Chapter 14: Pronoun-Binding Revisited

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A. Adverbial Quantifiers

1. Introduction

David Lewis (1975) introduced what he calls *adverbs of quantification*, listing the following groups as primary examples.\(^\text{2}\)

1. always, invariably, universally, without exception
2. sometimes, occasionally
3. never
4. usually, mostly, generally, almost always, with few exceptions
5. often, frequently, commonly
6. seldom, infrequently, rarely, almost never

The basic idea is that these have quantificational readings, illustrated by the following equivalences.

1. dogs are *always* friendly = all dogs are friendly
2. dogs are *usually* friendly = most dogs are friendly
3. dogs are *never* friendly = no dogs are friendly
4. my dogs are *always* friendly ≠ all my dogs are friendly
5. some dogs are *always* friendly ≠ all some dogs are friendly
6. Penny is *always* friendly ≠ all Penny is friendly

We propose that whereas a *temporal-adverb* is a sentential-adverb, an *adverbial-quantifier* is a quantifier, which moreover operates scopally at the front of its containing clause. The latter is syntactically prohibited, however, if the clause is already headed by a determiner, as illustrated in the following.

7. dogs are *always* friendly ⇒ all dogs are friendly
8. some dogs are *always* friendly ⇒ all some dogs are friendly
9. Penny is *always* friendly ⇒ all Penny is friendly

The next obvious question is: what do adverbial-quantifiers quantify over? Lewis proposes that they quantify over *cases*, which he analyzes as follows.

A case may be regarded as the 'tuple of its participants; and these participants are values of the variables that occur free in the open sentence modified by the adverb.

This corresponds very closely to a *state-of-affairs*, which we define as follows.

A state-of-affairs is an item of the form,

\[
\Sigma \{ \alpha \mid \Phi \}
\]

where \(\Phi\) is a formula, and \(\alpha\) is a sequence of \(D\)-terms with or without case-markers.\(^\text{4}\)

Thus, a state-of-affairs is oftentimes an aggregate of all cases such that \(\Phi\).\(^\text{5}\)

With this in hand, we propose the following renderings of the standard adverbial-quantifiers.\(^\text{6}\)

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\(^\text{1}\) Circulated in 1972 under the title “Adverbs that Quantify”.

\(^\text{2}\) Lewis lists, in brackets, some other examples whose quantificational reading is less obvious.

\(^\text{3}\) These sentences nonetheless involve quantification. See later section for account.

\(^\text{4}\) Note that we include temporal-indices as items of type \(D\). See later examples. Also note carefully that \(\otimes\) and \(\times\) are interchangeable when the constituents have different types.

\(^\text{5}\) Sometimes, it is more complicated – when some of the \(D\)-terms have no free variables, which occurs when sigma-extraction if applied. In this connection, for these purposes, an expression of type \(\Sigma D\) counts as a \(D\)-term

\(^\text{6}\) The schematic type expression ‘\(D^*\)’ indicates a \(D\)-phrase with or without a case-marker.
always $\Sigma D^* \rightarrow \land D^*$  
$\Sigma \alpha \Phi \rightarrow \land \alpha \Phi$  
abbr: $\Sigma \rightarrow \land$  
sometimes $\Sigma D^* \rightarrow \lor D^*$  
$\Sigma \alpha \Phi \rightarrow \lor \alpha \Phi$  
abbr: $\Sigma \rightarrow \lor$  
never $\Sigma D^* \rightarrow \bigcirc D^*$  
$\Sigma \alpha \Phi \rightarrow \bigcirc \alpha \Phi$  
abbr: $\Sigma \rightarrow \bigcirc$

$\Sigma \alpha \Phi$ is a state-of-affairs.

Note that these correspond exactly to our (revised) accounts of ‘all’, ‘some’, and ‘no’.

By way of illustration, consider the following example.

10. dogs are always friendly

<table>
<thead>
<tr>
<th>dogs +1</th>
<th>always are friendly</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma { x \mid Dx } \Sigma \rightarrow \land$</td>
<td></td>
</tr>
<tr>
<td>$\land { x \mid Dx }$</td>
<td></td>
</tr>
<tr>
<td>$\land { Fx \mid Dx }$</td>
<td></td>
</tr>
<tr>
<td>$\forall x (Dx \rightarrow Fx)$</td>
<td></td>
</tr>
</tbody>
</table>

Note that, whereas ‘always’ is syntactically an adverb, it is semantically a quantifier, a discrepancy that occasionally requires adjustments to the left-right order in a sentence; for example, in the above derivation, ‘always’ is semantically relocated so as to be adjacent to ‘dogs’.

Note also that the state-of-affairs $\Sigma \{ x \mid Dx \}$ contains cases with only one participant each. The following example contains cases involving pairs of participants. It also does not require semantic relocation, since the adverbial-quantifier is already adjacent to the CNP.

11. owners of dogs never beat them

<table>
<thead>
<tr>
<th>owners</th>
<th>of dogs –1</th>
<th>+1</th>
<th>never beat them</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda y, \Sigma { x \mid Oxy }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma { y \mid x, y \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma { x \times y \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma { x \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma { y \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma { x \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lor { x \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lor { x \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lor { x \mid Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\neg \exists x \exists y { \exists Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\neg \exists x \exists y { \exists Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\neg \exists x \exists y { \exists Dx }$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

there is no $x$ and $y$ such that $y$ is a dog, and $x$ owns $y$, and $x$ beats $y$

Note that, ‘never’ automatically binds as many variables as it needs to.

2. Incorporating Time into Cases

According to Lewis, case-participants often include temporal-indices. This certainly makes sense given ordinary usage of the word ‘case’. For example, the following sentences involve temporal considerations.

12. cases of dog-owners beating their dogs are rare
13. cases of flu killing its victims are rare

So, if a dog-owner beats his dog on two separate occasions, these count (for forensic purposes) as two cases of dog-beating. Similarly, if a person survives two flu-infections, but dies from a later flu-infection, these episodes count (for epidemiological purposes) as three cases of flu.

---

7 Some transformations are OK. What we reject are transformations that involve traces.

8 Lewis posits a class of unselective quantifiers, i.e., quantifiers that bind multiple variables. Our quantifiers (junctions) have this feature automatically.

9 Surprisingly, even during the 1918 flu pandemic, the 50 million deaths were a small percentage of the total infections, which the CDC estimates to be 500 million!
Nevertheless, as Lewis points out, counting cases involving time is not completely trivial. Consider the following examples.

14. Jay rarely smokes
15. Jay rarely eats meat

The former says roughly that Jay spends little time smoking, and the latter says that Jay spends little time eating meat. Little time compared to what? All time? Jay’s life? Opportunities? We discuss this further in later sections.

In order to encode temporal-modifiers, we introduce the following notation.\(^{10}\)

\[ [\Phi/\tau] = \Phi \text{ at (time)} \tau \]

The syntactic rule is: any formula can be appended by a temporal index. The semantic rules include distribution and redundancy principles.\(^{11}\) Also, an unmodified sentence is understood to be eternal, for time-independent sentences, or "now" for time-dependent sentences.\(^{12}\)

Next, we interpret a time-dependent sentence of English as a common-noun over the domain of temporal-indices, which we officially notate as follows,

\[ \Sigma \{ \tau \mid \Phi/\tau \} \]

and which we abbreviate as follows.\(^{13}\)

\[ \Sigma[\Phi/\tau] \]

For example, ‘it is raining’ translates as follows.\(^{14}\)

\[ \Sigma[R/\tau] \]

The temporal-modifiers and their corresponding simplification rules apply mutatis mutandis to these forms, as in the following examples.

