C** = event the first and last tosses are the same.
List the outcomes that comprise the event (**B and C**).
a. HTTH, THHT
b. HTTH, THHT, TTTT, HHHH
c. The event (B & C) cannot happen
d. HHHH, HHTH, HTHH, HTTH, THHT, THTT, TTHT, TTTT
e. HHTT, HTHT, HTTH, THHT, THTH, TTHH
f. HHHH, HHTH, HHTT, HTHH, HTHT, HTTH, THHT, THTH, THTT, TTHH, TTHT, TTTT

9. Which of the following statements reflects a joint probability?
a. The probability is 0.043 that a student selected randomly earned a grade < 60.
b. The probability is 0.03 that a randomly selected student was a freshman and earned a grade of 90 and above.
c. 12.6% of the students taking the exam were seniors.
d. 15.2 % of the students earned a grade of 70 < 80.
e. 38 % of the sophomores earned grades of 80 and above.

10. Which of the following statements is a conditional probability?
a. The probability is 0.127 that a student selected randomly earned a grade < 60.
b. The probability that a student was a freshman and earned a grade of 90 and above is 0.006.
c. 10.8% of the students taking the exam were seniors.
d. 27.1 % of the students earned a grade of 70 < 80.
e. 10.9 % of the sophomores earned grades of 90 and above.

11. Approximate $P(2 < C < 5)$ using the probability distribution at the right. Choose the closest answer below.
a. 0.30
b. 0.40
c. 0.50
d. 0.60
e. 0.70



Name: ______________________ Student Number:

12. A lab orders 100 rats a week for each of the 52 weeks in the year for experiments that the lab conducts. Suppose the mean cost of rats used in lab experiments turned out to be $13.00 per week. Interpret this value.

a. The rat cost that occurs more often than any other is $13.00.
b. The expected or average cost per week for all weekly rat purchases is $13.00.
c. Most of the weeks resulted in rat costs of $13.00.
d. The median cost for the distribution of rat costs is $13.00.
e. None of the above.

13. When the outcome of each trial can assume only two possibilities, and the trials are statistically independent, then the random variable that tracks the number of successes in those trials will follow which distribution?

a. Factorial
b. Binomial
c. Uniform
d. Normal
e. None of the above

14. A binomial probability distribution with .25 as the probability of a success will have a distribution with a shape best described by:

a. Uniform
b. right skewed
c. symmetric
d. left skewed
e. None of the above

15. The variable z is normally distributed with $\mu_z = 0$ and $\sigma_z = 1$. Identify the two values that contain 68.26% of the values for z.

a. Quartiles 1 and 2.
b. - 1 and + 1
c. - 2 and + 2
d. - 3 and + 3
e. We need more information - there are an infinite number of possible intervals that contain 68.26% of the values.

Name: ______________________ Student Number:

**Part C: Please answer all questions in this section.** Point totals are provided in the left margin.

***Show all work: No formulas, no credit. No calculations, no credit.***

1. The probability that a new advertising campaign will increase sales is assessed as being 0.80. The probability that the cost of developing the new ad campaign can be kept within the original budget allocation is 0.40. Assume that these two events are independent. **Answer each of the questions below using this information.** ***Round probabilities to 2 decimal places.***

(4) a. The information above gives two probabilities and implies two more. What are those four probabilities? What kinds of probabilities are given? (You may then find the table helpful in answering questions b and c.)

Given are two **marginal probabilities**:

**P(I) = 0.80 and P(B) = 0.40**.

But from these, we can get two more **marginal probabilities** as shown in the table at the right:

**P(not I) = 0.20 and P(not B) = 0.60**

| | **I** | **not I** | |
|---|---|---|---|
| **B** | **0.32** | 0.08 | 0.40 |
| **not B** | 0.48 | 0.12 | 0.60 |
| | 0.80 | 0.20 | |

(3) b. Determine the probability that the cost is kept within budget and the campaign will increase sales. What kind of probability is this?

The assumption that these two events (I and B) are independent makes this easy. To solve for the **joint probability** that is asked, just apply the **special multiplication rule:**

**P(B & I) = P(B) · P(I) = (0.80) · (0.40) = 0.32**

(I completed the table above using the special multiplication rule to calculate all joint probabilities.

