Background
Sound is a longitudinal wave in which the particles in the transmitting medium oscillate parallel to the direction of propagation of the wave. This results in alternating regions where the density is higher (compression) and lower (rarefaction) than average. The speed of propagation of the wave depends only on the properties of the medium, and not on the frequency of the wave. If the medium is air, the speed depends is given by $v = \sqrt{\gamma RT / M} \approx 20.1\sqrt{T}$ m/s, where $T$ is the absolute temperature on the Kelvin scale, $R$ the gas constant, $M$ the molar mass and $\gamma \approx 1.4$.

As with all waves, the frequency and wavelength of sound waves are related to the speed of propagation by $v = f \lambda$. We will determine the speed of propagation $v$ by using the interference of sound waves to measure the wavelength $\lambda$ for waves of known frequency $f$.

Speed of Sound in Air with a Resonating Air Column
If a glass tube is partially filled with water, a standing wave can be established in the air column above the water. In order to obtain a standing wave, the wave must have a displacement node at the closed end at the bottom of the air column. It must also have a displacement antinode at (or near) the open end at the top of the air column. (Recall that a displacement node is a location in the air column where the molecules composing the air are not displaced, whereas at the antinode the molecules are moving sinusoidally with maximum amplitude. Pressure nodes are located where the pressure does not change, and pressure antinodes are located where the pressure changes maximally. With simple arguments you can show that pressure nodes are located at displacement antinodes, and pressure antinodes are located at displacement nodes. Why do we expect a displacement node at the closed end of the air column? Why do we expect a displacement antinode (or equivalently, a pressure node) near the top of the air column?)

For any particular column length, we could set up a standing wave if we had a device that could generate standing waves of any frequency. We would just tune the frequency until we were generating sound waves with a displacement node at the bottom of the air column, and an antinode near the top. Lacking such a device, we will use tuning forks to generate a fixed frequency, then set up the conditions for a standing wave by adjusting the length of the air column. In this experiment, the length of the air column is adjusted by changing the height of the water reservoir.

The distance between an antinode and the nearest node is one-fourth of a wavelength, $L = \lambda/4$. This is the minimum length of the air column for a standing wave. Generally, the antinode is not exactly at the open end of the column, so we write $L + c = \lambda/4$ as the minimum length of the air column for a standing wave. The constant $c$ is a correction for the fact that the antinode is not exactly at the open end of the air column of length $L$. In general, we can have a standing wave if $L + c = \lambda/4$, or $3\lambda/4$, or $5\lambda/4$, etc.

If the frequency is held constant, and the column length is varied, the change of column length from one standing wave to the next is $\lambda/2$, allowing a measurement of wavelength.
Measurements

Strike a tuning fork over the air column above the water, with the water level near the top of the cylinder. Raise the water reservoir, increasing the length of the air column, until the sound is loudest. This is referred to as a resonance, and should be fairly sharp. Record the position of the water level at this resonance. Lengthen the air column further, until another resonance is reached. Record the water level. Repeat for as many resonances as you can obtain.

Repeat the above measurements several times, so that you have multiple determinations for each resonance. Record the air temperature using the Kelvin scale.

Calculations

For each resonance found, calculate the average measurement of the air column length, \( \langle L \rangle \). Also calculate the uncertainty in the average air column length for each resonance as the standard deviation of the individual measurements.

The air column length for each resonance is stated as \( \langle L \rangle \pm \sigma_L \) with appropriate units.

From your measurements of air column lengths, calculate:
- the wavelength and its uncertainty,
- the speed of sound and its uncertainty,
- the distance between the antinode and the end of the cylinder.
  (Is the antinode inside the cylinder or out?)

You may calculate the wavelength either graphically or numerically, but you must state why you think your method is the better of the two choices.

Some questions to consider:
(a) Would you expect sound to travel faster in pure helium or pure xenon? What ratio of velocities would you expect? (Assume that \( \gamma \) is comparable for the two gases)
(b) Is the speed of sound in air at a fixed temperature sensitive to the composition of air? Can you think of any applications of this effect?
(c) If there were severe budget cutbacks, and the class was shivering in an unheated lab at 0°C, by what fraction would the velocity of sound we measure have changed compared to a measurement at 20°C? How could we measure air temperature without a thermometer?