<table>
<thead>
<tr>
<th>always</th>
<th>it is raining</th>
<th>it is always raining</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \forall[\tau][R/\tau] )</td>
<td>( \Sigma \rightarrow \wedge )</td>
<td>( \Sigma \rightarrow \land )</td>
</tr>
<tr>
<td>( \wedge[R/\tau] )</td>
<td>( \Sigma[R/\tau] )</td>
<td>( \Sigma[R/\tau] )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sometimes</th>
<th>it is raining</th>
<th>it is sometimes raining</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \forall[\tau][R/\tau] )</td>
<td>( \Sigma \rightarrow \vee )</td>
<td>( \Sigma \rightarrow \lor )</td>
</tr>
<tr>
<td>( \vee[R/\tau] )</td>
<td>( \Sigma[R/\tau] )</td>
<td>( \Sigma[R/\tau] )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>never</th>
<th>it is raining</th>
<th>it is never raining</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \forall[\tau][R/\tau] )</td>
<td>( \Sigma \rightarrow \neg )</td>
<td>( \Sigma \rightarrow \neg )</td>
</tr>
<tr>
<td>( \neg \exists[\tau][R/\tau] )</td>
<td>( \Sigma[R/\tau] )</td>
<td>( \Sigma[R/\tau] )</td>
</tr>
</tbody>
</table>

In particular, the semantic rules are expanded to include the following simplification principles.\(^{15}\)

<table>
<thead>
<tr>
<th>always</th>
<th>it is raining</th>
<th>it is always raining</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \forall[\tau][\Phi/\tau] )</td>
<td>( \land[\Phi/\tau] )</td>
<td>( \wedge[\Phi/\tau] )</td>
</tr>
<tr>
<td>( \neg \forall[\tau][\Phi/\tau] )</td>
<td>( \forall[\tau][\Phi/\tau] )</td>
<td>( \neg \forall[\tau][\Phi/\tau] )</td>
</tr>
</tbody>
</table>

With this new machinery in hand, let's reconsider the following example.

\(^{10}\) More generally, we propose to append a series of indices, including time, place, degree, and possible world. Degree is employed in assessing comparatives.

\(^{11}\) See Appendix.

\(^{12}\) Apparently, this is not obvious to everyone. For example, once when he was asked what time it was, the legendary baseball player (and legendary inadvertent comedian) Yogi Berra is reported to have said “you mean now?”

\(^{13}\) Note that \( \Sigma \) only binds \( \tau \).

\(^{14}\) The evaluation of ‘it is raining’ (simpliciter) is based on the context, which includes a time of utterance (“now”). If “now” is a member of \( \{ \tau \mid R \} \), then “it is raining” is true; otherwise, it is false.

\(^{15}\) What counts as the domain of temporal-indices will vary with context. Usually, they are finite intervals of time, like minutes, hours, or days. The coarseness of the intervals will affect the truth-conditions for many sentences. For example, to say a person is always smoking/eating/sleeping/etc. is to say that, for any minute/hour/day/etc., that person is smoking/eating/sleeping/etc. every minute/hour/day/etc. One can smoke every day without smoking every minute.
Note first that ‘never’ can modify either ‘beat them’ or ‘owners of dogs’. If we treat ‘never’ as a temporal-adverb, modifying ‘beat them’, then we have the following derivation.

<table>
<thead>
<tr>
<th>owners</th>
<th>of dogs –1</th>
<th>+1</th>
<th>never</th>
<th>beat them</th>
</tr>
</thead>
</table>
| \[ \lambda y_6 \{ x \mid \text{Oxy} \} \] | \[ \{ y_6 \times y_1 \mid \text{Dy} \} \] | \[ +1 \] | \[ \text{never} \] | \[ \Sigma \rightarrow \Omega \{ \lambda y_1 \lambda x_1 \Sigma \text{Bxy/\tau} \} \]
| \[ \Sigma \{ x \mid \text{Oxy} \} \times y_1 \mid \text{Dy} \] \[ \Sigma \{ x \times y_1 \mid \text{Dy} \} \text{ & Oxy} \] | \[ \lambda x, x_1 \] | \[ \lambda y_1 \lambda x_1 \odot \text{Bxy/\tau} \] |

In this derivation, the generality is supplied by ‘never’, which promotes \( \Sigma \) to \( \Pi \). Alternatively, we can preëmptively insert a (tacit) generality-operator \( \text{GEN} \) as follows.

<table>
<thead>
<tr>
<th>[ \text{GEN} ]</th>
<th>owners</th>
<th>of dogs –1</th>
<th>+1</th>
<th>never</th>
<th>beat them</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \lambda y_6 { x \mid \text{Oxy} } ]</td>
<td>[ { y_6 \times y_1 \mid \text{Dy} } ]</td>
<td>[ \Sigma \rightarrow \Pi ]</td>
<td>[ \lambda y_1 \lambda x_1 \odot \text{Bxy/\tau} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \Pi { x \mid \text{Dy} } \text{ &amp; Oxy} ] [ { x \mid \text{Dy} } \text{ &amp; Oxy} ]</td>
<td>[ \lambda x, x_1 ]</td>
<td>[ \lambda y_1 \lambda x_1 \odot \text{Bxy/\tau} ]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can also treat ‘never’ as an adverbial-quantifier, modifying ‘owners of dogs’, which renders the \( \text{GEN} \)-operator otiose (and also odious!), as in the following derivation.

<table>
<thead>
<tr>
<th>owners</th>
<th>of dogs –1</th>
<th>+1</th>
<th>never</th>
<th>beat them</th>
</tr>
</thead>
</table>
| \[ \lambda y_6 \{ x \mid \text{Oxy} \} \] | \[ \{ y_6 \times y_1 \mid \text{Dy} \} \] | \[ +1 \] | \[ \text{never} \] | \[ \Sigma \rightarrow \Omega \{ \lambda y_1 \lambda x_1 \Sigma \text{Bxy/\tau} \} \]
| \[ \Sigma \{ x \mid \text{Oxy} \} \times y_1 \mid \text{Dy} \] \[ \Sigma \{ x \times y_1 \mid \text{Dy} \} \text{ & Oxy} \] | \[ \lambda x, x_1 \] | \[ \lambda y_1 \lambda x_1 \odot \text{Bxy/\tau} \] |
| \[ \Sigma \{ x_1 \times y_1 \mid \text{Dy} \} \text{ & Oxy} \] | \[ \Sigma \{ x_1 \times y_1 \mid \text{Dy} \} \text{ & Oxy} \] | \[ \Sigma \rightarrow \Omega \] | \[ \lambda y_1 \lambda x_1 \Sigma \text{Bxy/\tau} \] |

Notice that these three derivations all produce the same reading.

### 3. Events, Processes, States, Habits, Traits, and Types

Not all semantically-admissible readings of sentences containing adverbs, whether temporal or quantificational, are plausible. It is accordingly worthwhile to examine what makes the difference. Consider the difference between:

- smokes, walks, cooks, ...
- is smoking, is walking, is cooking, ...

If one says
Kay smokes, Kay walks, Kay cooks, one presumably means that Kay does these things habitually. But if one says Kay is smoking, Kay is walking, Kay is cooking, one presumably means that Kay is doing these things right now. Whereas ‘smokes’ denotes a property/state one has over long stretches of time, ‘is smoking’ denotes a property/state/activity that one participates in occasionally and perhaps repeatedly.

This distinction, between static-states and dynamic-states, influences our readings of indefinites. For example, sentences like

17. the owner of a dog smokes
18. the owner of a dog is smoking
don’t always agree on word-meaning [just like people!]

Besides the difference between static and dynamic states, there is also a distinction between states and events, and hence a difference between stative-verbs and eventive-verbs. Consider dog-training. First note that you can train your dog to do something (an event), and you can train your dog to be something (a state).

things a dog can do: sit, bark, shake hands, roll over, play dead
things a dog can be: quiet, attentive, affectionate, sitting, staying

Dogs can also be four-legged, warm-blooded, mammalian, and sentient, but humans rarely train their dogs to be these sorts of things. Rather, these are traits that a dog has purely in virtue of being a certain type of entity – in this case a (properly-functioning) dog.

A common problem with dog-training is that people and dogs don't always agree on word-meaning [just like people!] Consider the word ‘sit’. When you issue the command “sit!”, you probably understand sitting as a state, but your dog may understand it as an event. This makes sense to your dog; it sits down, then gets right back up, so it can do its next trick, perhaps even sitting again, in order to receive its next treat!