(3) c. Determine the probability that the cost is not kept within budget or the campaign will not increase sales..

Apply the general addition rule these two events are not mutually exclusive.

**P(not B or not I) = P(not B) + P(not I) – P(not B & not I)**
**P(not B or not I) = (0.60) + (0.20) – (0.12) = 0.68**

(4) 2. One Stats professor is very clever. To encourage students to practice using the Z-Table, he only reports exam scores as a z-scores. Students know that the mean of the exam was **67.84** and that the standard deviation was **17.52**. Determine the actual exam score for a student whose z-score was **1.55**. (**Round your final answer to 1 decimal place.)**

Determine an X-value: $X = \mu_X + z \cdot \sigma_X$

$\boldsymbol{X}$ **= 67.48 + (1.55) · (17.52) = 94.996 = 95.0** is her exam score.

Name: ______________________ Student Number:

3. As reported in Trends in Television, the proportion of US households who have at least one DVD player is **0.55**. If **9** households are selected at random, without replacement, from all US households, what is the (approximate) probability that the number of households having at least one DVD is …

(6) a. no more than **8** but at least **6**. (**Round your final answer to three decimal places**.)

$$P(6 \le X \le 8) = P(X = 6) + P(X = 7) + P(X = 7);$$ we need three binomial probabilities.

From above, we know that n = 9 and p = 0.55.

$$P(X = 6) = C_6^9 \, p^6 \, (1 - p)^3 = (84)(0.55)^6(0.45)^3 = 0.212$$

$$P(X = 7) = C_7^9 \, p^7 \, (1 - p)^2 = (36)(0.55)^7(0.45)^2 = 0.111$$

$$P(X = 8) = C_8^9 \, p^8 \, (1 - p) = (9)(0.55)^8(0.45) = 0.034$$

$$\mathbf{P(6 \le X \le 8) = 0.212 + 0.111 + 0.034 = 0.357}$$

(3) b. Suppose you now select **30 households** at random. Determine the expected number of US households with at least one DVD player. (**Round to 1 decimal place and report units**.)

$$\mu_X = E[X] = n \cdot p = (30)(0.55) = \mathbf{16.5 \text{ households}}$$

(3) c. Determine the standard deviation for the number of US households with at least one DVD player, when 30 US households are selected. (**Round to 1 decimal place and report units**.)

$$\sigma_X = \sqrt{n \cdot p \cdot (1 - p)} = \sqrt{(30)(0.55)(0.45)} = \mathbf{2.7 \text{ households}}$$

Name: ______________________ Student Number:

4. The survey of students in this class found the following probability distribution for the number of siblings (***S***).

| s | P(***S = s***) | s · P(***S = s***) | $s^2$ | $s^2$ · P(***S = s***) | |
|---|---|---|---|---|---|
| 0 | 0.066 | 0.000 | 0 | 0.000 | |
| 1 | 0.423 | 0.423 | 1 | 0.423 | |
| 2 | 0.313 | 0.626 | 4 | 1.252 | |
| 3 | 0.101 | 0.303 | 9 | 0.909 | |
| 4 | 0.044 | 0.176 | 16 | 0.704 | |
| 5 | 0.022 | 0.110 | 25 | 0.550 | |
| 6 | 0.018 | 0.108 | 36 | 0.648 | |
| 7 | 0.009 | 0.063 | 49 | 0.441 | |
| 8 | 0.004 | 0.032 | 64 | 0.256 | |
| | 1.000 | 1.841 | | 5.183 | |

(6) a. **Complete the table above**. You must **include and label columns required showing the work required for parts b and c**. You may or may not need all the columns depending upon the approach you take for part c.

(3) b. Determine the expected number of siblings for students in this stats course. (**Round to 2 decimal places and report units**.)

$$E[S] = \sum s \cdot P(S = s) = \mathbf{1.84 \ \ siblings}$$

(3) c. Determine the standard deviation of the number of siblings for the students in this course. (**Round to 2 decimal places and report units**.)

$$\sigma_X = \sqrt{\sum s^2 \cdot P(S = s) - (1.841)^2} = \mathbf{1.34 \ \ siblings}$$

(4) d. Determine the probability that a student has 2 or more siblings. (**Round to 3 decimal places**.)

