2. Relation of Marginal Revenue to Elasticity
   (in Approximation-of-Derivative Form)

From the definition of arc-price elasticity $\varepsilon_{\alpha\beta}$ between points $\alpha$ and $\beta$ on the demand curve shown above, and using the selections for $x$ and $p_x$ employed in class,

$$
\varepsilon_{\alpha\beta} = -\left(\frac{x' - x''}{p_x' - p_x''}\right)\left(\frac{p_x'}{x''}\right) = -p_x' \left[\frac{1}{\frac{p_x' x'' - p_x'' x''}{x' - x''}}\right].
$$

(1)

Now

$$
\frac{p_x' x'' - p_x'' x''}{x' - x''} = \frac{p_x' x'' - p_x' x' + p_x' x' - p_x'' x''}{x' - x''} = \frac{p_x' x'' - p_x' x'}{x' - x''} + \frac{TR(x') - TR(x'')}{x' - x''},
$$

where $p_x' x'$ has been both added in and subtracted from the numerator of the fraction on the left, and $TR(x)$ represents total revenue at $x$. It follows from the definition of marginal revenue that

$$
\frac{p_x' x'' - p_x'' x''}{x' - x''} = -p_x' + MR,
$$

where $MR$ is the marginal revenue in moving between $x'$ and $x''$. Substituting the right-hand expression for the denominator in the fraction of (1) that is between the square brackets, and solving for $MR$, results in

$$
MR = p_x' \left(1 - \frac{1}{\varepsilon_{\alpha\beta}}\right).
$$