A similar problem hounds ‘stay’, although the problem is not ambiguity (event versus state) so much as vagueness. How long does a dog have to stay in order to qualify as staying? It seems that dogs and people have differing opinions about this.

By way of concluding this brief digression, it is important to note that, although natural languages track these distinctions, the underlying metaphysical distinctions don't seem so clear-cut. Rather, it seems that there is a spectrum, ranging from very short-lived states-of-affairs to very long-lived states-of-affairs, with (infinitely-) many degrees in between.

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16 Present indicative also has a tableau-reading. For example, each of these could be the title of a painting in a series depicting mundane Kay-activities. The term ‘tableau’ seems like a literary counterpart of ‘state of affairs’.

17 Things are complicated. Two points about ‘ing’ verbs.
1. If I ask you whether your toddler is talking/walking/reading yet, I mean the toddler's skill, not its current activity.
2. We must carefully distinguish standard English from African American English and Caribbean Patois. For example, ‘hey mon, we be jammin’ [a t-shirt I saw in the Bahamas in the 80’s. No, really! Google it!] says we jam as a matter of habit or trait or type, not we are currently jamming. **REFERENCE**

18 It makes the phrase ‘quit smoking’ ambiguous between putting out one's (say) cigarette, and putting out one's habit!

19 This is similar to the distinction between entity-properties and stage-properties. **REFERENCE**

20 Note the difference between static and stative; the former is a species of the latter.

21 A characteristic of eventive-verbs is that they admit the modifier ‘repeatedly’. On the other hand, a characteristic of stative-verbs is that they admit the modifier ‘forever’. Notice that a dog can sit repeatedly [eventive reading], and it can also sit forever [stative reading]. At least it is logically possible!

22 Not just dogs! In 2008, in response to rumors of an outside offer, the UMass Amherst basketball coach announced he was staying, but on the very next day, he announced he was leaving. He stayed for a day, but left forever!
4. Non-Standard (Adverbial) Quantifiers

Many adverbial-quantifiers correspond to standard quantifiers.

(1) always ↷ every
(2) occasionally ↷ some
(3) never ↷ no

Others correspond to non-standard quantifiers.

(4) frequently ↷ most
(5) rarely ↷ few

The latter are semantically rendered as follows, in a manner exactly parallel to the standard quantifiers.

<table>
<thead>
<tr>
<th>frequently</th>
<th>most</th>
<th>( \Sigma D^* \to MD^* )</th>
<th>( \Sigma \alpha \Phi \to M \alpha \Phi )</th>
<th>abbr: ( \Sigma \to M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>rarely</td>
<td>few</td>
<td>( \Sigma D^* \to \emptyset D^* )</td>
<td>( \Sigma \alpha \Phi \to F \alpha \Phi )</td>
<td>abbr: ( \Sigma \to F )</td>
</tr>
</tbody>
</table>

\( \Sigma \alpha \Phi \) is a state-of-affairs.

The new junctions \( M \) (most) and \( F \) (few) are governed by the following semantic rules.

| \( M \{ \psi | \Phi \} \) | \( \vdash \) | \( \{ \nu | \Phi \& \psi \} > \{ \nu | \Phi \& \neg \psi \} \) | > : is more than |
| \( M \{ \psi / \tau \} \) | \( \vdash \) | \( \{ \tau | \psi \} > \{ \tau | \neg \psi \} \) |
| \( F \{ \psi | \Phi \} \) | \( \vdash \) | \( \{ \nu | \Phi \& \psi \} << \{ \nu | \Phi \& \neg \psi \} \) | << : is much less than^2^3 |
| \( F \{ \psi / \tau \} \) | \( \vdash \) | \( \{ \tau | \psi \} << \{ \tau | \neg \psi \} \) |

\( \nu \) is a sequence all the variables C-free in \( \Phi \& \psi \).

[Note carefully that we later revise our account of ‘most’ and ‘few’.]

The following is an example.

19. owners of dogs rarely smoke

If we treat ‘rarely’ as a temporal-adverb, and we treat the indefinite as general, then we obtain the following derivation.\(^{24}\)

<table>
<thead>
<tr>
<th>GEN</th>
<th>owners of dogs +1 rarely smoke</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda y_y_0 \cdot SyDy )</td>
<td>( \Sigma \to F \lambda \chi_1 \Sigma [Sx/\tau] )</td>
</tr>
<tr>
<td>( \lambda y_0 \cdot SxOxy )</td>
<td>( \Sigma { y_0</td>
</tr>
<tr>
<td>( \Sigma { SxOxy</td>
<td>Dy } )</td>
</tr>
<tr>
<td>( \Pi { x_1</td>
<td>\exists y(Dy &amp; Oxy) } )</td>
</tr>
<tr>
<td>( \Pi { F[Sx/\tau] } )</td>
<td>( \forall x { \exists y(Dy &amp; Oxy) \to { \tau</td>
</tr>
</tbody>
</table>

\( \forall x : if \ x \ owns \ at \ least \ one \ dog, \ then \ the \ time \ x \ spends \ smoking \ is \ much \ less \ than \ time \ x \ spends \ not \ smoking \)

Note that an existential-reading of the indefinite is implausible given the static character of the VP. Compare this, however, to the following.

---

^23 Note that ‘much’ is vague, so pragmatic considerations are involved in evaluating it.

^24 Recall that ‘rarely’ is inherently vague/ambiguous based on how we measure time. If we measure in micro-seconds we get a different result than if we measure in days.
20. there are owners of dogs who rarely smoke

<table>
<thead>
<tr>
<th>owners</th>
<th>of dogs</th>
<th>who+1</th>
<th>rarely</th>
<th>smoke</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda y, x_0$</td>
<td>$\Sigma x_0 O x y$</td>
<td>$\lambda y, x_0$</td>
<td>$\Sigma y D y$</td>
<td>$\lambda x_1: \Sigma [S x/\tau]$</td>
</tr>
</tbody>
</table>

There are owners of dogs who rarely smoke

\[ \Sigma \{ x_0 \mid \exists y (D y & O x y) \} \]

\[ \lambda y, x_1 \]

\[ \lambda x_1 [S x/\tau] \]

Whereas 19 leans towards a general-reading, the opposite intuition applies to the following. The existential-reading is available, but not the general reading, given the dynamic character of the VP.

21. owners of dogs are smoking

<table>
<thead>
<tr>
<th>owners</th>
<th>of dogs</th>
<th>+1</th>
<th>are smoking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda y, x_0$</td>
<td>$\Sigma x_0 O x y$</td>
<td>$\lambda y, x_0$</td>
<td>$\Sigma y D y$</td>
</tr>
</tbody>
</table>

There are owners of dogs who rarely smoke

\[ \Sigma \{ x \mid \exists y (D y & O x y) \} \]

\[ \lambda x_1 x_1 \]

Next, we consider treating ‘rarely’ as an adverbial-quantifier. In this case, we can treat smoking as a habit (static-state), in which case we obtain the following derivation, in which ‘rarely’ serves as a quantifier.

<table>
<thead>
<tr>
<th>owners</th>
<th>of dogs</th>
<th>+1</th>
<th>rarely</th>
<th>smoke</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda y, x_0$</td>
<td>$\Sigma x_0 O x y$</td>
<td>$\lambda y, x_0$</td>
<td>$\Sigma y D y$</td>
<td>$\lambda x_1 x_1$</td>
</tr>
</tbody>
</table>

The number of dog-owners who smoke is much less than the number of dog-owners who don’t smoke.