P(S ≥ 2) = P(S = 2) + … + P(S = 8) = 1 – P(S = 0) – P(S = 1)

**P(S ≥ 2) = 1 – 0.066 – 0.423 = 0.511**

Name: ______________________ Student Number:

5. Suppose you open a bag of M&Ms that contains 10 M&Ms in total, 3 of which are green. You draw M&Ms from the bag, one at a time, record whether it is green (**G**), or not green (**not G**), and eat them. Completely label the tree diagram below, then answer the following questions.

(3) a. Determine the probability that you eat a green one on the second draw from the bag, given that you did not eat a green one on your first draw. What kind of probability is this? (**Round to 3 decimal places.**)

**P(G | not G) =** $\frac{3}{9}$**. This is a conditional probability and is the probability on the second branch of the tree.**

(3) b. Determine the probability for the following outcome: (**G** & **not G** & **G**). (**Round to 3 decimal places.**)

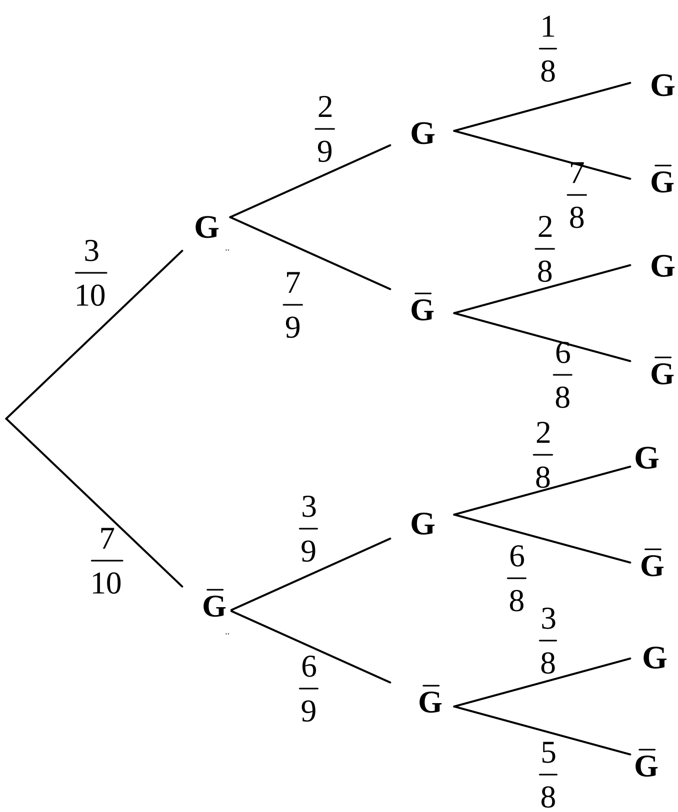
Apply the special multiplication rule:

**P(G & not G & G) =** $\frac{3}{10}\cdot\frac{7}{9}\cdot\frac{2}{8} = \frac{42}{720}$ **= 0.058**

(6) c. Determine the probability that you eat one green M&M in your three draws from the bag. (**Round to 3 decimal places.**)

The tree diagram outcomes are labeled and probabilities included. (Remember on each set of branches, the conditional probabilities must sum to 1.) Use the special multiplication rule to determine the probabilities of outcomes at the end of the sets of branches. Then **use the special addition rule to determine the probability in question**:

**P( G = 1 ) = 0.175 + 0.175 + 0.175 = 0.525**




**G** & $\bar{\mathbf{G}}$ & $\bar{\mathbf{G}}$ P(**G** & $\bar{\mathbf{G}}$ & $\bar{\mathbf{G}}$) = 0.175

$\bar{\mathbf{G}}$ & **G** & $\bar{\mathbf{G}}$ P($\bar{\mathbf{G}}$ & **G** & $\bar{\mathbf{G}}$) = 0.175

$\bar{\mathbf{G}}$ & $\bar{\mathbf{G}}$ & **G** P($\bar{\mathbf{G}}$ & $\bar{\mathbf{G}}$ & **G**) = 0.175