5. Problems Counting Cases

According to Lewis, (adverbial) quantifiers quantify over cases, which are participant-tuples. In the previous example, we take the relevant participants to be dog-owners, described thus:

\{ x \mid x \text{ owns at least one dog} \}

But on a fairly straightforward reading of Lewis's account, the relevant participants are dog-owner pairs, described thus:

\{ x \otimes y \mid y \text{ is a dog, and } x \text{ owns } y \}

Does this influence the semantics of case-binding quantifiers? Yes! Consider the following example. If we pursue our analysis, we obtain the following derivation.
22. most owners of dogs are happy

<table>
<thead>
<tr>
<th>most</th>
<th>owners of</th>
<th>dogs</th>
<th>+1 are happy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda y, \Sigma x Oxy)</td>
<td>(\lambda y, y_6)</td>
<td>(\Sigma yDy)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma { x</td>
<td>\exists y(Dy &amp; Oxy) })</td>
<td>(\lambda x: x_1)</td>
<td></td>
</tr>
</tbody>
</table>

\[\Sigma \rightarrow M\]

\[M \{ x | \exists y(Dy & Oxy) \}\]

\[\lambda x: Hx\]

But if we take the Lewis proposal at face-value, we must count cases of the following sort.

\[\{ x \otimes y | y \text{ is a dog, and } x \text{ owns } y \}\]

in which case we have the following as our final evaluation.\(^{25}\)

\[\{ x \otimes y | D_y \& Oxy \& H_x \}\] > \[\{ x \otimes y | D_y \& Oxy \& \sim H_x \}\]

the number of dog-owner pairs such that the owner is happy

is greater than

the number of dog-owner pairs such that the owner isn't happy

The difference between these two readings is subtle but critical. The first one measures all dog-owners equally; the second one measures dog-owners according to how many dogs they own. To see the difference, suppose there are 10 dog-owners.

Jay, Kay\(_1\), ..., Kay\(_9\)

Suppose the 9 Kays are happy, but Jay is not; in this case most dog-owners are happy. And according to the first reading, the sentence is true, since 9 is greater than 1. Now, further suppose that Jay owns 10 dogs, and the 9 Kays own one dog each. This has no effect on the first reading, but according to the second reading, the sentence is false, since 9 is not greater than 10.

We accordingly have a counter-example to Lewis's account.

Nevertheless, it is not a counter-example to the basic idea that quantifiers quantify over cases, but perhaps only a simple-minded account as to which cases are relevant. In particular, according to our account, cases don't attach \textit{willy-nilly}\(^{26}\) to semantic derivations, but are rather \textit{constructed by} (and \textit{within}) semantic derivations as parts of states-of-affairs.

The question then is whether the cases

\[\{ x \otimes y | y \text{ is a dog, and } x \text{ owns } y \}\]

can be constructed \textit{inside} our account. Although we have no proof that no construction is possible, we have made any efforts and failed, and accordingly surmise that we cannot construct the dyadic cases needed to produce the counterexample.

But what about examples in which we \textit{can} construct polyadic cases, such as the following?

23. most owners of dogs feed them

If we treat dog-feeding as a habit (static state), then we have the following derivation.

---

\(^{25}\) This problem was originally examined in detail by Nerit Kadmon (1987).

\(^{26}\) This is a rare usage of 'willy-nilly' that plays on both its original ('literal') meaning and its derivative ('vulgar') meaning. The original meaning is “whether one wishes to or not”; the derivative meaning is “in a disorganized or unplanned manner; sloppily”.

most owners of dogs feed them.

\[ \lambda y. \Sigma x Oxy \]
\[ \Sigma \{ y \times y, y \mid D_y \} \]
\[ \lambda y, \lambda x, \lambda x Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
\[ \Sigma \{ x \times y, y \mid D_y \} \]
\[ \lambda x, \lambda x, \lambda Fxy \]
24. most owners of dogs feed them

<table>
<thead>
<tr>
<th>most owners of dogs</th>
<th>-1</th>
<th>+1 feed (–1) them</th>
</tr>
</thead>
<tbody>
<tr>
<td>λy6 ΣxOxy</td>
<td></td>
<td>λx.x1</td>
</tr>
<tr>
<td>Σ{ x × y1</td>
<td>Dy &amp; Oxy }</td>
<td></td>
</tr>
<tr>
<td>Σ{ x × Σy(Dy &amp; Oxy),1</td>
<td>y</td>
<td>Σy(Dy &amp; Oxy) }</td>
</tr>
<tr>
<td>Σ{ x × Σy(Dy &amp; Oxy),1</td>
<td>y</td>
<td>Σy(Dy &amp; Oxy) }</td>
</tr>
</tbody>
</table>

Σ→M

M{ x1 × Σy(Dy & Oxy),1 | Σy(Dy & Oxy) } λy6,λx,Fxy

M{ F(x, Σy(Dy & Oxy)) | Σy(Dy & Oxy) }]

{ x | Σy(Dy & Oxy) & F[x, Σy(Dy & Oxy)] } >

{ x | Σy(Dy & Oxy) & ~F[x, Σy(Dy & Oxy)] } }

the number of individuals who own dogs and feed the dogs they own is greater than the number of individuals who own dogs and don’t feed the dogs they own

For plural nouns, ΣxFx = 1xFx.

Compare this derivation with the following, which fails.

The following derivation, which applies sigma-extraction to ‘x’ rather than ‘y’, also fails.

The following derivation, which applies sigma-extraction to ‘x’ rather than ‘y’, also fails.

This runs afoot of our theta-marking restriction, introduced in the previous chapter, which prohibits applying a theta-marker to a sigma-extracted phrase.

8. Examples with Definite Descriptions

Delia Graff Fara (2006) examines several examples of adverbial-quantifiers involving definite descriptions, including examples like the following.27

25. the owner of a dog rarely smokes

Key to her account is her claim that, contrary to orthodox opinion, definite-descriptions are indefinite-noun-phrases.

27 Her example: (26) The owner of an espresso machine rarely goes to bed early. She does examples involving smoking, but no examples involving dogs.
Notice, in particular, that if we take ‘the’ to be a determiner, then ‘rarely’ cannot also serve as a determiner modifying ‘owner of a dog’, since this role is already filled by ‘the’.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{the} & \text{owner} & \text{of a dog} & +1 \text{ rarely} & \text{smokes} \\
\hline
\lambda y_0 \Sigma xO \! x \! y & \lambda y_0 \Sigma y \! D y & \lambda x_1 \Sigma x_1 & \Sigma_+ \{ y_0 | Dy \} \\
\Sigma x P_\pi \rightarrow 1 \times P_\pi & \Sigma_+ \{ \Sigma xO \! x \! y | Dy \} & \Sigma x_1 \exists y \{Dy \& O \! x \! y\} & \Sigma_+ \{\} \\
\hline
1 \times \exists y \{Dy \& O \! x \! y\} & \Sigma_+ \{\} & \lambda x_1 \Sigma[Sx/\tau] \\
\hline
\end{array}
\]

But, if the phrase ‘the…’ is itself indefinite, then ‘rarely’ can modify this phrase, as in the following schema.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{the} & \text{owner} & \text{of a dog} & +1 \text{ rarely} & \text{smokes} \\
\hline
\Sigma_+ \{ v_1 \mid \Phi \} & \Sigma_+ \{\} & \lambda x_1 \Sigma[Sx/\tau] & \Sigma_+ \{\} \\
F_\pi \{ v_1 \mid \Phi \} & \lambda x_1 \Sigma[Sx/\tau] & \lambda x_1 \Sigma[Sx/\tau] & \Sigma_+ \{\} \\
F \{ \Sigma[Sx/\tau] \mid \Phi \} & \Sigma_+ \{\} & \Sigma_+ \{\} & \Sigma_+ \{\} \\
\hline
\end{array}
\]

Since Graff Fara’s approach is a key part of our approach, it is useful to see how our account reproduces her results. The main thing the reader will notice is that examples involving definite descriptions look just like examples involving simple indefinites; the only difference is the presence of the !-operator, which is part of our account of (Graff’s account of) ‘the’.

She proposes that the sentence

26. the owner of a dog rarely smokes

has three readings.

1. **existential** some sole owner of a dog rarely smokes
2. **general** in general, sole owners of dogs rarely smoke
3. **adverbial-Q** rare is the dog-owner who smokes

1. **existential**

This reading seems implausible, since ‘rarely smokes’ is a static-VP. See later for an example in which the existential reading is plausible.

2. **general**

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{the} & \text{owner} & \text{of a dog} & +1 & \text{rarely} & \text{smokes} \\
\hline
P \rightarrow P_! \lambda y_0 \Sigma xO \! x \! y & \lambda y_0 \Sigma y \! D y & \Sigma_+ \{ y_0 | Dy \} & \Sigma_+ \{\} & \lambda x_1 \Sigma[Sx/\tau] \\
\Sigma \rightarrow \Pi & \Sigma_+ \{ \Sigma xO \! x \! y | Dy \} & \Sigma \{ x \mid \exists y(Dy \& O \! x \! y) \} & \lambda x_1 \Sigma[Sx/\tau] & \Sigma_+ \{\} \\
\Pi_\pi \{ y_0 \mid Dy \& O \! x \! y \} & \lambda x_1 \Pi \{\} & \lambda x_1 \Pi \{\} & \lambda x_1 \Pi \{\} & \lambda x_1 \Pi \{\} \\
\hline
\end{array}
\]

\[\forall x \{ y_0(Dy \& O \! x \! y) \rightarrow \{ x \mid Sx/\tau \} <\{ x \mid Sx/\tau \} \} \]

\[\forall x : \text{if } x \text{ is the sole-owner of at least one dog, then the time } x \text{ spends smoking is much less than the time } x \text{ spends not smoking}\]

---

28 Indeed, inspiration!

29 Recall that our view on ‘the’ is intended to subsume (and reconcile) the three major views: Russell, Strawson, Predicative (Graff Fara).
3. adverbial-Q

<table>
<thead>
<tr>
<th>the</th>
<th>owner</th>
<th>of</th>
<th>a dog</th>
<th>+1</th>
<th>rarely</th>
<th>smokes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P→P!</td>
<td>λy6 ΣxOxy</td>
<td>λy6y6</td>
<td>ΣyDy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λy6 ΣxO!xy</td>
<td>Σ{ y6</td>
<td>Dy }</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Σ{ ΣxO!xy | Dy } | Σ{ x | ∃y(Dy & O!xy) } |
|----------|-----------------|
| λx,x1    |                  |

| Σ( y6 | Dy & O!xy ) |
|--------|
| λx,x1,Sx |

| Σ{ Sx | ∃y(Dy & O!xy) } |
|--------------------|
| λx,x1               |

| { x | ∃y(Dy & O!xy) & Sx } << { x | ∃y(Dy & O!xy) & ~Sx } |
|------------------|
| the number of individuals who (solely) own dogs and who smoke |
| is much less than |
| the number of individuals who (solely) own dogs and who do not smoke |

Earlier we rejected the existential-reading of this sentence. By contrast, the following readily admit an existential reading.

27. the owner of a dog is smoking
28. the owner of a dog is beating it

Let's concentrate on the second one. Suppose you hear a dog yelping, and wonder what is happening; the sentence above could be used to answer your question. It accordingly seems to be a claim about a current happening, which can be semantically analyzed as follows.

<table>
<thead>
<tr>
<th>the</th>
<th>owner</th>
<th>of</th>
<th>a dog</th>
<th>-1</th>
<th>+1</th>
<th>is beating (-1) it</th>
</tr>
</thead>
<tbody>
<tr>
<td>P→P!</td>
<td>λy6 ΣxOxy</td>
<td>λy6y6</td>
<td>ΣyDy</td>
<td>λy6y4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>λy6 ΣxO!xy</td>
<td>Σ{ y6 × y4</td>
<td>Dy }</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Σ{ ΣxO!xy × y4 | Dy } | Σ{ x × y4 | Dy & O!xy } |
|-----------------|-----------------|
| λx;x4          | λy4,λx4,Bxy |

| Σ{ Bxy | Dy & O!xy } |
|-----------------|
| Σ{ Bxy | τ } |

| ΣΠ{ Bxy/τ | Dy & O!xy } |
|-----------------|
| ∀x∀y{ Dy & O!xy | ~Bxy/τ } |

there is a dog whose (sole) owner is beating it (right now)

On the other hand, the following seems like a general claim about dogs and their owners.

29. the owner of a dog rarely beats it

<table>
<thead>
<tr>
<th>[GEN]</th>
<th>the</th>
<th>owner</th>
<th>of</th>
<th>a dog</th>
<th>-1</th>
<th>+1</th>
<th>rarely</th>
<th>beats (-1) it</th>
</tr>
</thead>
<tbody>
<tr>
<td>P→P!</td>
<td>λy6 ΣxOxy</td>
<td>λy6y6</td>
<td>ΣyDy</td>
<td>λy6y4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Σ{ x1 × y4 | Dy & O!xy } |
|----------------|
| λy4,λx4,Bxy |

We can also treat ‘rarely’ as an adverbial-quantifier, in which case ‘beats it’ is taken as a static-VP, as in the following derivation.
the owner of a dog rarely beats it

\[ P \to P! \quad \lambda y : \Sigma x O x y \quad \lambda y : y = \Sigma y D y \quad \lambda y : y \cdot 1 \quad \lambda x : x = 1 \quad \Sigma \to F \]

\[ F \{ x_1 \times y \mid D y \cdot O ! x y \} \]

\[ \lambda y : y \cdot 1 \quad \lambda x : x = 1 \quad B x y \]

Note that sigma-extraction \( \otimes \) is critical. Compare with the following.

9. How If-Clauses Interact with (Adverbial) Quantifiers

As noted in Chapter 8 [Pronouns], according to Lewis, when if-clauses are embedded in quantifier constructions, they behave as domain restrictors, which is based on the work of Belnap (1972) on Conditional Assertion. We incorporate this idea into our semantic theory, proposing that when a conditional is affiliated with a quantifier, it serves as a domain-restrictor, which means it is interpreted as conditional-assertion.

For this purpose, we introduce a two-place connective "slash" as follows.

\[ \text{if } [\text{CA}] \quad S \to (S \to S) \quad \lambda P \lambda Q [Q/P] \]

The logic of slash follows Belnap.\(^{30} \) To wit:

\[ A / B = A \quad \text{if } B \text{ is true;} \]
\[ A / B = \otimes \quad \text{if } B \text{ is false.} \]

In other words, \( A / B \) says \( A \), if \( A \) is true, but says nothing whatsoever if \( A \) is false.

The real value of slash arises in connection with the following.\(^{31} \)

\[ \text{Conditionalization Rule} \]

\[ \text{\( \text{\vdash} \{ \Omega / \Psi \mid \Phi \} \quad \text{iff} \quad \text{\( \text{\vdash} \{ \Omega / \Phi \& \Psi \} \)}} \]

\( \text{\( \Phi, \Psi, \Omega \) are formulas.} \)

By way of illustrating, we start by redoing examples with standard quantifiers.

---

\(^{30}\) Note that Belnap reads slash backwards from us. We follow conditional-probability notation.

\(^{31}\) This rule is derivable, based on treating \( A / B \) as conditional assertion \([B \text{ given } A]\).
30. every man is happy if he is virtuous

\[
\forall x \left( x \in M \implies H x \right) \\
\land \{ H x \times \lambda Q' x | M x \} \\
= \{ H x | M x \} \\
\forall x \{ M x & V x \rightarrow H x \} \\
\]

every man who is virtuous is happy

31. no man is happy if he is evil

\[
\forall x \left( x \in M \land E x \implies H x \right) \\
\land \{ H x \times \lambda Q E x | M x \} \\
= \{ H x | M x \} \\
\forall x \{ M x & E x \land H x \} \\
\land \{ H x | M x & V x \} \\
\forall x \{ M x & E x \rightarrow H x \} \\
\]

no man who is evil is happy

Next, we consider non-standard quantifiers.