0.525

Name: ______________________ Student Number:

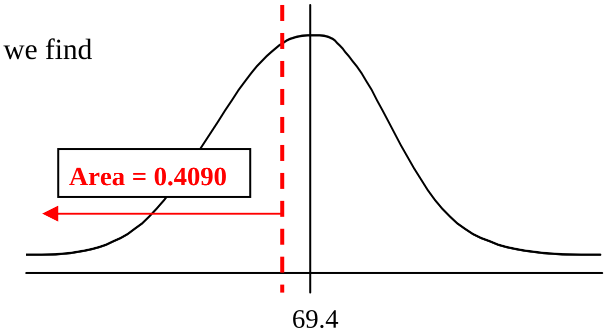
(6) 6. One Stats professor encourages students to practice using the Z-Table by only reporting exam scores as percentiles. Students know that the mean of the exam was **69.4** and that the standard deviation was **9.72**. ***Determine the actual exam score*** for a student whose score was at the **40.9** percentile. (**Round your final answer to 1 decimal place.**)

First, find the z-value such that $P(Z < z) = 0.4090$. From the table we find $z = -0.23$

Now, solve for X given this z-value:

$$X = \mu_X + z \cdot \sigma_X = 69.4 + (-0.23)(9.72)$$

$$\boldsymbol{X = 67.2}$$




7. The mean distance for tee shots on the 1999 men's PGA tour was 272.2 yards with a standard deviation of 8.12 yards. Assuming that the 1999 tee shots were normally distributed, determine …

(4) a. the percentage of tee shots that went between 260 and 280 yards. (**Round the percentage to 2 decimal places**.)

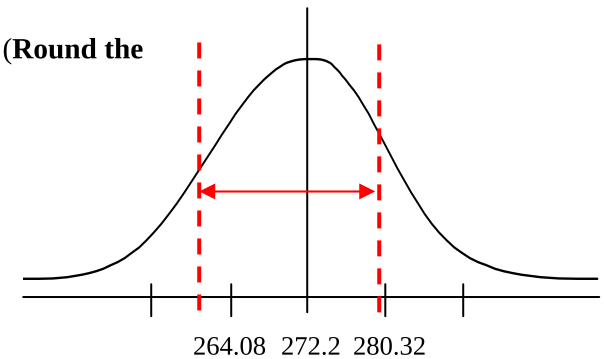
Looks like an area under the curve of about 75-80%

Determine z-values: $z = \dfrac{260 - 272.2}{8.12} = -1.50$

$$z = \frac{280 - 272.2}{8.12} = 0.96$$

$P(Z < 0.96) = 0.8315$
$P(Z < -1.50) = 0.0668$
0.7647

**76.47% of the tee shots fall**

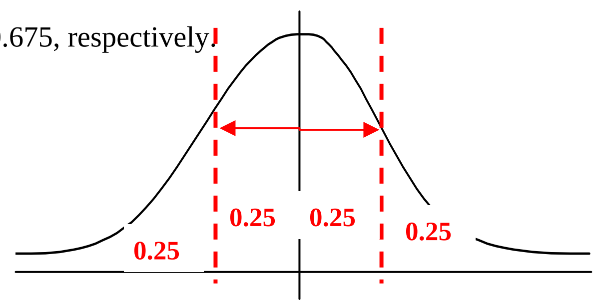



(4) b. quartiles 1 and 3 for the tee shot distances. (**Round the quartiles to 1 decimal place**.)

Quartiles break the distribution up into quarters – 25% chunks.
We know that the z-values for quartiles 1 and 3 are –0.675 and +0.675, respectively.

$$X_{Q1} = \mu_X + z \cdot \sigma_X = 272.2 + (-0.675)(8.12) = \mathbf{266.7}$$

$$X_{Q3} = \mu_X + z \cdot \sigma_X = 272.2 + (0.675)(8.12) = \mathbf{277.7}$$




(2) c. What percentage of tee shots fall between quartiles 1 and 3?

Obvious from the picture above – **50% of the tee shots fall between quartiles 1 and 3.**