32. most men are happy if they are virtuous

\[
\exists x \left( x \in M \land V x \implies H x \right) \\
\land \{ H x \times \lambda Q V x | M x \} \\
= \{ H x | M x \} \\
\forall x \{ M x & V x \land H x \} > \{ x | M x & V x & \neg H x \} \\
\]

the number of virtuous men who are happy is greater than the number of virtuous men who are not happy

33. few men are happy if they are evil

\[
\forall x \left( x \in M \land E x \implies H x \right) \\
\land \{ H x \times \lambda Q E x | M x \} \\
= \{ H x | M x \} \\
\forall x \{ M x & E x \land \neg H x \} \\
\land \{ H x | M x & V x \} \\
\forall x \{ M x & E x \rightarrow \neg H x \} \\
\]

the number of evil men who are happy is much less than the number of evil men who are not happy

Finally, we consider an example involving an adverbial-quantifier.
34. always, if a man owns a dog, he feeds it

<table>
<thead>
<tr>
<th>always</th>
<th>if</th>
<th>a man +1 –1</th>
<th>owns</th>
<th>a dog +2 –2</th>
<th>(–1) he feeds (–2) it</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝜅PQ/P</td>
<td>𝜅</td>
<td>𝜅 y, y x O</td>
<td>𝜅</td>
<td>y y x O y y</td>
<td>𝜅 y, y x F</td>
</tr>
</tbody>
</table>

\[ \Sigma \in \Lambda \]

\[ \Sigma \{ \lambda Q(Oxy) \times y y y | y y M y & D y \} \]

35. mostly, if a man owns a dog, he feeds it

<table>
<thead>
<tr>
<th>mostly</th>
<th>if</th>
<th>a man +1 –1</th>
<th>owns</th>
<th>a dog +2 –2</th>
<th>(–1) he feeds (–2) it</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝜅PQ/P</td>
<td>𝜅</td>
<td>𝜅 y, y x O</td>
<td>𝜅</td>
<td>y y x O y y</td>
<td>𝜅 y, y x F</td>
</tr>
</tbody>
</table>

\[ \Sigma \in \Lambda \]

\[ \Sigma \{ \lambda Q(Oxy) \times y y y | y y M y & D y \} \]

\[ \Sigma \{ F B y | M y & D y & O y \} \]

\[ \{ x | y y M y & \exists y (D y & O y) \} \]

\[ \{ x | y y M y & \exists y (D y & O y) & ~ B [ x, y y D y & O y ] \} \]

\[ \{ x | y y M y & \exists y (D y & O y) & ~ B [ x, y y D y & O y ] \} \]

Whereas \( \Lambda \) binds any number of variables, \( M \) can only bind one variable. So, in order to salvage this derivation, we must employ sigma-extraction, but two different extractions are admissible.

extract ‘a dog’ [‘a man’ wide]

<table>
<thead>
<tr>
<th>mostly</th>
<th>if</th>
<th>a man +1 –1</th>
<th>owns</th>
<th>a dog +2 –2</th>
<th>(–1) he beats (–2) it</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝜅PQ/P</td>
<td>𝜅</td>
<td>𝜅 y, y x O</td>
<td>𝜅</td>
<td>y y x O y y</td>
<td>𝜅 y, y x B</td>
</tr>
</tbody>
</table>

\[ \Sigma \in \Lambda \]

\[ \Sigma \{ B y y | M y & D y & O y \} \]

\[ \Sigma \{ B [ x, y y (D y & O y) ] | M y & \exists y (D y & O y) \} \]

\[ \{ x | y y M y & \exists y (D y & O y) & B [ x, y y D y & O y ] \} \]

\[ \{ x | y y M y & \exists y (D y & O y) & ~ B [ x, y y D y & O y ] \} \]

the number of men who own dogs and the dogs they own is greater than

\[ \Sigma \{ B [ x, y (M x & O y) ] , y ] | D y & \exists x (M x & O y) \} \]

\[ \{ y | D y & \exists x (M x & O y) & B [ x, y y (M x & O y) ] , y ] \} \]

\[ \{ y | D y & \exists x (M x & O y) & ~ B [ x, y y (M x & O y) ] , y ] \} \]

extract ‘a man’ [‘a dog’ wide]

<table>
<thead>
<tr>
<th>mostly</th>
<th>if</th>
<th>a man +1 –1</th>
<th>owns</th>
<th>a dog +2 –2</th>
<th>(–1) he beats (–2) it</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝜅PQ/P</td>
<td>𝜅</td>
<td>𝜅 y, y x O</td>
<td>𝜅</td>
<td>y y x O y y</td>
<td>𝜅 y, y x B</td>
</tr>
</tbody>
</table>

\[ \Sigma \in \Lambda \]

\[ \Sigma \{ B y y | M y & D y & O y \} \]

\[ \Sigma \{ B [ x, y y (M x & O y) ] , y ] | D y & \exists x (M x & O y) \} \]

\[ \{ y | D y & \exists x (M x & O y) & B [ x, y y (M x & O y) ] , y ] \} \]

\[ \{ y | D y & \exists x (M x & O y) & ~ B [ x, y y (M x & O y) ] , y ] \} \]

the number of dogs who are owned by men and beaten by the men who own them

\[ \Sigma \{ B [ x, y (M x & O y) ] , y ] | D y & \exists x (M x & O y) \} \]

\[ \{ y | D y & \exists x (M x & O y) & B [ x, y y (M x & O y) ] , y ] \} \]

\[ \{ y | D y & \exists x (M x & O y) & ~ B [ x, y y (M x & O y) ] , y ] \} \]

the number of dogs who are owned by men but not beaten by the men who own them

10. Another Anomaly Involving Most

The quantifiers ‘most’ and ‘few’ continue to be vexing. Consider the following example, which we analyze in a manner exactly parallel to the previous one.
Unlike examples in which ‘man’ or ‘dog’ is made prominent by theta-marking, this example does not clearly favor one over the other. We accordingly propose that both readings are admissible, that this is just another example of compositional ambiguity.

### Bi-Conditionalization

11. Bi-Conditionalization

According to our account above, which is based on Lewis, which is based on Belnap, ‘if’ serves as a domain-restrictor. Can we do the same with ‘if and only if’, said really fast, and sometimes spelled ‘iff’, and thought of as an idiomatic-unit. For this purpose, we introduce a bi-slash operator.

\[
\text{iff } S \rightarrow S \rightarrow S \lambda P \lambda Q [Q || P]
\]

The logic of double-slash is similar to slash. To wit:

\[
A \parallel B \text{ is true if } A \text{ and } B \text{ are true;}
A \parallel B = \emptyset \text{ otherwise.}
\]

In other words, A \parallel B says A&B, if A and B are both true, but says nothing whatsoever if either is false. Because of this, double-slash also has the following.

### Bi-Conditionalization Rule

\[
\text{Bi-Conditionalization Rule}
\]

\[
\text{倘 } \{ \Omega || \Psi | \Phi \} \text{ 与 } \text{文 } \{ \Psi | \Phi & \Omega \}
\]

\[
\text{倘 is any junction; } \Phi, \Psi, \Omega \text{ are formulas.}
\]
The following simple example illustrates.

36. a dog is happy if and only if it is well-fed

<table>
<thead>
<tr>
<th>a dog</th>
<th>+1</th>
<th>–1</th>
<th>is happy</th>
<th>iff</th>
<th>(–1) it is well-fed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma { x_1 \times x_1 \mid Dx } )</td>
<td>( \lambda x_1 Hx )</td>
<td>( \lambda P\lambda Q[Q|P] )</td>
<td>( \lambda x_1 Wx )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma { Hx \times x_1 \mid Dx } )</td>
<td>( \lambda x_1 \lambda Q[Q | Wx] )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\Pi \{ Hx \times \lambda Q[Q \| Wx] \mid Dx \} \\
\Pi \{ Hx \mid Wx \mid Dr \} \\
\Pi \{ Hx \mid Dx \& Wx \} \land \Pi \{ Wx \mid Dx \& Hx \} \\
\forall x \{ Dx \& Wx \rightarrow Hx \} \land \forall x \{ Dx \& Hx \rightarrow Wx \}
\]

\[
\forall x \{ Dx \& Wx . \rightarrow Hx \} \land \forall x \{ Dx \& Hx . \rightarrow Wx \}
\]

12. The Former and The Latter

The expressions ‘the former’ and ‘the latter’ can be understood anaphorically,\(^{33}\) the idea being that the former latches onto an earlier NP, and the latter latches onto a later NP. For example, in the last sentence, ‘the former’ latches onto “the former”, and “the latter” latches onto “the latter”\(^{34}\). Consider the following example.

37. if a man owns a dog, then the former feeds the latter

As it turns out, this can be analyzed quite simply using current semantic resources, including ordered-pairs.

<table>
<thead>
<tr>
<th>the-former</th>
<th>D⊙D → D</th>
<th>x⊙y → x</th>
</tr>
</thead>
<tbody>
<tr>
<td>the-latter</td>
<td>D⊙D → D</td>
<td>x⊙y → y</td>
</tr>
</tbody>
</table>

In other words, ‘the former’ takes an ordered-pair of entities and extracts the first entity, and ‘the latter’ takes an ordered-pair of entities and extracts the second entity

\[
\Sigma \rightarrow \Pi \{ x \times \mid Mx \& Dy \& Ox y \} \\
\Pi \{ x \times y \mid Mx \& Dy \& Ox y \} \\
\forall x \forall y \{ Mx \& Dy \& Ox y . \rightarrow Fxy \}
\]

---

\(^{33}\) They can also be used in connection with demonstratives. If one makes two demonstrations, then ‘the former’ can refer to the first demonstrated object, and ‘the latter’ can refer to the second demonstrated object.

\(^{34}\) Note the double application of single-quotes. See Appendix on Use-Mention.
B. Appendices

1. Appendix 1 – Summary of Pronoun-Binding

1. Alpha-Pronouns

Alpha-pronouns are essentially-anaphoric pronouns. According to our account, alpha-pronouns create alpha-roles. They include third-person pronouns ['he', 'she', 'it', 'they'], which are rendered as follows.

\[(\alpha) \ e \ D \alpha \rightarrow D \lambda x_{\alpha}:x\]

Here, \(e\) is a third-person pronoun, and \(\alpha\) is a negative-integer. Note the parentheses.

They also include anaphoric-descriptions of the form ‘the C’, where C is a common-noun-phrase, and are rendered as follows,

\[(\alpha) \text{ the } C \ D \alpha \rightarrow D \lambda x_{\alpha}/Cx/x\]

where the bracketed material corresponds to function-restriction.

2. Binding

Alpha-pronouns are bound by their antecedents, which accomplish this by being marked by the corresponding role-marking morpheme, \(\alpha D \alpha \rightarrow D \lambda x_{\alpha}:x\), which behaves exactly like a case-marking morpheme. Alpha-marked antecedents bind alpha-pronouns by filling the alpha-roles they create, which is done the same way theta-marked NPs fill theta-roles – by function-application.

3. Full-Stop

Full-stop (\(\bullet\)) is a morpheme, whose semantic function is to force calculation to type-S (\(\Rightarrow S\)).

If the expression does not simplify to a closed sentence, then the computation fails tout court.

4. Alpha-Marking is Restricted

Alpha-marking can only be applied to simple NPs. Non-simple NPs include:

- exactly N C
- at most N C
- only C

Here \(N\) is a counting number, and \(C\) is a common-noun-phrase.

5. Quantifier-Scope Restriction

The scope of a quantifier must go at least to its maximal theta-marker.

The maximal theta-marker of a phrase \(\phi\) is the theta-marker attached to the maximal-NP that contains \(\phi\).

---

35 More generally, an anaphoric pronoun is any pronoun that is not exophoric [indexical, demonstrative].
2. Appendix 2 – Summary of New Semantic Machinery

1. Duality

We have two interlocking duality principles.\(^\text{36}\) The first one connects C-phrases \([\text{type } D_0 \rightarrow S]\) and entity-sums \([\text{type } \Sigma D]\). It is founded on the intimate connection between sets and their characteristic-functions, and is formalized as follows.

\[
\lambda \nu \Phi \quad \| \quad \Sigma \nu \Phi
\]

Here, \(\nu\) is an entity-variable, and \(\Phi\) is a formula.

The second duality connects existential-claims with sums, equating an existential-claim with the collection of all its witnesses. It is formalized as follows.

\[
\exists \nu \Phi \quad \| \quad \Sigma \nu \Phi
\]

Here, \(\nu\) is a sequence of entity-variables.

2. States of Affairs (Collections of Cases)

A state-of-affairs is an item of the form

\[
\Sigma \{ \nu \mid \Phi \}
\]

where \(\nu\) is a sequence of \(D\)-terms, with or without case-markers.

3. Standard Quantifiers are Expanded

The standard quantifiers – ‘every’, ‘some’, and ‘no’ – are expanded as follows.

<table>
<thead>
<tr>
<th>Quantifier</th>
<th>Formula</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>every</td>
<td>(\Sigma D^* \rightarrow \land D^*)</td>
<td>(\Sigma \land \Phi)</td>
</tr>
<tr>
<td>some</td>
<td>(\Sigma D^* \rightarrow \lor D^*)</td>
<td>(\Sigma \lor \Phi)</td>
</tr>
<tr>
<td>no</td>
<td>(\Sigma D^* \rightarrow \ominus D^*)</td>
<td>(\Sigma \ominus \Phi)</td>
</tr>
</tbody>
</table>

\(\Sigma \nu \Phi\) is a state-of-affairs

4. Non-Standard Quantifiers

We consider the following subset of non-standard quantifiers.

<table>
<thead>
<tr>
<th>Quantifier</th>
<th>Formula</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>most</td>
<td>(\Sigma D^* \rightarrow \exists D^*)</td>
<td>(\Sigma \exists \Phi)</td>
</tr>
<tr>
<td>few</td>
<td>(\Sigma D^* \rightarrow \forall D^*)</td>
<td>(\Sigma \forall \Phi)</td>
</tr>
</tbody>
</table>

\(\Sigma \nu \Phi\) is a state-of-affairs

\(\{ \Psi \mid \Phi \}\) \(\leftarrow\) \(\{ \nu \mid \Phi \land \Psi \}\) > \(\{ \nu \mid \Phi \land \sim \Psi \}\) > : is more than

\(\{ \Psi / \tau \}\) \(\leftarrow\) \(\{ \tau \mid \Psi \}\) > \(\{ \tau \mid \sim \Psi \}\)

\(\{ \Psi \mid \Phi \}\) \(\leftarrow\) \(\{ \nu \mid \Phi \land \Psi \}\) << \(\{ \nu \mid \Phi \land \sim \Psi \}\) << : is much less than

\(\{ \Psi / \tau \}\) \(\leftarrow\) \(\{ \tau \mid \Psi \}\) << \(\{ \tau \mid \sim \Psi \}\)

\(\nu\) is the only variable \(C\)-free in \(\Phi\)

how "much" is ‘much’ is context-dependent

\(^{36}\) Or a single "triality" principle.

\(^{37}\) The expression ‘\(D^*\)’ is intended to convey that the items may or may not have a case-marker attached.
5. **Standard Adverbial-Quantifiers**

The adverbial-quantifiers mimic quantifiers.

<table>
<thead>
<tr>
<th></th>
<th>Always</th>
<th>Every</th>
<th>$\Sigma D^* \rightarrow \land D^*$</th>
<th>$\Sigma \alpha \Phi \rightarrow \land \alpha \Phi$</th>
<th>abbr: $\Sigma \rightarrow \land$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sometimes</td>
<td>Some</td>
<td>$\Sigma D^* \rightarrow \lor D^*$</td>
<td>$\Sigma \alpha \Phi \rightarrow \lor \alpha \Phi$</td>
<td>abbr: $\Sigma \rightarrow \lor$</td>
<td></td>
</tr>
<tr>
<td>Never</td>
<td>No</td>
<td>$\Sigma D^* \rightarrow \circ D^*$</td>
<td>$\Sigma \alpha \Phi \rightarrow \circ \alpha \Phi$</td>
<td>abbr: $\Sigma \rightarrow \circ$</td>
<td></td>
</tr>
</tbody>
</table>

- frequently | most | $\Sigma D^* \rightarrow \lor \circ D^*$ | $\Sigma \alpha \Phi \rightarrow \lor \circ \alpha \Phi$ |
- rarely     | few  | $\Sigma D^* \rightarrow \lor \circ \circ D^*$ | $\Sigma \alpha \Phi \rightarrow \lor \circ \circ \alpha \Phi$ |

$\Sigma \nu \Phi$ is a state-of-affairs

6. **Rules for Indexical-Adverbs**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lnot [\Phi/i]$</td>
<td>$\lnot [\Phi/i]$</td>
</tr>
<tr>
<td>$(\Phi \star \Psi)/i$</td>
<td>$[\Phi/i] \star [\Psi/i]$ $\star$ is a two-place connective</td>
</tr>
<tr>
<td>$[Q \nu \Phi]/i$</td>
<td>$Q \nu [\Phi/i]$ $Q$ is a quantifier</td>
</tr>
<tr>
<td>$[\Phi/i]/j$</td>
<td>$\Phi/j$ redundancy</td>
</tr>
<tr>
<td>$\land [\Phi/i]$</td>
<td>$\forall i[\Phi/i]$</td>
</tr>
<tr>
<td>$\lor [\Phi/i]$</td>
<td>$\exists i[\Phi/i]$ simplification rules</td>
</tr>
<tr>
<td>$\circ [\Phi/i]$</td>
<td>$\lnot \exists i[\Phi/i]$</td>
</tr>
</tbody>
</table>

7. **Sigma-Extraction**

$$\Sigma \{ \alpha[v] \mid \beta[\Phi] \} \vdash \Sigma \{ \alpha[\Sigma v \Phi/v] \mid \beta[\exists v \Phi] \}$$

Here, $\alpha[v]$ is an expression in which $v$ occurs free.
$\beta[\Phi]$ is an expression in which $\Phi$ occurs, and every occurrence of $v$ that is free in $\beta[\Phi]$ is in $\Phi$.
$\alpha[\Sigma v \Phi/v]$ is the result of replacing each free occurrence of $v$ by $\Sigma v \Phi$.
$\beta[\exists v \Phi]$ is the result of replace $\Phi$ by $\exists v \Phi$.

The idea is that sigma-extraction can be repeatedly applied.

In its simplest form, it may be written thus:

$$\Sigma \{ \ldots v \ldots \mid \ldots \Phi \ldots \} \vdash \Sigma \{ \ldots \Sigma v \Phi \ldots \mid \ldots \exists v \Phi \ldots \}$$

8. **Polyadic Sigma-Extraction**

The same as ordinary sigma-extraction, except $v$ is replaced by $v$, which is a sequence of variables.

9. **Iota-Extraction**

Iota-extraction is a special case of sigma-extraction involving the uniqueness-operator. In its simplest form, it is written thus.

$$\Sigma \{ \ldots v \ldots \mid \ldots \Phi ! v \ldots \} \vdash \Sigma \{ \ldots \nu \Phi \ldots \mid \ldots \exists ! v \Phi \ldots \}$$
10. Sigma-Stigma

A theta-marker cannot attach to a sigma-extracted phrase.

Sigma-extraction cannot be applied to a variable that derives from a nominative-marked variable.

11. Sequence (Parallel) Composition

Sequences of the same length combine component-wise; for example,

\[(F \otimes G)(\alpha \otimes \beta) = F(\alpha) \otimes G(\beta)\]

where \(F\) is a function and \(\alpha\) is an argument in its domain, and \(G\) is a function and \(\beta\) is an argument in its domain.

12. Plural Descriptions

\[\nu P \nu \vdash \Sigma \nu P \nu\]

\(\nu\) ranges over plural-entities

13. \(\times\)-Simplification

\[\Phi \times \Psi \vdash \Phi \& \Psi\]

14. Distribution of \(\times\) over \(\Sigma\)

\[\Sigma\{ \alpha | \Phi \} \times \beta \vdash \Sigma\{ \alpha \times \beta | \Phi \}\]

15. Association of \(\Sigma\)

\[\Sigma\{ \Sigma\{ \alpha | \Psi \} | \Phi \} \vdash \Sigma\{ \alpha | \exists \nu (\Phi \& \Psi) \}\]

\(\nu\) are all the variables C-free in \(\Phi \& \Psi\) but not C-free in \(\alpha\)

16. State-Extraction (derived rule)

\[\Sigma\{ \Sigma\{ \Omega | \Psi \} | \Phi \} \vdash \Sigma\{ \nu | \Phi \& \Psi \& \Omega \}\]

\(\Phi, \Psi, \Omega\) are formulas;

\(\nu\) is a sequence of all variables C-free in \(\Phi \& \Psi \& \Omega\).

17. Conditional Assertion

We introduce a two-place connective "slash" as follows.

\[\text{if (given)} \quad S \rightarrow S \rightarrow \lambda P \lambda Q [Q/P]\]

The logic of slash follows Belnap.\(^{38}\) To wit:

\(A/B = A\) if \(B\) is true;

\(A/B = \emptyset\) if \(B\) is false.

In other words, \(A/B\) says \(A\), if \(A\) is true, but says nothing whatsoever if \(A\) is false.

\(^{38}\) Note that Belnap reads slash backwards from us. We follow conditional-probability notation.
18. Biconditional Assertion

The corresponding biconditional-assertion involves "double-slash", defined as follows.

\[ \text{iff (given)} \quad S \rightarrow S \rightarrow S \rightarrow \lambda P \lambda Q [Q \| P] \]

The logic of double-slash goes as follows.

\[ A \| B = A \& B \quad \text{if A and B are both true;} \]
\[ A \| B = \Box \quad \text{otherwise.} \]

In other words, \( A \| B \) says A\&B, if A and B both true, but says nothing whatsoever otherwise.

19. Conditionalization

\[ \frac{\{ \Omega / \Psi | \Phi \}}{\{ \Omega | \Phi & \Psi \}} \]

\( \Psi \) is any junction; \( \Phi, \Psi, \Omega \) are formulas.

20. Bi-Conditionalization

\[ \frac{\{ \Omega \| \Psi | \Phi \}}{\{ \Omega | \Phi & \Psi \} \& \{ \Psi | \Phi & \Omega \}} \]

\( \Psi \) is any junction; \( \Phi, \Psi, \Omega \) are formulas.

21. The Former and The Latter

| the-former | \( D \otimes D \rightarrow D \) | \( x \otimes y \rightarrow x \) |
| the-latter | \( D \otimes D \rightarrow D \) | \( x \otimes y \rightarrow y \) |

22. The Many Locations of ‘!’

| \( P!\alpha \) | =_o | \( P \alpha & \sim \exists v \{ v \perp \alpha & P v \} \) | \( v \) not free in \( P \) or \( \alpha \) | \( \alpha \) is the only \( P \) |
| \( [\Phi!v] \) | =_o | \( [\lambda v \Phi]v \) | \( v \) is the only \( v \) such that \( \Phi \) |
| \( \exists!v\Phi \) | =_o | \( \exists v [\Phi!v] \) | there is exactly one \( v \) such that \( \Phi \) |
| \( R!\alpha\beta \) | =_o | \( [\lambda v R \beta]!\alpha \) | \( v \) not free in \( \beta \) | only \( \alpha \) \( R \)'s \( \beta \) |
| \( R\alpha!\beta \) | =_o | \( [\lambda v R \alpha \beta]!\beta \) | \( v \) not free in \( \alpha \) | \( \alpha \) \( R \)'s only \( \beta \) |

Here,

\( P \) is a one-place predicate; \( R \) is a two-place predicate;
\( \alpha \) and \( \beta \) are entity-terms; \( v \) is an entity-variable;
\( \Phi \) is a formula.

23. The Unity of Pronouns

We have seen many kinds of pronouns, but they all have the same overall semantic form.

| wh- restrictive | \( D_0 \rightarrow D \) | \( \lambda x_0 : x \) | narrow scope |
| wh- non-restrictive | \( D_0 \rightarrow D \) | \( \lambda x_1 : x \) | wide scope |
| wh- interrogative | \( D_2 \rightarrow D \) | \( \lambda x_2 : x \) |
| \( \alpha \)-e anaphoric | \( D_\alpha \rightarrow D \) | \( \lambda x_\alpha : x \) |

In other words, all these kinds of pronouns are allomorphs of each other; they differ from one another only with regard to inflectional-markers.